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CONVERGENCE AND STABILITY OF A NEW THREE-STEP ITERATIVE TECHNIQUE IN CONVEX METRIC SPACE

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Abstract: In order to approximate the fixed points of a contractive mapping, a new three-step iteration approach is proposed in this study. Additionally, the suggested scheme's stability and convergence are taken into account, and its efficacy for various problems is explored. The novel technique outperforms all the well-known three-step strategies that are currently accessible in the literature, according to several experiments that are described.

Keywords and Phrases: Contraction mapping, Stability, Fixed point, Convex metric, Convergence.

2020 Mathematics Subject Classification: 47H09, 47H10, 54H25.

1. Introduction

Fixed point (FP) problems may be used to create a wide range of mathematical issues, including systems of nonlinear equations, integral and differential equations, systems of linear equations, variational analysis problems and optimization theory problems. The reader is directed to ([5], [4], [3], [18]) for the latest recent work on FP issues and related problems. The approximate solution to these issues and the conditions under which they can be solved are investigated using FP theory. The FP of any given nonlinear function cannot be found using any universal closed-form formula. For this reason, FP iterative algorithms offer a sophisticated and effective method of computing them.

Numerous classical iteration techniques are used on metric spaces and normed linear spaces with a convexity structure in the FP theory literature.
Let $(H, \mathfrak{h})$ be a metric space and $W : H \times H \times [0,1] \to H$ be a mapping. If

$$\mathfrak{h}(\mathfrak{u}_1, W(\mathfrak{u}_2, \mathfrak{u}_3, \mu)) \leq \mu\mathfrak{h}(\mathfrak{u}_1, \mathfrak{u}_2) + (1-\mu)\mathfrak{h}(\mathfrak{u}_1, \mathfrak{u}_3),\ \forall\, \mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3 \in H \textit{ and } \mu \in [0,1], \tag{1.1}$$

then $(H, \mathfrak{h}, W)$ is called a convex metric space (CMS) [17].
The iteration technique that carries Picard's name, was established in 1890. A Picard sequence [13] of the beginning point $\{\mathfrak{u}_0\}$ is defined as

$$\mathfrak{u}_{n+1} = R\mathfrak{u}_n, \tag{1.2}$$

where $R : H \to H$ is a mapping.
Mann introduced the concept of the Mann iteration [12] in 1953. Mann iteration can be expressed as

$$\mathfrak{u}_{n+1} = W(R\mathfrak{u}_n, \mathfrak{u}_n, \mathfrak{a}_n), \tag{1.3}$$

where $\{\mathfrak{a}_n\}$ is a sequence in [0,1].
Ishikawa [11] proposed an extension of the Mann iteration, as described by

$$\begin{cases} \mathfrak{v}_n = W(R\mathfrak{u}_n, \mathfrak{u}_n, \mathfrak{b}_n), \\ \mathfrak{u}_{n+1} = W(R\mathfrak{v}_n, \mathfrak{u}_n, \mathfrak{a}_n) \end{cases} \tag{1.4}$$

where $\{\mathfrak{a}_n\}$ and $\{\mathfrak{b}_n\}$ are real sequences in [0, 1].
Multiple iteration strategies have been created over the years by different researchers to approximate fixed points of various non-linear operators in appropriate normed spaces and metric spaces ([2], [8], [9], [14], [16]).
In 2018, a novel three-step iteration approach was introduced by Abbas et al. [1]. It is as follows:

$$\begin{cases} \mathfrak{w}_n = R\mathfrak{u}_n, \\ \mathfrak{v}_n = R\mathfrak{w}_n, \\ \mathfrak{u}_{n+1} = \mathfrak{a}_n R\mathfrak{v}_n + (1-\mathfrak{a}_n)\mathfrak{v}_n \end{cases} \tag{1.5}$$

where $\{\mathfrak{a}_n\}$ is a sequence in (0,1).
The three-step iteration process that Ali and Ali [7] developed in 2020 is characterized by the sequence $\{\mathfrak{a}_n\}$, $\mathfrak{a}_n \in (0,1)$ and

$$\begin{cases} \mathfrak{w}_n = R(\mathfrak{a}_n R\mathfrak{u}_n + (1-\mathfrak{a}_n)\mathfrak{u}_n, \mathfrak{a}_n), \\ \mathfrak{v}_n = R\mathfrak{w}_n, \\ \mathfrak{u}_{n+1} = R\mathfrak{v}_n. \end{cases} \tag{1.6}$$

