

## ON MILLI CONTINUOUS FUNCTION AND ITS APPLICATIONS

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**Abstract:** This work focuses on rough approximation operators generated from milli topological spaces and examines their features. For this purpose, we show how milli continuous functions are constructed. Further, exploiting the concepts of milli contra continuous, milli perfectly continuous, milli strongly continuous functions and milli basis in milli topological spaces, a real-life application is presented. The essential food habits for a healthy lifestyle were identified. Here the concepts of

the topological reduction of attributes in a complete information system is used to get the key factors. A general algorithm for identifying the key factors is provided.

**Keywords and Phrases:** Milli topology, milli continuous, milli contra continuous, milli perfectly continuous, milli strongly continuous.

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## 1. Introduction

Rough set theory is a novel mathematical approach originated by Pawlak [3, 4] to manage inexplicit and uncertain data that cannot be addressed by the classical set theory. In recent years, rough set theory and its extended models have raised more and more scholars attention in various fields; especially, those who work in computer science and artificial intelligence. Also, an effective and robust tool to scope with imperfect knowledge problems. In many real-life issues that humans deal with computer networks, economics, medical sciences and engineering.

Lellis Thivagar et al. [5, 6], introduced nano continuous functions and derived their characterizations in terms of nano closed sets, nano closure and nano interior. There is also an attempt to define nano-open maps, nano closed maps and nano homeomorphism. Lellis Thivagar et al. [7, 9], introduced and studied the stronger form of nano continuity called nano contra continuity. Lellis Thivagar and Richard [8], applied the basis of nano topology to find the key environmental factors for the yield of plants using topological reduction of attributes in complete information system.

The notion of milli topology was introduced by Ittanagi et al. [1, 2], which was defined in terms of approximations, boundary region, approximations of edges and approximations of boundary regions of a subset of a universe using an equivalence relation on it. The milli topological space consists of maximum of nine elements. The authors also introduced some basic topological operators of the resultant milli topological spaces, such as milli open set, milli closed set, milli closure, milli interior and milli exterior. They also presented a new class of milli neighbourhood of a point, milli neighbourhood of a set and milli neighbourhood system in milli topological spaces. Moreover, they elucidated the relationship between them and investigated the conditions under which some of them are identical. Then create the notion of milli limit points, milli derived sets and milli dense sets. Also introduced new types of sets namely milli clopen sets, milli extremely disconnected sets and milli door space in milli topological spaces. They investigated some of their properties.

This paper is organized as follows. In Section 2, some preliminary concepts of rough set theory, milli topological space and nano topological space are recalled. In

Section 3, the notion of milli continuous and milli contra continuous functions are introduced and their properties are investigated. The milli continuous is compared with nano continuous function and also the milli contra continuous is compared with nano contra continuous functions. In Section 4, the notion of milli perfectly continuous and milli strongly continuous functions are introduced and their properties are investigated. These milli perfectly continuous and milli strongly continuous functions are compared with other milli continuous functions. In Section 5, we define real-life application that was produced by using the concepts of milli basis in milli topological spaces. In the future, this work will be extended with some real-life applications by using the corresponding set of milli open in milli topological spaces. The conclusion is given in Section 6.

## 2. Preliminaries

**Definition 2.1.** [4] *Let  $U$  be a non-empty set of objects called the universe and  $R$  be an equivalence relation on  $U$  named as the indiscernibility relation. The pair  $(U, R)$  is called the approximation space. Let  $X$  be a subset of  $U$ .*

- (i) *The lower approximation of  $X$  with respect to  $R$  is the set of all objects, which can be for certain classified as  $X$  with respect to  $R$  and it is denoted by  $\underline{R}(X)$  and defined as  $\underline{R}(X) = \{x \in U : [x]_R \subseteq X\}$ , where  $[x]_R$  denotes the equivalence class determined by  $x$ .*
- (ii) *The upper approximation of  $X$  with respect to  $R$  is the set of all objects, which can be possibly classified as  $X$  with respect to  $R$  and it is denoted by  $\overline{R}(X)$  and defined as  $\overline{R}(X) = \{x \in U : [x]_R \cap X \neq \phi\}$ .*

**Definition 2.2.** [3] *Let  $(U, R)$  be an approximation space and  $R$  be an equivalence relation on  $U$ . Let  $X$  be a subset of  $U$ .*

- (i) *The boundary region of  $X$  with respect to  $R$  is the set of all objects, which can be classified neither as  $X$  nor as not- $X$  with respect to  $R$  and it is denoted by  $BN(X)$  or  $B(X)$ . It is defined as  $B(X) = \overline{R}(X) - \underline{R}(X)$ .*
- (ii) *The set  $X$  is said to be rough with respect to  $R$  if  $\overline{R}(X) \neq \underline{R}(X)$ , that is, if  $B(X) \neq \phi$ .*

**Definition 2.3.** [3] *Let  $(U, R)$  be an approximation space and  $R$  be an equivalence relation on  $U$ . Let  $X$  be a subset of  $U$ .*

- (i) *The internal edge (lower edge) of  $X$  with respect to  $R$  and it is denoted by  $\underline{E}(X)$  and defined as  $\underline{E}(X) = X - \underline{R}(X)$*

- (ii) The external edge (upper edge) of  $X$  with respect to  $R$  and it is denoted by  $\overline{E}(X)$  and defined as  $\overline{E}(X) = \overline{R}(X) - X$ .

**Definition 2.4.** [1] Let  $U$  be a non-empty set of objects called the universe and  $R$  an equivalence relation on  $U$  named as the indiscernibility relation. The pair  $(U, R)$  is called the approximation space. Let  $X$  be a subset of  $U$ .

- (i) The lower boundary region of  $X$  with respect to  $R$  is denoted by  $\underline{B}(X)$  and is defined as  $\underline{B}(X) = \overline{R}(X) - \underline{E}(X)$ .
- (ii) The upper boundary region of  $X$  with respect to  $R$  is denoted by  $\overline{B}(X)$  and is defined as  $\overline{B}(X) = \overline{R}(X) - \overline{E}(X)$ .

**Definition 2.5.** [1] Let  $U$  be a universe of objects and  $R$  an equivalence relation on  $U$  and  $\tau_M(X) = \{U, \phi, \underline{R}(X), \overline{R}(X), B(X), \underline{E}(X), \overline{E}(X), \underline{B}(X), \overline{B}(X)\}$ , where  $X \subseteq U$  and  $\tau_M(X)$  satisfies the following axioms:

- (i)  $U$  and  $\phi \in \tau_M(X)$
- (ii) The union of the elements of any sub-collection of  $\tau_M(X)$  is in  $\tau_M(X)$ .
- (iii) The intersection of the elements of any finite sub-collection of  $\tau_M(X)$  is in  $\tau_M(X)$ .

$\tau_M(X)$  forms a topology on  $U$  called the milli topology on  $U$  with respect to  $X$ . We call  $(U, \tau_M(X))$  as the milli topological space. The elements of  $\tau_M(X)$  are called milli open sets and denoted as  $MO(X)$ .

**Definition 2.6.** [5] Let  $U$  be a non-empty, universe of objects and  $R$  an equivalence relation on  $U$ . Let  $X \subseteq U$  and  $\tau_R(X) = \{U, \phi, \underline{R}(X), \overline{R}(X), B(X)\}$ . Then  $\tau_R(X)$  is a topology on  $U$  called the nano topology with respect to  $X$ .

**Definition 2.7.** [6] Let  $(U, \tau_R(X))$  and  $(V, \mu_{R'}(Y))$  be nano topological spaces. Then a mapping  $f : (U, \tau_R(X)) \rightarrow (V, \mu_{R'}(Y))$  is nano continuous on  $U$  if the inverse image of every nano open set in  $V$  is nano open set in  $U$ .

**Definition 2.8.** [7] Let  $(U, \tau_R(X))$  and  $(V, \mu_{R'}(Y))$  be nano topological spaces. Then the mapping  $f : (U, \tau_R(X)) \rightarrow (V, \mu_{R'}(Y))$  is nano contra continuous, if the inverse image of every nano open set in  $V$  is nano closed set in  $U$ .

**Definition 2.9.** [7] A function  $f : (U, \tau_R(X)) \rightarrow (V, \mu_{R'}(Y))$  is said to be

- (i) nano perfectly continuous, if  $f^{-1}(B)$  nano clopen set in  $U$  for every nano open set  $B$  in  $V$ .

- (ii) nano strongly continuous, if  $f^{-1}(B)$  nano clopen set in  $U$  for every subset set  $B$  in  $V$ .

**Definition 2.10.** [8] Let  $(U, A)$  be an information system, where  $A$  is divided into a set  $C$  of condition attributes and a set  $D$  of decision attribute. A subset  $R$  of  $C$  is said to be a core, if  $\mathcal{B}_R = \mathcal{B}_C$  and  $\mathcal{B}_R \neq \mathcal{B}_{R-\{r\}}$  for all  $r \in R$ , where  $\mathcal{B}_R$  is the basis of nano topology corresponding to  $R \subseteq C$ . That is, a core is a minimal subset of attributes such that none of its elements can be removed without affecting the classification power of attributes.

