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SPECTRUM OF THE GENERALIZED ZERO-DIVISOR GRAPHS

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Abstract: The generalized zero-divisor graph of a ring R, denoted by $\Gamma'(R)$, is a simple (undirected) graph with a vertex set consisting of all nonzero zero-divisors in R, and two distinct vertices x and y are adjacent if $x^n y = 0$ or $y^n x = 0$, for some positive integer n. If $R = \prod_{i=1}^{k} R_i$ is a direct product of finite commutative local rings R_i with $|R_i| = p_i^{\alpha_i}$, then we express $\Gamma'(R)$ as a H-generalized join of a family $\mathcal{F}$ of a complete graph and null graphs, where H is a graph obtained from $\Gamma'(S^k)$ by contraction of edges of all nonzero nilpotents at a single vertex $\mathbf{0}$, and $S = \{0, 1, 2\}$ is a multiplicative submonoid of a ring $\mathbb{Z}_4$. Also, we prove that the adjacency spectrum of $\Gamma'(R)$ is $\left\{(-1)^{(\beta-1)}, 0^{(\gamma-3^k+2^k+1)}\right\} \cup \sigma(NA(H))$, where β is the number of nonzero nilpotent elements, γ is the number of non-nilpotent zero-divisors in R and N is a diagonal matrix whose rows (columns) are indexed with vertices $e \in \Gamma'(H)$ with e^{th} diagonal entry is the cardinality of e^{th} graph in the family $\mathcal{F}$.

Keywords and Phrases: Eigenvalue, generalized zero-divisor graph, complete graph, regular graph, adjacency matrix.

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