Sharma et al. [15] presented an innovative three-step iteration strategy in 2023. It looks like this:

$$\begin{cases} \mathfrak{w}_n = \dfrac{k}{k+1}\mathfrak{u}_n + \dfrac{1}{k+1}R\mathfrak{u}_n, \\ \mathfrak{v}_n = R\mathfrak{w}_n, \\ \mathfrak{u}_{n+1} = R\mathfrak{v}_n, \end{cases} \tag{1.7}$$

where $k \geq 0$ is a real number.
In context of CMS, (1.5), (1.6) and (1.7) respectively can be written as:

$$\begin{cases} \mathfrak{w}_n = R\mathfrak{u}_n, \\ \mathfrak{v}_n = R\mathfrak{w}_n, \\ \mathfrak{u}_{n+1} = W(R\mathfrak{v}_n, \mathfrak{v}_n, \mathfrak{a}_n). \end{cases} \tag{1.8}$$

$$\begin{cases} \mathfrak{w}_n = R(W(R\mathfrak{u}_n, \mathfrak{u}_n, \mathfrak{a}_n)), \\ \mathfrak{v}_n = R\mathfrak{w}_n, \\ \mathfrak{u}_{n+1} = R\mathfrak{v}_n. \end{cases} \tag{1.9}$$

$$\begin{cases} \mathfrak{w}_n = W\left(\mathfrak{u}_n, R\mathfrak{u}_n, \dfrac{k}{k+1}\right), \\ \mathfrak{v}_n = R\mathfrak{w}_n, \\ \mathfrak{u}_{n+1} = R\mathfrak{v}_n. \end{cases} \tag{1.10}$$

2. Preliminaries

The terminologies and research covered in this part will be used to support our main conclusions.

Definition 2.1. [6] *Let $H : R \to R$ be an operator and $(R, \mathfrak{h})$ be a metric space. Then H is said to be ς contraction if there exists $\varsigma \in [0, 1)$, such that :*

$$\mathfrak{h}(H\mathfrak{u}_1, H\mathfrak{u}_2) \leq \varsigma\mathfrak{h}(\mathfrak{u}_1, \mathfrak{u}_2), \text{ for all } \mathfrak{u}_1, \mathfrak{u}_2 \in R. \tag{2.1}$$

Definition 2.2. [6] *Let $\{s_n\}$ and $\{t_n\}$ be two sequences of positive numbers that converge to $\mathfrak{s}$ and $\mathfrak{t}$, respectively. Assume that there exists the following limit:*

$$\lim_{n\to\infty} \frac{|\mathfrak{s}_n - \mathfrak{s}|}{|\mathfrak{t}_n - \mathfrak{t}|} = \mathfrak{b}.$$

(i) If $\mathfrak{b} = 0$, then it is said that $\{\mathfrak{s}_n\}$ converges faster to $\mathfrak{s}$ than $\{\mathfrak{t}_n\}$ to $\mathfrak{t}$.
(ii) If $0 < \mathfrak{b} < 1$, then it is said that $\{\mathfrak{s}_n\}$ and $\{\mathfrak{t}_n\}$ have the same rate of convergence.

Definition 2.3. [6] *Assume that we have two iteration sequences,* $\{\mathfrak{d}_n\}$ *and* $\{\mathfrak{e}_n\}$, *both converging to a FP* $\mathfrak{c}$.
Let $\{\mathfrak{s}_n\}$ *and* $\{\mathfrak{t}_n\}$ *be two sequences of positive numbers, such that:*

$$\mathfrak{h}(\mathfrak{d}_n, \mathfrak{c}) \leq \mathfrak{s}_n, \text{ for all } n \in \mathbb{N},$$

$$\mathfrak{h}(\mathfrak{e}_n, \mathfrak{c}) \leq \mathfrak{t}_n, \text{ for all } n \in \mathbb{N},$$

where $\{\mathfrak{s}_n\}$ *and* $\{\mathfrak{t}_n\}$ *converge to 0. If* $\{\mathfrak{s}_n\}$ *converges faster than* $\{\mathfrak{t}_n\}$ *in the sense of Definition 2.2, then* $\{\mathfrak{d}_n\}$ *is convergent faster than* $\{\mathfrak{e}_n\}$ *to* $\mathfrak{c}$.