### 3. Milli continuous and Milli contra continuous functions

In this section, the notion of milli continuous and milli contra continuous functions are introduced and their properties are investigated. The milli continuity is compared with nano continuous function and also the milli contra continuity is compared with nano continuous and milli continuous functions.

**Definition 3.1.** Let  $(U, \tau_M(X))$  and  $(V, \mu_M(Y))$  be milli topological spaces. A mapping  $f : (U, \tau_M(X)) \rightarrow (V, \mu_M(Y))$  is said to be

- (i) milli continuous function if  $f^{-1}(B)$  is milli open set in  $U$  for every milli open set  $B$  in  $V$ .
- (ii) milli contra continuous function if  $f^{-1}(B)$  is milli closed set in  $U$  for every milli open set  $B$  in  $V$ .

**Example 3.2.** Let  $U = \{a, b, c, d\}$ ,  $U/R = \{\{a, c\}, \{b\}, \{d\}\}$  and  $X = \{a, d\}$ . Then the milli topology on  $U$  with respect to  $X$  is  $\tau_M(X) = \{U, \phi, \{d\}, \{a, c, d\}, \{a, c\}, \{a\}, \{c\}, \{c, d\}, \{a, d\}\}$ . Let  $V = \{x, y, z, w\}$ ,  $V/R' = \{\{x\}, \{y, z\}, \{w\}\}$  and  $Y = \{x, z\}$ . Then the milli topology on  $V$  with respect to  $Y$  is  $\mu_M(Y) = \{V, \phi, \{x\}, \{x, y, z\}, \{y, z\}, \{z\}, \{y\}, \{x, y\}, \{x, z\}\}$ . Define  $f : U \rightarrow V$  as  $f(a) = z$ ,  $f(b) = w$ ,  $f(c) = x$ ,  $f(d) = y$ . Then  $f^{-1}(V) = U$ ,  $f^{-1}(\phi) = \phi$ ,  $f^{-1}(\{x\}) = \{c\}$ ,  $f^{-1}(\{x, y, z\}) = \{a, c, d\}$ ,  $f^{-1}(\{y, z\}) = \{a, d\}$ ,  $f^{-1}(\{z\}) = \{a\}$ ,  $f^{-1}(\{y\}) = \{d\}$ ,  $f^{-1}(\{x, y\}) = \{c, d\}$ ,  $f^{-1}(\{x, z\}) = \{a, c\}$ . That is, the inverse image of every milli open set in  $V$  is milli open set in  $U$ . Hence,  $f$  is milli continuous.

**Example 3.3.** Let  $U = \{a, b, c, d\}$ ,  $U/R = \{\{a\}, \{b, d\}, \{c\}\}$  and  $X = \{a, c\} \subset U$ . Then  $\tau_M(X) = \{U, \phi, \{a, c\}\}$ . The milli closed sets of  $U$  are:  $\phi, U, \{b, d\}$ . Let  $V = \{x, y, z, w\}$ ,  $V/R' = \{\{x\}, \{y, z\}, \{w\}\}$ . Let  $Y = \{y, z\} \subset V$ . Then  $\mu_M(Y) = \{V, \phi, \{y, z\}\}$ . Define  $f : U \rightarrow V$  as  $f(a) = x$ ,  $f(b) = y$ ,  $f(c) = w$ ,  $f(d) = z$ . Then  $f^{-1}(V) = U$ ,  $f^{-1}(\phi) = \phi$ ,  $f^{-1}(\{y, z\}) = \{b, d\}$ . Thus, the inverse image of every milli open set in  $V$  is milli closed set in  $U$ . Hence,  $f$  is milli contra continuous.

**Theorem 3.4.** *If  $f : (U, \tau_M(X)) \rightarrow (V, \mu_M(Y))$  is nano continuous then it is milli continuous, but the converse need not be true.*

**Proof.** Assume that  $f$  is nano continuous. Let  $B$  be any nano open set in  $V$ . Since every nano open set is milli open set, then  $B$  is milli open set in  $V$ . By the fact that  $f$  is nano continuous,  $f^{-1}(B)$  is nano open set in  $U$ . Since every nano open set is milli open set, so  $f^{-1}(B)$  is milli open set in  $U$ . Thus, the inverse image of every milli open set in  $V$  is milli open set in  $U$ . Hence  $f$  is milli continuous on  $U$ .

**Example 3.5.** From Example 3.2,  $f$  is milli continuous but not nano continuous. We know that the set  $\{y, z\}$  is nano open set in  $V$ , but  $f^{-1}(\{y, z\}) = \{a, d\}$  is not a nano open set in  $U$ .

**Theorem 3.6.** *If  $f : (U, \tau_M(X)) \rightarrow (V, \mu_M(Y))$  is nano contra continuous then it is milli contra continuous, but the converse need not be true.*

**Proof.** Assume that  $f$  is nano contra continuous. Let  $B$  be any nano open set in  $V$ . Since every nano open set is milli open set, it follows that  $B$  is milli open set in  $V$ . Now  $f$  is nano contra continuous, thus  $f^{-1}(B)$  is nano closed set in  $U$ . Since every nano-closed set is milli closed set, so  $f^{-1}(B)$  is milli closed set in  $U$ . Thus, the inverse image of every milli open set in  $V$  is milli closed set in  $U$ . Hence  $f$  is milli contra continuous on  $U$ .

**Example 3.7.** Let  $U = \{a, b, c, d\}$ ,  $U/R = \{\{a\}, \{b, d\}, \{c\}\}$  and  $X = \{b\} \subset U$ . Then  $\tau_M(X) = \{U, \phi, \{b, d\}, \{b\}, \{d\}\}$ . The milli closed sets of  $U$  are:  $\phi, U, \{a, c\}, \{a, c, d\}, \{a, b, c\}$ . Let  $V = \{x, y, z, w\}$ ,  $V/R' = \{\{x\}, \{y, z\}, \{w\}\}$ . Let  $Y = \{x, y, z\} \subset V$ . Then  $\mu_M(Y) = \{V, \phi, \{x, y, z\}\}$ . Define  $f : U \rightarrow V$  as  $f(a) = x$ ,  $f(b) = y$ ,  $f(c) = z$ ,  $f(d) = w$ . Then  $f^{-1}(V) = U$ ,  $f^{-1}(\phi) = \phi$ ,  $f^{-1}(\{x, y, z\}) = \{a, b, c\}$ . Observe that, the inverse image of every milli open set in  $V$  is milli closed set in  $U$ . Hence,  $f$  is milli contra continuous, but not nano contra continuous. Since  $\{x, y, z\}$  is nano-open set in  $V$ , then  $f^{-1}(\{x, y, z\}) = \{a, b, c\}$  is not a nano-closed set in  $U$ .

**Remark 3.8.** *The concept of milli continuity and milli contra continuity are independent of each other as shown in the following example.*

**Example 3.9.** From Example 3.2, the milli closed sets of  $U$  are:  $\phi, U, \{a, b, c\}, \{b\}, \{b, d\}, \{b, c, d\}, \{a, b, d\}, \{a, b\}, \{b, c\}$ . The milli open sets of  $V$  are:  $V, \phi, \{x\}, \{x, y, z\}, \{y, z\}, \{z\}, \{y\}, \{x, y\}, \{x, z\}$ . Define  $f : U \rightarrow V$  as  $f(a) = z$ ,  $f(b) = w$ ,  $f(c) = x$ ,  $f(d) = y$ . Then  $f$  is milli continuous, but not milli contra continuous function. Observe that,  $f^{-1}(\{y, z\}) = \{a, d\}$  is not milli closed set in  $U$ , where  $\{y, z\}$  is milli open set in  $V$ . Hence  $f$  is milli continuous but not milli contra con-

tinuous.

**Example 3.10.** From Example 3.3, the milli open sets of  $U$  are:  $U, \phi, \{a, c\}$  and milli closed sets of  $U$  are:  $\phi, U, \{b, d\}$ . Also the milli open sets of  $V$  are:  $V, \phi, \{y, z\}$ . Define  $f : U \rightarrow V$  as  $f(a) = x, f(b) = y, f(c) = w, f(d) = z$ . Then  $f$  is milli contra continuous but not milli continuous function. Because,  $f^{-1}(\{y, z\}) = \{b, d\}$  is not milli open set in  $U$ , but  $\{y, z\}$  is milli open set in  $V$ .

**Theorem 3.11.** *A function  $f : (U, \tau_M(X)) \rightarrow (V, \mu_M(Y))$  is milli continuous if and only if the inverse image of every milli closed set in  $V$  is milli closed set in  $U$ .*

**Proof.** Let  $f$  be milli continuous and  $B$  be any milli closed set in  $V$ . Thus the set  $(V - B)$  is a milli open set in  $V$ . Since  $f$  is milli continuous,  $f^{-1}(V - B)$  is milli open set in  $U$ . That is,  $f^{-1}(V - B) = U - f^{-1}(B)$  is milli open set in  $U$ . Therefore,  $f^{-1}(B)$  is milli closed set in  $U$ . Thus, the inverse image of every milli closed set in  $V$  is milli closed set in  $U$ .

Conversely, the inverse image of every milli closed set is milli closed. Let  $G$  be milli open set in  $V$ . Thus  $(V - G)$  is milli closed set in  $V$ . Then,  $f^{-1}(V - G)$  is milli closed set in  $U$ . We have,  $f^{-1}(V - G) = U - f^{-1}(G)$  which is milli closed set in  $U$ . Therefore,  $f^{-1}(G)$  is milli open set in  $U$ . Thus, the inverse image of every milli open set in  $V$  is milli open set in  $U$ . Hence,  $f$  is milli continuous on  $U$ .

**Theorem 3.12.** *A function  $f : (U, \tau_M(X)) \rightarrow (V, \mu_M(Y))$  is milli contra continuous if and only if the inverse image of every milli closed set in  $V$  is milli open in  $U$ .*

**Proof.** Proof is similar to that of Theorem 3.11.

**Theorem 3.13.** *A function  $f : (U, \tau_M(X)) \rightarrow (V, \mu_M(Y))$  is milli continuous if and only if  $f(Mcl(A)) \subseteq Mcl(f(A))$  for every subset  $A$  of  $U$ .*

**Proof.** Let  $f$  be milli continuous and  $A \subseteq U$ . Then  $f(A) \subseteq V$ . Since  $f$  is milli continuous and  $Mcl(f(A))$  is milli closed set in  $V$ ,  $f^{-1}(Mcl(f(A)))$  is milli closed set in  $U$ . Since  $f(A) \subseteq Mcl(f(A))$ ,  $A \subseteq f^{-1}(Mcl(f(A)))$ . Thus  $f^{-1}(Mcl(f(A)))$  is a milli closed set containing  $A$ . But,  $Mcl(A)$  is the smallest milli closed set containing  $A$ . Therefore  $Mcl(A) \subseteq f^{-1}(Mcl(f(A)))$ . This means that  $f(Mcl(A)) \subseteq Mcl(f(A))$ .

Conversely, let  $f(Mcl(A)) \subseteq Mcl(f(A))$  for every subset  $A$  of  $U$ . If  $B$  is milli closed set in  $V$ , since  $f^{-1}(B) \subseteq U$ ,  $f(Mcl(f^{-1}(B))) \subseteq Mcl(f(f^{-1}(B))) = Mcl(B)$ . Thus,  $Mcl(f^{-1}(B)) \subseteq f^{-1}(Mcl(B)) = f^{-1}(B)$ , since  $B$  is milli closed. Therefore  $Mcl(f^{-1}(B)) \subseteq f^{-1}(B)$ . But  $f^{-1}(B) \subseteq Mcl(f^{-1}(B))$ . Thus we have,  $Mcl(f^{-1}(B)) = f^{-1}(B)$ . Therefore,  $f^{-1}(B)$  is milli closed set in  $U$  for every milli closed set  $B$  in  $V$ . Hence,  $f$  is milli continuous.

**Remark 3.14.** *If  $f : (U, \tau_M(X)) \rightarrow (V, \mu_M(Y))$  is milli continuous then  $f(Mcl(A))$*

is not necessarily equal to  $Mcl(f(A))$  for every subset  $A$  of  $U$ , as it is shown in the following example.

**Example 3.15.** Let  $U = \{a, b, c, d\}$ ,  $U/R = \{\{a\}, \{b, d\}, \{c\}\}$ . Let  $X = \{a, c, d\} \subset U$ . Then  $\tau_M(X) = \{U, \phi, \{a, c\}, \{b, d\}, \{d\}, \{b\}, \{a, b, c\}, \{a, c, d\}\}$ . Let  $V = \{x, y, z, w\}$ ,  $V/R' = \{\{x, z\}, \{y\}, \{w\}\}$ . Let  $Y = \{x, y\} \subset V$ . Then  $\mu_M(Y) = \{V, \phi, \{y\}, \{x, y, z\}, \{x, z\}, \{x\}, \{z\}, \{y, z\}, \{x, y\}\}$ . Define  $f : U \rightarrow V$  as  $f(a) = y$ ,  $f(b) = x$ ,  $f(c) = y$ ,  $f(d) = x$ . Then  $f^{-1}(V) = U$ ,  $f^{-1}(\phi) = \phi$ ,  $f^{-1}(\{y\}) = \{a, c\}$ ,  $f^{-1}(\{x, y, z\}) = U$ ,  $f^{-1}(\{x, z\}) = \{b, d\}$ ,  $f^{-1}(\{x\}) = \{b, d\}$ ,  $f^{-1}(\{z\}) = \phi$ ,  $f^{-1}(\{y, z\}) = \{a, c\}$ ,  $f^{-1}(\{x, y\}) = U$ . The inverse image of every milli open set in  $V$  is milli open set in  $U$ . Hence,  $f$  is milli continuous. Let  $A = \{a, c\} \subseteq U$  and  $Mcl(A) = Mcl(\{a, c\}) = \{a, c\}$ . Then  $f(Mcl(A)) = f(Mcl(\{a, c\})) = f(\{a, c\}) = \{y\}$ . But,  $Mcl(f(A)) = Mcl(f(\{a, c\})) = Mcl(\{y\}) = \{y, w\}$ . Thus,  $f(Mcl(A)) \neq Mcl(f(A))$ , even though  $f$  is milli continuous. Thus the, equality does not hold in the previous Theorem 3.14 when  $f$  is milli continuous.

**Theorem 3.16.** Let  $(U, \tau_M(X))$  and  $(V, \mu_M(Y))$  be two milli topological spaces where  $X \subseteq U$  and  $Y \subseteq V$ . Then  $\mu_M(Y) = \{V, \phi, \underline{R}(Y), \overline{R}(Y), B(Y), \underline{E}(Y), \overline{E}(Y), \underline{B}(Y), \overline{B}(Y)\}$  and its basis is given by  $\beta_M(Y) = \{V, \underline{R}(Y), \underline{E}(Y), \overline{E}(Y)\}$ . A function  $f : (U, \tau_M(X)) \rightarrow (V, \mu_M(Y))$  is milli continuous if and only if the inverse image of every member of basis  $\beta_M(Y)$  is milli open set in  $U$ .

**Proof.** Let  $B \in \beta_M(Y)$ . Then  $B$  is milli open set in  $V$ . Since  $f$  is milli continuous,  $f^{-1}(B)$  is milli open set in  $U$ . That is, the inverse image of every member of  $\beta_M(Y)$  is milli open set in  $U$ .

Conversely, let the inverse image of every member of  $\beta_M(Y)$  is milli open set in  $U$ . Let  $G$  be a milli open set in  $V$ . Then  $G = \bigcup \{B : B \in B_1\}$ , where  $B_1 \subset \beta_M(Y)$ . Then  $f^{-1}(G) = f^{-1}(\bigcup \{B : B \in B_1\}) = \bigcup \{f^{-1}(B) : B \in B_1\}$ , where each  $f^{-1}(B)$  is milli open set in  $U$  and hence their union  $f^{-1}(G)$  is milli open set in  $U$ . Thus,  $f$  is milli continuous.

**Theorem 3.17.** A function  $f : (U, \tau_M(X)) \rightarrow (V, \mu_M(Y))$  is milli continuous if and only if  $Mcl(f^{-1}(B)) \subseteq f^{-1}(Mcl(B))$  for every subset  $B$  of  $V$ .

**Proof.** If  $f$  is milli continuous and  $B \subseteq V$ , then  $Mcl(B)$  is milli closed set in  $V$  and hence  $f^{-1}(Mcl(B))$  is milli closed set in  $U$ . Therefore,  $Mcl\{f^{-1}(Mcl(B))\} = f^{-1}(Mcl(B))$ . Since  $B \subseteq Mcl(B)$ ,  $f^{-1}(B) \subseteq f^{-1}(Mcl(B))$ . Therefore,  $Mcl(f^{-1}(B)) \subseteq Mcl(f^{-1}(Mcl(B))) = f^{-1}(Mcl(B))$ . So we have,  $Mcl(f^{-1}(B)) \subseteq f^{-1}(Mcl(B))$ .

Conversely, let  $Mcl(f^{-1}(B)) \subseteq f^{-1}(Mcl(B))$  for every  $B \subseteq V$ . If  $B$  is milli closed set in  $V$ , then  $Mcl(B) = B$ . By assumption,  $Mcl f^{-1}(B) \subseteq f^{-1}(Mcl(B)) = f^{-1}(B)$ . Thus,  $Mcl f^{-1}(B) \subseteq f^{-1}(B)$ . But  $f^{-1}(B) \subseteq Mcl(f^{-1}(B))$ . Therefore,  $Mcl(f^{-1}(B)) = f^{-1}(B)$ . The set  $f^{-1}(B)$  is milli closed set in  $U$  for every milli



closed set  $B$  in  $V$ . Thus,  $f$  is milli continuous.