Definition 2.4. [10] *Let* $\mathfrak{c}$ *be a FP of* $R : H \to H$. *Let* $\{\mathfrak{u}_n\}$ *be a sequence defined as* $\mathfrak{u}_{n+1} = \mu(R, \mathfrak{u}_n)$, *where* $n = 0, 1, 2...$, *converge to* $\mathfrak{c}$ *for each* $\mathfrak{u}_0 \in H$. *Consider a random sequence* $\{\mathfrak{v}_n\}$, *and define* $\epsilon_n = \mathfrak{h}(\mathfrak{u}_n, \mathfrak{v}_n)$. *Then, the* $\{\mathfrak{u}_n\}$ *is referred as T-stable if and only if* $\lim_{n\to\infty} \epsilon_n = 0 \Rightarrow \lim_{n\to\infty} \mathfrak{v}_n = \mathfrak{c}$.
The literature contains a relatively limited quantity of effective three-step FP methods. This fact serve as our inspiration for creating a new, three-step scheme that outperforms the majority of FP methods that are already in use.

3. Main Results

We introduce a new three-step iterative approach in this portion, which is described as follows:
Let $(R, \mathfrak{h}, W)$ be a CMS and $R : H \to H$ be a self-mapping.

$$\begin{cases} \mathfrak{w}_n = R\Big(W\Big(\mathfrak{u}_n, R\mathfrak{u}_n, \dfrac{k}{k+1}\Big)\Big), \\ \mathfrak{v}_n = R\mathfrak{w}_n, \\ \mathfrak{u}_{n+1} = R\mathfrak{v}_n, \end{cases} \tag{3.1}$$

where $k \geq 0$ is a real number.

3.1. Convergence

In the present subsection, we will demonstrate the convergence of (3.1).

Theorem 3.1. *Let* M *be a closed convex subset of a complete CMS* $(H, \mathfrak{h}, W)$, *and let* $R : M \to M$ *be a mapping satisfying (2.1) with FP* $\mathfrak{c}$. *Let* $\{\mathfrak{u}_n\}$ *be defined by (3.1) and* $\mathfrak{u}_0 \in M$. *Then* $\{\mathfrak{u}_n\}$ *converges to the unique FP of* R.
Proof. We will prove that $\lim_{n\to\infty} \mathfrak{h}(\mathfrak{u}_n, \mathfrak{c}) = 0$.

$$\mathfrak{h}(\mathfrak{w}_n, \mathfrak{c}) = \mathfrak{h}\Big(R\Big(W\Big(\mathfrak{u}_n, R\mathfrak{u}_n, \frac{k}{k+1}\Big)\Big), \mathfrak{c}\Big)$$

$$
\begin{aligned}
&\le \varsigma\mathfrak{h}\left(\left(W\left(\mathfrak{u}_n, R\mathfrak{u}_n, \frac{k}{k+1}\right)\right), \mathfrak{c}\right)\\
&\le \varsigma\left(\frac{k}{k+1}\mathfrak{h}(\mathfrak{u}_n, \mathfrak{c}) + \left(1-\frac{k}{k+1}\right)\mathfrak{h}(R\mathfrak{u}_n, \mathfrak{c})\right)\\
&\le \varsigma\left(\frac{k}{k+1}\mathfrak{h}(\mathfrak{u}_n, \mathfrak{c}) + \left(\frac{\varsigma}{k+1}\right)\mathfrak{h}(\mathfrak{u}_n, \mathfrak{c})\right), \qquad (3.2)
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{h}(\mathfrak{v}_n, \mathfrak{c}) &= \mathfrak{h}(R\mathfrak{w}_n, \mathfrak{c})\\
&\le \varsigma\mathfrak{h}(\mathfrak{w}_n, \mathfrak{c}), \qquad (3.3)\\
\mathfrak{h}(\mathfrak{u}_{n+1}, \mathfrak{c}) &= \mathfrak{h}(R\mathfrak{v}_n, \mathfrak{c})\\
&\le \varsigma\mathfrak{h}(\mathfrak{v}_n, \mathfrak{c}). \qquad (3.4)
\end{aligned}
$$

Using (3.2), (3.3) and (3.4), we get

$$
\begin{aligned}
\mathfrak{h}(\mathfrak{u}_{n+1}, \mathfrak{c}) &\le (\varsigma)^3\left(\frac{k}{k+1} + \frac{\varsigma}{k+1}\right)\mathfrak{h}(\mathfrak{u}_n, \mathfrak{c}),\\
\mathfrak{h}(\mathfrak{u}_n, \mathfrak{c}) &\le (\varsigma)^{3(n+1)}\left(\frac{k}{k+1} + \frac{\varsigma}{k+1}\right)^{n+1}\mathfrak{h}(\mathfrak{u}_0, \mathfrak{c}). \qquad (3.5)
\end{aligned}
$$

Hence, $\lim_{n\to\infty} \mathfrak{h}(\mathfrak{u}_n, \mathfrak{c}) = 0$.