**Remark 3.18.** *If  $f : (U, \tau_M(X)) \rightarrow (V, \mu_M(Y))$  is milli continuous then  $Mcl(f^{-1}(B))$  is not necessarily equal to  $f^{-1}(Mcl(B))$  for every subset  $B$  of  $V$ , as it is shown by the following example.*

**Example 3.19.** Let  $U = \{a, b, c, d\}$ ,  $U/R = \{\{a, d\}, \{b\}, \{c\}\}$ . Let  $X = \{a, c\} \subset U$ . Then  $\tau_M(X) = \{U, \phi, \{c\}, \{a, c, d\}, \{a, d\}, \{a\}, \{d\}, \{c, d\}, \{a, c\}\}$ . Let  $V = \{x, y, z, w\}$ ,  $V/R' = \{\{x\}, \{y\}, \{z\}, \{w\}\}$ . Let  $Y = \{x, w\} \subset V$ . Then  $\mu_M(Y) = \{V, \phi, \{x, w\}\}$ . Define  $f : U \rightarrow V$  as  $f(a) = x$ ,  $f(b) = y$ ,  $f(c) = z$ ,  $f(d) = w$ . Then  $f^{-1}(V) = U$ ,  $f^{-1}(\phi) = \phi$ ,  $f^{-1}(\{x, w\}) = \{a, d\}$ . That is, the inverse image of every milli open set in  $V$  is milli open set in  $U$ . Hence,  $f$  is milli continuous. Let  $B = \{y\} \subset V$ . Then  $f^{-1}(Mcl(B)) = f^{-1}(\{y, z\}) = \{b, c\}$  and  $Mcl(f^{-1}(B)) = Mcl(\{b\}) = \{b\}$ . Thus,  $Mcl(f^{-1}(B)) \neq f^{-1}(Mcl(B))$ , even though  $f$  is milli continuous. The equality does not hold in the Theorem 3.17 when  $f$  is milli continuous.

**Theorem 3.20.** *A function  $f : (U, \tau_M(X)) \rightarrow (V, \mu_M(Y))$  is milli continuous on  $U$  if and only if  $f^{-1}(Mint(B)) \subseteq Mint(f^{-1}(B))$  for every subset  $B$  of  $V$ .*

**Proof.** Let  $f$  be milli continuous and  $B \subseteq V$ . Then  $Mint(B)$  is a milli open set in  $V$ . Since  $f$  be milli continuous then  $f^{-1}(Mint(B))$  is a milli open set in  $U$ . Therefore  $f^{-1}(Mint(B)) = Mint\{f^{-1}(Mint(B))\}$ . Also,  $Mint(B) \subseteq B$  implies that  $f^{-1}(Mint(B)) \subseteq f^{-1}(B)$ . Therefore  $Mint\{f^{-1}(Mint(B))\} \subseteq Mint(f^{-1}(B))$ . That is,  $f^{-1}(Mint(B)) \subseteq Mint(f^{-1}(B))$ .

Conversely, let  $f^{-1}(Mint(B)) \subseteq Mint(f^{-1}(B))$  for every subset  $B$  of  $V$ . If  $B$  is a milli open set in  $V$ ,  $Mint(B) = B$ . Also,  $f^{-1}(Mint(B)) \subseteq Mint(f^{-1}(B))$ . We have,  $f^{-1}(B) \subseteq Mint(f^{-1}(B))$ . But  $Mint(f^{-1}(B)) \subseteq f^{-1}(B)$ . Therefore,  $f^{-1}(B) = Mint(f^{-1}(B))$ . Thus,  $f^{-1}(B)$  is a milli open set in  $U$  for every milli open set  $B$  in  $V$ . Hence,  $f$  is milli continuous.

**Remark 3.21.** *If  $f : (U, \tau_M(X)) \rightarrow (V, \mu_M(Y))$  is milli continuous then  $f^{-1}(Mint(B))$  is not necessarily equal to  $Mint(f^{-1}(B))$  for every subset  $B$  of  $V$ .*

**Example 3.22.** From Example 3.19,  $f$  is milli continuous on  $U$ . Let  $A = \{x, z, w\} \subseteq V$ ,  $f^{-1}(Mint(A)) = f^{-1}(\{x, w\}) = \{a, d\}$  but  $Mint(f^{-1}(A)) = Mint(\{a, c, d\}) = \{a, c, d\}$ . Thus,  $f^{-1}(Mint(A)) \neq Mint(f^{-1}(A))$ , even though  $f$  is milli continuous. That is, the equality does not hold in the previous Theorem 3.20 when  $f$  is milli continuous.

**Theorem 3.23.** *If  $(U, \tau_M(X))$  and  $(V, \mu_M(Y))$  are milli topological spaces with respect to  $X \subseteq U$  and  $Y \subseteq V$  respectively, then for any function  $f : (U, \tau_M(X)) \rightarrow (V, \mu_M(Y))$  the following are equivalent:*

- (i)  $f$  is milli continuous.
- (ii) The inverse image of every milli closed set in  $V$  is milli closed set in  $U$ .
- (iii)  $f(Mcl(A)) \subset Mcl(f(A))$  for very subset  $A$  of  $V$ .
- (iv) The inverse image of every member of the basis  $B_M(Y)$  of  $(V, \mu_M(Y))$  is milli open set in  $U$ .
- (v)  $Mcl(f^{-1}(B)) \subseteq f^{-1}(Mcl(B))$  for every subset  $B$  of  $V$ .
- (vi)  $f^{-1}(Mint(B)) \subset Mint(f^{-1}(B))$  for every subset  $B$  of  $V$ .

**Proof.** It follows from Theorems 3.11, 3.13, 3.16, 3.17 and 3.20.

**Theorem 3.24.** Let  $f : (U, \tau_M(X)) \rightarrow (V, \mu_M(Y))$  be one-one, onto and milli continuous function. If  $A$  is milli dense in  $U$ , then  $f(A)$  is milli dense in  $V$ .

**Proof.** Since  $A$  is milli dense in  $U$ ,  $Mcl(A) = U$ . Then  $f(Mcl(A)) = f(U) = V$ , by the fact that  $f$  is one-one, onto and milli continuous. Since  $f$  is milli continuous,  $f(Mcl(A)) \subseteq Mcl(f(A))$ . Therefore,  $V \subseteq Mcl(f(A))$ . But  $Mcl(f(A)) \subseteq V$ . This means that,  $Mcl(f(A)) = V$ . Therefore,  $f(A)$  is milli dense in  $V$ . Hence the result.

**Theorem 3.25.** Let  $(U, \tau_M(X))$ ,  $(V, \mu_M(Y))$  and  $(W, \eta_M(Z))$  be milli topological spaces. Let  $f : U \rightarrow V$  and  $g : V \rightarrow W$  be milli continuous functions, then their composition  $g \circ f : U \rightarrow W$  is also milli continuous function.

**Proof.** Let  $B$  be any milli open set in  $W$ . Since  $g$  is milli continuous.  $g^{-1}(B)$  is a milli open set in  $V$ . Since  $f$  is milli continuous, so  $f^{-1}\{g^{-1}(B)\}$  is a milli open set in  $U$ . Therefore  $f^{-1}\{g^{-1}(B)\} = (g \circ f)^{-1}(B)$  is a milli open set in  $U$ . Hence  $g \circ f$  is also milli continuous function.

**Remark 3.26.** The composition of two milli contra continuous function need not be milli contra continuous, as it is shown in the following example.

**Example 3.27.** Let  $U = \{a, b, c, d\}$ ,  $U/R = \{\{a\}, \{b, d\}, \{c\}\}$ . Let  $X = \{a, c\} \subset U$ . Then  $\tau_M(X) = \{U, \phi, \{a, c\}\}$ . Milli closed sets of  $U$  are:  $\phi, U, \{b, d\}$ . Let  $V = \{x, y, z, w\}$ ,  $V/R' = \{\{x\}, \{y, z\}, \{w\}\}$ . Let  $Y = \{y, z\} \subset V$ . Then  $\mu_M(Y) = \{V, \phi, \{y, z\}\}$ . Milli closed sets of  $V$  are:  $\phi, V, \{x, w\}$ . Let  $W = \{p, q, r, s\}$ ,  $W/R'' = \{\{p\}, \{q, s\}, \{r\}\}$ . Let  $Z = \{q, s\} \subset W$ . Then  $\eta_M(Z) = \{W, \phi, \{q, s\}\}$ . Then  $f : (U, \tau_M(X)) \rightarrow (V, \mu_M(Y))$ , define by  $f(a) = x$ ,  $f(b) = y$ ,  $f(c) = w$ ,  $f(d) = z$ . Therefore  $f^{-1}(V) = U$ ,  $f^{-1}(\{\phi\}) = \phi$ ,  $f^{-1}(\{y, z\}) = \{b, d\}$  is milli closed set in  $U$ . Therefore,  $f$  is milli contra continuous. Also  $g : (V, \mu_M(Y)) \rightarrow (W, \eta_M(Z))$ , define by  $g(x) = q$ ,  $g(y) = p$ ,  $g(z) = r$ ,  $g(w) = s$ . Therefore  $g^{-1}(W) = V$ ,  $g^{-1}(\{\phi\}) = \phi$ ,  $g^{-1}(\{q, s\}) = \{x, w\}$  is a milli closed set in  $U$ . Thus,  $g$

is milli contra continuous. Here  $f$  and  $g$  are milli contra continuous functions. But  $f^{-1}\{g^{-1}(\{q, s\})\} = f^{-1}\{x, w\} = \{a, c\}$  which is not milli closed set in  $U$ . Hence  $g \circ f$  is not milli contra continuous.