3.2. Stability

The stability of (3.1) will be shown in this subsection.

Theorem 3.2. *Let M be a closed convex subset of a complete CMS $(H, \mathfrak{h}, W)$, and let R be a ς operator with $F(R) \ne \phi$. Assume that $\mathfrak{u}_0 \in M$, and the sequence $\{\mathfrak{u}_n\}$ is defined by (3.1). Then, the iteration (3.1) is T stable.*

Proof. Assume $\{\mathfrak{p}_n\}_{n=0}^{\infty} \subset M$ is an arbitrary sequence, $\epsilon_n = \mathfrak{h}(\mathfrak{p}_n, R\mathfrak{q}_{n-1})$, where $\mathfrak{q}_{n-1} = R\mathfrak{r}_{n-1}$, $\mathfrak{r}_{n-1} = R\left(W\left(\mathfrak{p}_{n-1}, R\mathfrak{p}_{n-1}, \frac{k}{k+1}\right)\right)$ and let $\lim_{n\to\infty} \epsilon_n = 0$.

Now,

$$
\begin{aligned}
\mathfrak{h}(\mathfrak{p}_n, \mathfrak{c}) &\le \mathfrak{h}(\mathfrak{p}_n, R\mathfrak{q}_{n-1}) + \mathfrak{h}(R\mathfrak{q}_{n-1}, \mathfrak{c})\\
&\le \epsilon_n + \varsigma\mathfrak{h}(\mathfrak{q}_{n-1}, \mathfrak{c}), \qquad (3.6)
\end{aligned}
$$

$$
\begin{aligned}
\mathfrak{h}(\mathfrak{q}_{n-1}, \mathfrak{c}) &\le \mathfrak{h}(R\mathfrak{r}_{n-1}, \mathfrak{c})\\
&\le \varsigma\mathfrak{h}(\mathfrak{r}_{n-1}, \mathfrak{c}), \qquad (3.7)
\end{aligned}
$$

$$\begin{aligned}\mathfrak{h}(\mathfrak{r}_{n-1},\mathfrak{c}) &\le \mathfrak{h}\left(R\left(W\left(\mathfrak{p}_{n-1},R\mathfrak{p}_{n-1},\frac{k}{k+1}\right)\right),\mathfrak{c}\right)\\ &\le \varsigma\mathfrak{h}\left(\left(W\left(\mathfrak{p}_{n-1},R\mathfrak{p}_{n-1},\frac{k}{k+1}\right)\right),\mathfrak{c}\right)\\ &\le \varsigma\left(\frac{k}{k+1}\mathfrak{h}(\mathfrak{p}_{n-1},c)+\left(1-\frac{k}{k+1}\right)\varsigma\mathfrak{h}(\mathfrak{p}_{n-1},c)\right)\\ &= \varsigma\left(\frac{k}{k+1}\mathfrak{h}(\mathfrak{p}_{n-1},c)+\left(\frac{\varsigma}{k+1}\right)\mathfrak{h}(\mathfrak{p}_{n-1},c)\right).\end{aligned} \tag{3.8}$$

By (3.6), (3.7) and (3.8), we get

$$\begin{aligned}\mathfrak{h}(\mathfrak{p}_n,\mathfrak{c}) &\le \epsilon_n+(\varsigma)^2\mathfrak{h}(\mathfrak{r}_{n-1},\mathfrak{c})\\ &\le \epsilon_n+(\varsigma)^3.\left(\frac{k+\varsigma}{k+1}\right)\mathfrak{h}(\mathfrak{p}_{n-1},\mathfrak{c}).\end{aligned} \tag{3.9}$$

Thus, $\lim_{n\to\infty}\mathfrak{h}(\mathfrak{p}_n,\mathfrak{c})=0$ by [[8], Lemma 4].
Conversely, assume $\lim_{n\to\infty}\mathfrak{h}(\mathfrak{p}_n,\mathfrak{c})=0$.