**Theorem 3.28.** *Let  $f : (U, \tau_M(X)) \rightarrow (V, \mu_M(Y))$  and  $g : (V, \mu_M(Y)) \rightarrow (W, \eta_M(Z))$  be the functions then  $g \circ f : (U, \tau_M(X)) \rightarrow (W, \eta_M(Z))$  is milli contra continuous if  $g$  is milli continuous and  $f$  is milli contra continuous.*

**Proof.** Let  $f$  and  $g$  be a milli contra continuous function and a milli continuous function respectively. Suppose  $B$  is a milli open set in  $W$ . Since  $g$  is milli continuous  $g^{-1}(B)$  is a milli open set in  $V$ . Since  $f$  is milli contra continuous,  $f^{-1}\{g^{-1}(B)\} = (g \circ f)^{-1}(B)$  is a milli closed set in  $U$ . That is  $(g \circ f)^{-1}(B)$  is a milli closed set in  $U$  for every milli open set  $B$  in  $W$ . Hence  $g \circ f$  is milli contra continuous.

#### 4. Milli perfectly and Milli strongly continuous functions

In this section, the notion of milli perfectly continuous and milli strongly continuous functions are introduced and their properties are investigated. These milli perfectly continuous and milli strongly continuous functions are compared with other milli continuous functions.

**Definition 4.1.** *Let  $(U, \tau_M(X))$  and  $(V, \mu_M(Y))$  be milli topological spaces. A function  $f : (U, \tau_M(X)) \rightarrow (V, \mu_M(Y))$  is said to be*

- (i) *milli perfectly continuous, if  $f^{-1}(B)$  is a milli clopen set in  $U$  for every milli open set  $B$  in  $V$ .*
- (ii) *milli strongly continuous, if  $f^{-1}(B)$  is a milli clopen set in  $U$  for every subset set  $B$  in  $V$ .*

**Theorem 4.2.** *Every milli perfectly continuous function is milli continuous, but converse need not be true.*

**Proof.** Let  $B$  be a milli open set in  $V$ . Since  $f$  is milli perfectly continuous,  $f^{-1}(B)$  is a milli clopen set in  $U$ . That is  $f^{-1}(B)$  is both a milli open set and as well as milli closed set in  $U$ . Thus, the inverse image of every milli open set in  $V$  is a milli open set in  $U$ . Hence  $f$  is milli continuous.

**Example 4.3.** From Example 3.2, the milli open sets of  $U$  are:  $U, \phi, \{d\}, \{a, c, d\}, \{a, c\}, \{a\}, \{c\}, \{c, d\}, \{a, d\}$  and the milli closed sets of  $U$  are:  $\phi, U, \{a, b, c\}, \{b\}, \{b, d\}, \{b, c, d\}, \{a, b, d\}, \{a, b\}, \{b, c\}$ . The milli open sets of  $V$  are:  $V, \phi, \{x\}, \{x, y, z\}, \{y, z\}, \{z\}, \{y\}, \{x, y\}, \{x, z\}$ . Define  $f : U \rightarrow V$  as  $f(a) = z, f(b) = w, f(c) = x, f(d) = y$ . Then  $f$  is milli continuous, but not milli perfectly continuous. Since,  $f^{-1}(\{y, z\}) = \{a, d\}$  is not a milli clopen set in  $U$ , where  $\{y, z\}$  is milli open

set in  $V$ .

**Theorem 4.4.** *Every milli perfectly continuous function is milli contra continuous, but the converse need not be true.*

**Proof.** By the same token as that of Theorem 4.2.

**Example 4.5.** From Example 3.3, the milli open sets of  $U$  are:  $U, \phi, \{a, c\}$  and the milli closed sets of  $U$  are:  $\phi, U, \{b, d\}$ . The milli open sets of  $V$  are:  $\phi, V, \{y, z\}$ . Define  $f : U \rightarrow V$  as  $f(a) = x, f(b) = y, f(c) = w, f(d) = z$ . Then  $f$  is milli contra continuous but not milli perfectly continuous. Because,  $f^{-1}(\{y, z\}) = \{b, d\}$ , is not milli clopen in  $U$ , but  $\{y, z\}$  is milli open set in  $V$ .

**Theorem 4.6.** *For a function  $f : (U, \tau_M(X)) \rightarrow (V, \mu_M(Y))$  the following conditions are equivalent*

(i)  $f$  is milli perfectly continuous.

(ii)  $f$  is milli continuous and milli contra continuous.

**Proof.**  $i) \Rightarrow ii)$ . Let  $B$  be a milli open set in  $V$ . Since  $f$  is milli perfectly continuous.  $f^{-1}(B)$  is a milli clopen set in  $U$ . That is  $f^{-1}(B)$  is milli open set and also milli closed set in  $U$ . Hence  $f$  is both milli continuous and milli contra continuous.  $ii) \Rightarrow i)$ . Let  $B$  be a milli open set in  $V$ . Since  $f$  is milli continuous and milli contra continuous.  $f^{-1}(B)$  is a milli open set and also milli closed set in  $U$ . That is  $f^{-1}(B)$  is both milli open set and also milli closed set in  $U$ . Hence  $f$  is milli perfectly continuous.

**Theorem 4.7.** *Every milli strongly continuous function is milli continuous, but the converse need not be true.*

**Proof.** Let  $B$  be an arbitrary set in  $V$ . Since  $f$  is milli strongly continuous.  $f^{-1}(B)$  is a milli clopen set in  $U$ . Hence  $f^{-1}(B)$  is both milli open set and as well as milli closed set in  $U$ . Since it holds for every subset of  $V$ , it is also true for all the milli open sets in  $V$ . Thus every inverse image of milli open set in  $V$  is milli open set in  $U$ . Hence  $f$  is milli continuous.

**Example 4.8.** From Example 3.2,  $f$  is milli continuous, but not milli strongly continuous. Since,  $f^{-1}(\{y, z\}) = \{a, d\}$  is not milli clopen set in  $U$ , where  $\{y, z\}$  is a subset in  $V$ .

**Theorem 4.9.** *Every milli strongly continuous function is milli contra continuous, but the converse need not be true.*

**Proof.** By the same token as that of Theorem 4.7.

**Example 4.10.** From Example 3.5,  $f$  is milli contra continuous but not milli

strongly continuous, since  $f^{-1}(\{y, z\}) = \{a, d\}$ , is not milli clopen set in  $U$ , whereas  $\{y, z\}$  is a subset in  $V$ .

**Theorem 4.11.** *Let  $(U, \tau_M(X))$ ,  $(V, \mu_M(Y))$  and  $(W, \eta_M(Z))$  be milli topological spaces. Let  $f : U \rightarrow V$  and  $g : V \rightarrow W$  are milli perfectly (milli strongly) continuous functions, then their composition  $g \circ f : U \rightarrow W$  is also milli perfectly ( milli strongly) continuous function.*

**Proof.** By the same token as that of Theorem 3.28.

**Remark 4.12.** *From the above theorems and examples we have the following diagram,  $A \rightarrow B$  represents  $A$  implies  $B$  but converse is not true.*

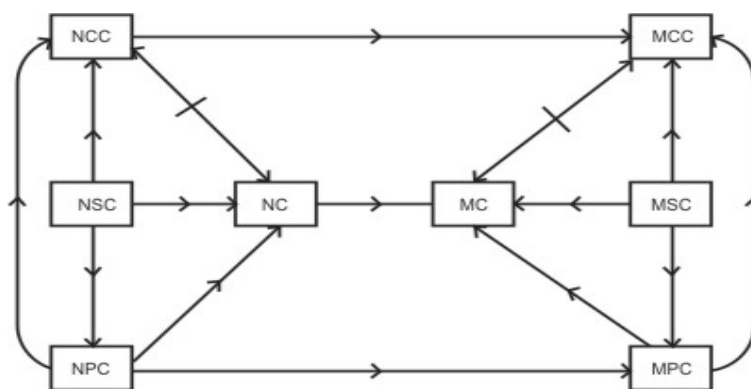


Figure 1: Relations between existing continuous functions

- |                                  |                                 |
|----------------------------------|---------------------------------|
| MC: Milli Continuous,            | NC: Nano Continuous,            |
| MCC: Milli Contra Continuous,    | NCC: Nano Contra Continuous,    |
| MSC: Milli Strongly Continuous,  | NSC: Nano Strongly Continuous,  |
| MPC: Milli Perfectly Continuous, | NPC: Nano Perfectly Continuous. |

### 5. Applications of milli Topology

In this section, we define real-life application that was produced by using the concepts of milli basis in milli topological space. The essential food habits for a healthy lifestyle were identified. In the future, this work will be extended with some real-life applications by using the corresponding set of milli open in milli topological spaces.

**Definition 5.1.** *Let  $(U, A)$  be an information system, where  $A$  is divided into a set  $C$  of condition attributes and a set  $D$  of decision attributes. A subset  $R$  of  $C$  is said to be a core, if  $\beta_M^R = \beta_M^C$  and  $\beta_M^R \neq \beta_M^{R-\{r\}}$  for all  $r \in R$ , where  $\beta_M^R$  is the milli basis of milli topology corresponding to  $R \subseteq C$ . That is, a core is a minimal*

subset of attributes such that none of its elements can be removed without affecting the classification power of attributes.

### To Find the Key Factors for a Healthy Life

In this section we discussed a new application in data analysis by using the milli basis of milli topological space. We found the essential key factors for a healthy lifestyle. Here the concepts of the topological reduction of attributes in a complete information system is used to get the key factors. The general algorithm for identify the key factors is given below.

#### Algorithm

**Step 1.** Let  $U$  be the finite universe,  $A$  be the finite set of attributes which is divided into two classes of condition attributes and decision attributes where condition attributes are denoted by  $C$  and decision attributes are denoted by  $D$ . Then  $R$  is the equivalence relation on  $U$  corresponding to the condition attributes  $C$  and a subset  $X$  of  $U$ . The given tabular column was represented by the data, in which columns are the attributes and rows are the objects. The entries of the table are known as the attribute values.

**Step 2.** Find the lower approximation  $\underline{R}(X)$ , upper approximation  $\overline{R}(X)$ , boundary region  $B(X)$ , lower edge  $\underline{E}(X)$ , upper edge  $\overline{E}(X)$ , lower boundary region  $\underline{B}(X)$  and upper boundary region  $\overline{B}(X)$  of  $X$  with respect to  $R$ .

**Step 3.** Find the milli topology  $\tau_M^C(X)$  on  $U$  and its basis  $\beta_M^C(X)$  corresponding to the conditional attribute set  $C$ .

**Step 4.** Remove an attribute  $x$  from  $C$  and find the lower approximation, upper approximation, boundary region, lower edge, upper edge, lower boundary region and upper boundary region of  $X$  with respect to the equivalence relation on  $C - \{x\}$ .

**Step 5.** Find the milli topology  $\tau_M^{C-\{x\}}(X)$  on  $U$  and its basis  $\beta_M^{C-\{x\}}(X)$ .

**Step 6.** Repeat steps 4 and 5 for all attributes in  $C$ .

**Step 7.** Those attributes in  $C$  for which  $\beta_M^{C-\{x\}}(X) \neq \beta_M^C(X)$  form the CORE.

**Example 5.2.** Food habits are the most important thing for a healthy life. Here we collect the data of food habits for the working people. In their busy morning routine they have much time to cook their breakfast. And also they move on to the instant foods due to their less interest in cooking. Sometimes they skip their breakfast or ordered foods in online too. Based on their daily food habits we find out the essential food habits for their healthy life.

Persons	Ordered Foods	Fruits and Vegetables	Beverages	Instant Foods	Properly Cooked Foods	Healthy or Unhealthy
$P_1$	✓	✓	✓	×	×	HEALTHY
$P_2$	×	✓	×	✓	✓	HEALTHY
$P_3$	✓	×	✓	×	✓	HEALTHY
$P_4$	✓	✓	✓	×	×	UNHEALTHY
$P_5$	✓	×	✓	×	×	UNHEALTHY
$P_6$	✓	×	✓	×	✓	UNHEALTHY
$P_7$	✓	✓	×	×	✓	HEALTHY
$P_8$	✓	✓	×	✓	×	UNHEALTHY

From the above table we got the different types of food habits. Here  $U = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8\}$  is the set of persons and  $A = \{ \text{Ordered Foods (OF), Fruits and Vegetables (F\&V), Beverages (B), Instant Foods (IF) and Properly Cooked Foods (PCF)} \}$  are the set of attributes. Yes (✓) or No (×) are the entries in the table which known as attribute values. The attributes in  $A$  are the condition attributes and Healthy (H)/Unhealthy (UH) is the decision attribute. Condition attribute is denoted by  $C$  such as

$C = \{ \text{OF, F\&V, B, IF, PCF} \}$  and decision attribute is denoted by  $D = \{ \text{H/UH} \}$ . The family of classes,  $U/R^C$  corresponding to  $C$  is given by  $U/R^C = \{ \{P_1, P_4\}, \{P_2\}, \{P_3, P_6\}, \{P_5\}, \{P_7\}, \{P_8\} \}$ .

**Case 1.** (Persons who are healthy)

Let  $X = \{P_1, P_2, P_3, P_7\}$ , the set of persons who are healthy. Then  $\underline{R}(X) = \{P_2, P_7\}$ ,  $\overline{R}(X) = \{P_1, P_2, P_3, P_4, P_6, P_7\}$ ,  $B(X) = \{P_1, P_3, P_4, P_6\}$ ,  $\underline{E}(X) = \{P_1, P_3\}$ ,  $\overline{E}(X) = \{P_4, P_6\}$ ,  $\underline{B}(X) = \{P_2, P_4, P_6, P_7\}$  and  $\overline{B}(X) = \{P_1, P_2, P_3, P_7\}$ . Therefore the milli topology,  $\tau_M^C(X) = \{U, \phi, \{P_2, P_7\}, \{P_1, P_2, P_3, P_4, P_6, P_7\}, \{P_1, P_3, P_4, P_6\}, \{P_1, P_3\}, \{P_4, P_6\}, \{P_2, P_4, P_6, P_7\}, \{P_1, P_2, P_3, P_7\}\}$  and the milli basis of  $\tau_M^C(X)$  is given by  $\beta_M^C(X) = \{U, \{P_2, P_7\}, \{P_1, P_3\}, \{P_4, P_6\}\}$ .

**Step 1.** i) Remove the attribute Ordered Foods from  $C$ :

$U/R^{(C-\{OF\})} = \{ \{P_1, P_4\}, \{P_2\}, \{P_3, P_6\}, \{P_5\}, \{P_7\}, \{P_8\} \}$ . Then  $\underline{R}(X) = \{P_2, P_7\}$ ,  $\overline{R}(X) = \{P_1, P_2, P_3, P_4, P_6, P_7\}$ ,  $B(X) = \{P_1, P_3, P_4, P_6\}$ ,  $\underline{E}(X) = \{P_1, P_3\}$ ,  $\overline{E}(X) = \{P_4, P_6\}$ ,  $\underline{B}(X) = \{P_2, P_4, P_6, P_7\}$  and  $\overline{B}(X) = \{P_1, P_2, P_3, P_7\}$ . Therefore the milli topology,  $\tau_M^{(C-\{OF\})}(X) = \{U, \phi, \{P_2, P_7\}, \{P_1, P_2, P_3, P_4, P_6, P_7\}, \{P_1, P_3, P_4, P_6\}, \{P_1, P_3\}, \{P_4, P_6\}, \{P_2, P_4, P_6, P_7\}, \{P_1, P_2, P_3, P_7\}\}$  and the milli basis of

$\tau_M^{(C-\{OF\})}(X)$  is given by

$$\beta_M^{(C-\{OF\})}(X) = \{U, \{P_2, P_7\}, \{P_1, P_3\}, \{P_4, P_6\}\} = \beta_M^C(X).$$

ii) Remove the attribute Fruits and Vegetables from  $C$ :

$U/R^{(C-\{F\&V\})} = \{\{P_1, P_4, P_5\}, \{P_2\}, \{P_3, P_6\}, \{P_7\}, \{P_8\}\}$ . Then  $\underline{R}(X) = \{P_2, P_7\}$ ,  $\overline{R}(X) = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}$ ,  $B(X) = \{P_1, P_3, P_4, P_5, P_6\}$ ,  $\underline{E}(X) = \{P_1, P_3\}$ ,  $\overline{E}(X) = \{P_4, P_5, P_6\}$ ,  $\underline{B}(X) = \{P_2, P_4, P_5, P_6, P_7\}$  and  $\overline{B}(X) = \{P_1, P_2, P_3, P_7\}$ . Therefore the milli topology,  $\tau_M^{(C-\{F\&V\})}(X) = \{U, \phi, \{P_2, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_3, P_4, P_5, P_6\}, \{P_1, P_3\}, \{P_4, P_5, P_6\}, \{P_2, P_4, P_5, P_6, P_7\}, \{P_1, P_2, P_3, P_7\}\}$  and the milli basis of  $\tau_M^{(C-\{F\&V\})}(X)$  is given by  $\beta_M^{(C-\{F\&V\})}(X) = \{U, \{P_2, P_7\}, \{P_1, P_3\}, \{P_4, P_5, P_6\}\} \neq \beta_M^C(X)$ .