$$\begin{aligned}\epsilon_n &= \mathfrak{h}(\mathfrak{p}_n,R\mathfrak{q}_{n-1})\\ &\le \mathfrak{h}(\mathfrak{p}_n,\mathfrak{c})+\mathfrak{h}(\mathfrak{c},R\mathfrak{q}_{n-1})\\ &\le \mathfrak{h}(\mathfrak{p}_n,\mathfrak{c})+\varsigma\mathfrak{h}(\mathfrak{q}_{n-1},\mathfrak{c})\\ &\le \mathfrak{h}(\mathfrak{p}_n,\mathfrak{c})+(\varsigma)^2\mathfrak{h}(\mathfrak{r}_{n-1},\mathfrak{c})\\ &\le \mathfrak{h}(\mathfrak{p}_n,\mathfrak{c})+(\varsigma)^3.\left(\frac{k+\varsigma}{k+1}\right)\mathfrak{h}(\mathfrak{p}_{n-1},\mathfrak{c}).\end{aligned}$$

Hence, $\lim_{n\to\infty}\epsilon_n=0$.

3.3. Comparison with various iterative scheme

In the present subsection, we will compare the rate of convergence of various iterative schemes with our scheme (3.1).

Theorem 3.3. *Let M be a nonempty, closed, and convex subset of a complete CMS $(H,\mathfrak{h},W)$. Let $\{\mathfrak{s}_n\}$ and $\{\mathfrak{u}_n\}$ be the sequences defined by iteration (1.6) and (3.1) respectively with the same initial guess $\{\mathfrak{s}_0\}$ and R be a ς-contraction. Additionally, suppose that $0<\mathfrak{a}\le\mathfrak{a}_n<1$, and $k>0$ is a real number such that $\mathfrak{a}(k+1)<1$. Then, iteration (3.1) converges faster than (1.6).*

Proof. By (3.5) in Theorem 3.1, the following inequality can be obtained:

$$\mathfrak{h}(\mathfrak{u}_n,\mathfrak{c})\le(\varsigma)^{3(n+1)}\left(\frac{k}{k+1}+\frac{\varsigma}{k+1}\right)^{n+1}\mathfrak{h}(\mathfrak{u}_0,\mathfrak{c}). \tag{3.10}$$

It is simple to see that the iteration process (1.6) takes the following form by using similar reasoning as in Theorem 3.2

$$\mathfrak{h}(\mathfrak{s}_n, \mathfrak{c}) \leq (\varsigma)^{3(n+1)}(1-(1-\varsigma)\mathfrak{a}_n)\mathfrak{h}(\mathfrak{s}_0, \mathfrak{c}). \tag{3.11}$$

As, $\mathfrak{a} \leq \mathfrak{a}_n < 1$

$$(1-(1-\varsigma)\mathfrak{a}_n) \leq (1-(1-\varsigma)\mathfrak{a}),$$
$$\prod_{i=0}^{n}(1-(1-\varsigma)\mathfrak{a}_n) \leq (1-(1-\varsigma)\mathfrak{a})^{n+1}.$$

Using (3.11),we have

$$\mathfrak{h}(\mathfrak{s}_n, \mathfrak{c}) \leq (\varsigma)^{3(n+1)}(1-(1-\varsigma)\mathfrak{a})^{n+1}\mathfrak{h}(\mathfrak{s}_0, \mathfrak{c}). \tag{3.12}$$

Assume

$$\mathfrak{C}_n = (\varsigma)^{3(n+1)}\left(\frac{k}{k+1}+\frac{\varsigma}{k+1}\right)^{n+1}\mathfrak{h}(\mathfrak{u}_0, \mathfrak{c}),$$
$$\mathfrak{D}_n = (\varsigma)^{3(n+1)}(1-(1-\varsigma)\mathfrak{a})^{n+1}\mathfrak{h}(\mathfrak{s}_0, \mathfrak{c})$$

and

$$\Upsilon_n = \frac{\mathfrak{C}_n}{\mathfrak{D}_n} = \frac{(\varsigma)^{3(n+1)}\left(\frac{k}{k+1}+\frac{\varsigma}{k+1}\right)^{n+1}\mathfrak{h}(\mathfrak{u}_0, \mathfrak{c})}{(\varsigma)^{3(n+1)}(1-(1-\varsigma)\mathfrak{a})^{n+1}\mathfrak{h}(\mathfrak{s}_0, \mathfrak{c})}. \tag{3.13}$$

Since, $\mathfrak{u}_0 = \mathfrak{s}_0$

$$\Upsilon_n = \frac{\mathfrak{C}_n}{\mathfrak{D}_n} = \frac{(\varsigma)^{3(n+1)}\left(\frac{k}{k+1}+\frac{\varsigma}{k+1}\right)^{n+1}}{(\varsigma)^{3(n+1)}(1-(1-\varsigma)\mathfrak{a})^{n+1}}. \tag{3.14}$$

By using $a(k+1) < 1$, we get

$$(1-(1-\varsigma)\mathfrak{a}) > \left(\frac{k}{k+1}+\frac{\varsigma}{k+1}\right) \Rightarrow \frac{\left(\frac{k}{k+1}+\frac{\varsigma}{k+1}\right)}{(1-(1-\varsigma)\mathfrak{a})} < 1. \tag{3.15}$$

Hence, $\lim_{n\to\infty} \Upsilon_n = 0$. It can be concluded that $\{\mathfrak{u}_n\}$ converges faster than $\{\mathfrak{s}_n\}$ by applying Definitions 2.2 and 2.3.
By following the same procedure, it can be demonstrated that (3.1) converges more quickly than (1.5) and (1.7).

Example 3.1. Let $H : [0, 1] \to [0, 1]$ be a mapping defined by $H\mathfrak{u} = \frac{1}{3}e^{\mathfrak{u}}$. Suppose $\mathfrak{a}_n = \dfrac{1}{n+6}$ and $k = 0.5$, with $\mathfrak{u}_0 = 0$. Iteration (3.1) requires 20 number of iterations to converge to FP $\mathfrak{c} = 0.619061286735945$, while (1.5), (1.6), and (1.7) need 35, 24, and 28 number of iterations, respectively.
The following table compares the rate of convergence of different iterations with the same initial guess.

n	$\{\mathfrak{u}_n\}$			
	Iteration (1.5)	Iteration (1.6)	Iteration (1.7)	Iteration (3.1)
1	0.474572084240672	0.534842211762562	0.505438286356170	0.552570637926451
2	0.572039708402158	0.601513769685729	0.589545513659980	0.608044973133556
3	0.602450809915476	0.615142918377830	0.610836522400626	0.617133195225655
4	0.613020688199426	0.618170552952124	0.616726656911940	0.618720694366165
5	0.616837509890812	0.618857461812156	0.618395161418170	0.619001024242677
6	0.618237609459136	0.619014474448451	0.618870946249506	0.619050621171085
7	0.618755079680769	0.619050506599855	0.619006875536929	0.619059398993802
8	0.618947158574880	0.619058798797368	0.619045730755306	0.619060952613706
9	0.619018663363014	0.619060711471222	0.619056839180713	0.619061227597652
10	0.619045341353993	0.619061153505531	0.619060015138771	0.619061276268702
11	0.619055312888449	0.619061255835707	0.619060923173536	0.619061284883284
12	0.619059045799359	0.619061279560101	0.619061182789716	0.619061286408031
13	0.619060445146567	0.619061285067630	0.619061257016643	0.619061286677906
14	0.619060970351249	0.619061286347681	0.619061278238889	0.619061286725672
15	0.619061167685166	0.619061286645502	0.619061284306549	0.619061286734127
16	0.619061241901309	0.619061286714859	0.619061286041355	0.619061286735623
17	0.619061269838243	0.619061286731025	0.619061286537355	0.619061286735888
18	0.619061280362910	0.619061286734796	0.619061286679166	0.619061286735935
19	0.619061284330779	0.619061286735677	0.619061286719711	0.619061286735943
20	0.619061285827702	0.619061286735882	0.619061286731304	0.619061286735945

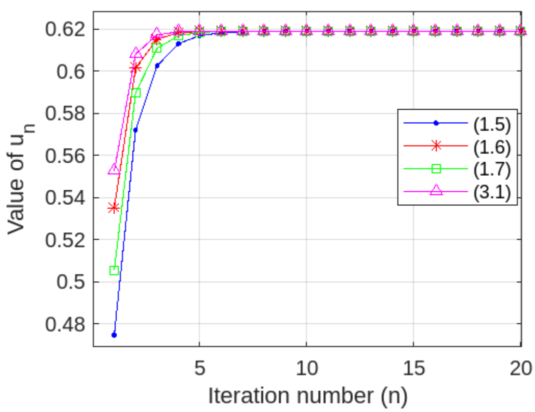

Figure 1: Comparison of rate of convergence of Various Iterative Techniques

4. Conclusion

This study presents an iterative method for solving fixed point problems numerically. In order to compare the performance of the suggested approach with that of the current methods, a nonlinear issue example is taken into consideration. The results of the studies show that this approach consistently outperforms other well-known techniques, leading to a faster convergence speed and improved efficiency in locating fixed points.

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