iii) Remove the attribute Beverages from  $C$ :

$U/R^{(C-B)} = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_6\}, \{P_5\}, \{P_7\}, \{P_8\}\}$ . Then  $\underline{R}(X) = \{P_2, P_7\}$ ,  $\overline{R}(X) = \{P_1, P_2, P_3, P_4, P_6, P_7\}$ ,  $B(X) = \{P_1, P_3, P_4, P_6\}$ ,  $\underline{E}(X) = \{P_1, P_3\}$ ,  $\overline{E}(X) = \{P_4, P_6\}$ ,  $\underline{B}(X) = \{P_2, P_4, P_6, P_7\}$  and  $\overline{B}(X) = \{P_1, P_2, P_3, P_7\}$ . Therefore the milli topology,  $\tau_M^{(C-B)}(X) = \{U, \phi, \{P_2, P_7\}, \{P_1, P_2, P_3, P_4, P_6, P_7\}, \{P_1, P_3, P_4, P_6\}, \{P_1, P_3\}, \{P_4, P_6\}, \{P_2, P_4, P_6, P_7\}, \{P_1, P_2, P_3, P_7\}\}$  and the milli basis of  $\tau_M^{(C-B)}(X)$  is given by  $\beta_M^{(C-B)}(X) = \{U, \{P_2, P_7\}, \{P_1, P_3\}, \{P_4, P_6\}\} = \beta_M^C(X)$ .

iv) Remove the attribute Instant Foods from  $C$ :

$U/R^{(C-\{IF\})} = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_6\}, \{P_5\}, \{P_7\}, \{P_8\}\}$ . Then  $\underline{R}(X) = \{P_2, P_7\}$ ,  $\overline{R}(X) = \{P_1, P_2, P_3, P_4, P_6, P_7\}$ ,  $B(X) = \{P_1, P_3, P_4, P_6\}$ ,  $\underline{E}(X) = \{P_1, P_3\}$ ,  $\overline{E}(X) = \{P_4, P_6\}$ ,  $\underline{B}(X) = \{P_2, P_4, P_6, P_7\}$  and  $\overline{B}(X) = \{P_1, P_2, P_3, P_7\}$ . Therefore the milli topology,  $\tau_M^{(C-\{IF\})}(X) = \{U, \phi, \{P_2, P_7\}, \{P_1, P_2, P_3, P_4, P_6, P_7\}, \{P_1, P_3, P_4, P_6\}, \{P_1, P_3\}, \{P_4, P_6\}, \{P_2, P_4, P_6, P_7\}, \{P_1, P_2, P_3, P_7\}\}$  and the milli basis of  $\tau_M^{(C-\{IF\})}(X)$  is given by  $\beta_M^{(C-\{IF\})}(X) = \{U, \{P_2, P_7\}, \{P_1, P_3\}, \{P_4, P_6\}\} = \beta_M^C(X)$ .

v) Remove the attribute Properly Cooked Foods from  $C$ :

$U/R^{(C-\{PCF\})} = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_5, P_6\}, \{P_7\}, \{P_8\}\}$ . Then  $\underline{R}(X) = \{P_2, P_7\}$ ,  $\overline{R}(X) = \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}$ ,  $B(X) = \{P_1, P_3, P_4, P_5, P_6\}$ ,  $\underline{E}(X) = \{P_1, P_3\}$ ,  $\overline{E}(X) = \{P_4, P_5, P_6\}$ ,  $\underline{B}(X) = \{P_2, P_4, P_5, P_6, P_7\}$  and  $\overline{B}(X) = \{P_1, P_2, P_3, P_7\}$ . Therefore the milli topology,  $\tau_M^{(C-\{PCF\})}(X) = \{U, \phi, \{P_2, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_3, P_4, P_5, P_6\}, \{P_1, P_3\}, \{P_4, P_5, P_6\}, \{P_2, P_4, P_5, P_6, P_7\}, \{P_1, P_2, P_3, P_7\}\}$  and the milli basis of  $\tau_M^{(C-\{PCF\})}(X)$  is given by  $\beta_M^{(C-\{PCF\})}(X) = \{U, \{P_2, P_7\}, \{P_1, P_3\}, \{P_4, P_5, P_6\}\} \neq \beta_M^C(X)$ .

From (i) to (v):  $\beta_M^{(C-\{OF\})}(X) = \beta_M^C(X)$ ;  $\beta_M^{(C-\{F\&V\})}(X) \neq \beta_M^C(X)$ ;  $\beta_M^{(C-B)}(X) = \beta_M^C(X)$ ;  $\beta_M^{(C-\{IF\})}(X) = \beta_M^C(X)$ ;  $\beta_M^{(C-\{PCF\})}(X) \neq \beta_M^C(X)$ .



**Step 2.** If  $N = C - \{OF, B, IF\} = \{F\&V, PCF\}$ , then  $\beta_M^N(X) = \beta_M^C(X)$ . From step 1, we get,

i)  $U/R^{(C-\{F\&V\})} = \{\{P_1, P_4, P_5\}, \{P_2\}, \{P_3, P_6\}, \{P_7\}, \{P_8\}\}$ . Then  $\tau_M^{(C-\{F\&V\})}(X) = \{U, \phi, \{P_2, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_3, P_4, P_5, P_6\}, \{P_1, P_3\}, \{P_4, P_5, P_6\}, \{P_2, P_4, P_5, P_6, P_7\}, \{P_1, P_2, P_3, P_7\}\}$  and the milli basis  $\beta_M^{(C-\{F\&V\})}(X) = \{U, \{P_2, P_7\}, \{P_1, P_3\}, \{P_4, P_5, P_6\}\} \neq \beta_M^C(X)$ .

ii)  $U/R^{(C-\{PCF\})} = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_5, P_6\}, \{P_7\}, \{P_8\}\}$ . Then  $\tau_M^{(C-\{PCF\})}(X) = \{U, \phi, \{P_2, P_7\}, \{P_1, P_2, P_3, P_4, P_5, P_6, P_7\}, \{P_1, P_3, P_4, P_5, P_6\}, \{P_1, P_3\}, \{P_4, P_5, P_6\}, \{P_2, P_4, P_5, P_6, P_7\}, \{P_1, P_2, P_3, P_7\}\}$  and the milli basis  $\beta_M^{(C-\{PCF\})}(X) = \{U, \{P_2, P_7\}, \{P_1, P_3\}, \{P_4, P_5, P_6\}\} \neq \beta_M^C(X)$ .

From (i) and (ii) we get,  $\beta_M^N(X) = \beta_M^C(X)$  and  $\beta_M^{(N-\{x\})}(X) \neq \beta_M^C(X)$  for every  $x$  in  $N$ . Since  $\text{CORE}(A) = \{\text{Fruits and Vegetables, Properly Cooked Foods}\}$ .

**Result.** From the core we conclude that the essential key factors for a healthy life is Fruits and Vegetables and Properly Cooked Foods.

**Case 2.** (Persons who are Unhealthy)

Let  $X = \{P_4, P_5, P_6, P_8\}$  be the set of persons who are unhealthy. The family of classes,  $U/R^C$  corresponding to  $C$  is given by  $U/R^C = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_6\}, \{P_5\}, \{P_7\}, \{P_8\}\}$ . Then  $\underline{R}(X) = \{P_5, P_8\}$ ,  $\overline{R}(X) = \{P_1, P_3, P_4, P_5, P_6, P_8\}$ ,  $B(X) = \{P_1, P_3, P_4, P_6\}$ ,  $\underline{E}(X) = \{P_4, P_6\}$ ,  $\overline{E}(X) = \{P_1, P_3\}$ ,  $\underline{B}(X) = \{P_1, P_3, P_5, P_8\}$  and  $\overline{B}(X) = \{P_4, P_5, P_6, P_8\}$ . Therefore the milli topology,  $\tau_M^C(X) = \{U, \phi, \{P_5, P_8\}, \{P_1, P_3, P_4, P_5, P_6, P_8\}, \{P_1, P_3, P_4, P_6\}, \{P_4, P_6\}, \{P_1, P_3\}, \{P_1, P_3, P_5, P_8\}, \{P_4, P_5, P_6, P_8\}\}$  and the milli basis of  $\tau_M^C(X)$  is given by  $\beta_M^C(X) = \{U, \{P_5, P_8\}, \{P_4, P_6\}, \{P_1, P_3\}\}$ .

**Step 1.** i) Remove the attribute Ordered Foods from  $C$ :

$U/R^{(C-\{OF\})} = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_6\}, \{P_5\}, \{P_7\}, \{P_8\}\}$ . Then  $\underline{R}(X) = \{P_5, P_8\}$ ,  $\overline{R}(X) = \{P_1, P_3, P_4, P_5, P_6, P_8\}$ ,  $B(X) = \{P_1, P_3, P_4, P_6\}$ ,  $\underline{E}(X) = \{P_4, P_6\}$ ,  $\overline{E}(X) = \{P_1, P_3\}$ ,  $\underline{B}(X) = \{P_1, P_3, P_5, P_8\}$  and  $\overline{B}(X) = \{P_4, P_5, P_6, P_8\}$ . Therefore the milli topology,  $\tau_M^{(C-\{OF\})}(X) = \{U, \phi, \{P_5, P_8\}, \{P_1, P_3, P_4, P_5, P_6, P_8\}, \{P_1, P_3, P_4, P_6\}, \{P_4, P_6\}, \{P_1, P_3\}, \{P_1, P_3, P_5, P_8\}, \{P_4, P_5, P_6, P_8\}\}$  and the milli basis of  $\tau_M^{(C-\{OF\})}(X)$  is given by  $\beta_M^{(C-\{OF\})}(X) = \{U, \{P_5, P_8\}, \{P_4, P_6\}, \{P_1, P_3\}\} = \beta_M^C(X)$ .

ii) Remove the attribute Fruits and Vegetables from  $C$ :

$U/R^{(C-\{F\&V\})} = \{\{P_1, P_4, P_5\}, \{P_2\}, \{P_3, P_6\}, \{P_7\}, \{P_8\}\}$ . Then  $\underline{R}(X) = \{P_8\}$ ,  $\overline{R}(X) = \{P_1, P_3, P_4, P_5, P_6, P_8\}$ ,  $B(X) = \{P_1, P_3, P_4, P_5, P_6\}$ ,  $\underline{E}(X) = \{P_4, P_5, P_6\}$ ,  $\overline{E}(X) = \{P_1, P_3\}$ ,  $\underline{B}(X) = \{P_1, P_3, P_8\}$  and  $\overline{B}(X) = \{P_4, P_5, P_6, P_8\}$ . Therefore

the milli topology,  $\tau_M^{(C-\{F\&V\})}(X) = \{U, \phi, \{P_8\}, \{P_1, P_3, P_4, P_5, P_6, P_8\}, \{P_1, P_3, P_4, P_5, P_6\}, \{P_4, P_5, P_6\}, \{P_1, P_3\}, \{P_1, P_3, P_8\}, \{P_4, P_5, P_6, P_8\}\}$  and the milli basis of  $\tau_M^{(C-\{F\&V\})}(X)$  is given by

$$\beta_M^{(C-\{F\&V\})}(X) = \{U, \{P_8\}, \{P_4, P_5, P_6\}, \{P_1, P_3\}\} \neq \beta_M^C(X).$$

iii) Remove the attribute Beverages from  $C$ :

$U/R^{(C-B)} = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_6\}, \{P_5\}, \{P_7\}, \{P_8\}\}$ . Then  $\underline{R}(X) = \{P_5, P_8\}$ ,  $\overline{R}(X) = \{P_1, P_3, P_4, P_5, P_6, P_8\}$ ,  $B(X) = \{P_1, P_3, P_4, P_6\}$ ,  $\underline{E}(X) = \{P_4, P_6\}$ ,  $\overline{E}(X) = \{P_1, P_3\}$ ,  $\underline{B}(X) = \{P_1, P_3, P_5, P_8\}$  and  $\overline{B}(X) = \{P_4, P_5, P_6, P_8\}$ . Therefore the milli topology,  $\tau_M^{(C-B)}(X) = \{U, \phi, \{P_5, P_6\}, \{P_1, P_3, P_4, P_5, P_6, P_8\}, \{P_1, P_3, P_4, P_6\}, \{P_4, P_6\}, \{P_1, P_3\}, \{P_1, P_3, P_5, P_8\}, \{P_4, P_5, P_6, P_8\}\}$  and the milli basis of  $\tau_M^{(C-B)}(X)$  is given by

$$\beta_M^{(C-B)}(X) = \{U, \{P_5, P_8\}, \{P_4, P_6\}, \{P_1, P_3\}\} = \beta_M^C(X).$$

iv) Remove the attribute Instant Foods from  $C$ :

$U/R^{(C-IF)} = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_6\}, \{P_5\}, \{P_7\}, \{P_8\}\}$ . Then  $\underline{R}(X) = \{P_5, P_8\}$ ,  $\overline{R}(X) = \{P_1, P_3, P_4, P_5, P_6, P_8\}$ ,  $B(X) = \{P_1, P_3, P_4, P_6\}$ ,  $\underline{E}(X) = \{P_4, P_6\}$ ,  $\overline{E}(X) = \{P_1, P_3\}$ ,  $\underline{B}(X) = \{P_1, P_3, P_5, P_8\}$  and  $\overline{B}(X) = \{P_4, P_5, P_6, P_8\}$ . Therefore the milli topology,  $\tau_M^{(C-IF)}(X) = \{U, \phi, \{P_5, P_6\}, \{P_1, P_3, P_4, P_5, P_6, P_8\}, \{P_1, P_3, P_4, P_6\}, \{P_4, P_6\}, \{P_1, P_3\}, \{P_1, P_3, P_5, P_8\}, \{P_4, P_5, P_6, P_8\}\}$  and the milli basis of  $\tau_M^{(C-IF)}(X)$  is given by

$$\beta_M^{(C-IF)}(X) = \{U, \{P_5, P_8\}, \{P_4, P_6\}, \{P_1, P_3\}\} = \beta_M^C(X).$$

v) Remove the attribute Properly Cooked Foods from  $C$ :

$U/R^{(C-PCF)} = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_5, P_6\}, \{P_7\}, \{P_8\}\}$ . Then  $\underline{R}(X) = \{P_8\}$ ,  $\overline{R}(X) = \{P_1, P_3, P_4, P_5, P_6, P_8\}$ ,  $B(X) = \{P_1, P_3, P_4, P_5, P_6\}$ ,  $\underline{E}(X) = \{P_4, P_5, P_6\}$ ,  $\overline{E}(X) = \{P_1, P_3\}$ ,  $\underline{B}(X) = \{P_1, P_3, P_8\}$  and  $\overline{B}(X) = \{P_4, P_5, P_6, P_8\}$ . Therefore the milli topology,  $\tau_M^{(C-PCF)}(X) = \{U, \phi, \{P_8\}, \{P_1, P_3, P_4, P_5, P_6, P_8\}, \{P_1, P_3, P_4, P_5, P_6\}, \{P_4, P_5, P_6\}, \{P_1, P_3\}, \{P_1, P_3, P_8\}, \{P_4, P_5, P_6, P_8\}\}$  and the milli basis of  $\tau_M^{(C-PCF)}(X)$  is given by

$$\beta_M^{(C-PCF)}(X) = \{U, \{P_8\}, \{P_4, P_5, P_6\}, \{P_1, P_3\}\} \neq \beta_M^C(X).$$

From (i) to (v):  $\beta_M^{(C-\{OF\})}(X) = \beta_M^C(X)$ ;  $\beta_M^{(C-\{F\&V\})}(X) \neq \beta_M^C(X)$ ;  $\beta_M^{(C-B)}(X) = \beta_M^C(X)$ ;  $\beta_M^{(C-IF)}(X) = \beta_M^C(X)$ ;  $\beta_M^{(C-PCF)}(X) \neq \beta_M^C(X)$ .

**Step 2.** If  $N = C - \{OF, B, IF\} = \{F\&V, PCF\}$ , then  $\beta_M^N(X) = \beta_M^C(X)$ . From step 1, we get,

i)  $U/R^{(C-\{F\&V\})} = \{\{P_1, P_4, P_5\}, \{P_2\}, \{P_3, P_6\}, \{P_7\}, \{P_8\}\}$ . Then  $\tau_M^{(C-\{F\&V\})}(X) = \{U, \phi, \{P_8\}, \{P_1, P_3, P_4, P_5, P_6, P_8\}, \{P_1, P_3, P_4, P_5, P_6\}, \{P_4, P_5, P_6\}, \{P_1, P_3\}, \{P_1, P_3, P_8\}, \{P_4, P_5, P_6, P_8\}\}$  and the milli basis

$$\beta_M^{(C-\{F\&V\})}(X) = \{U, \{P_8\}, \{P_4, P_5, P_6\}, \{P_1, P_3\}\} \neq \beta_M^C(X).$$

ii)  $U/R^{(C-\{PCF\})} = \{\{P_1, P_4\}, \{P_2\}, \{P_3, P_5, P_6\}, \{P_7\}, \{P_8\}\}$ . Then

$$\tau_M^{(C-\{PCF\})}(X) = \{U, \phi, \{P_8\}, \{P_1, P_3, P_4, P_5, P_6, P_8\}, \{P_1, P_3, P_4, P_5, P_6\}, \{P_4, P_5, P_6\}, \{P_1, P_3\}, \{P_1, P_3, P_8\}, \{P_4, P_5, P_6, P_8\}\}$$
 and the milli basis

$$\beta_M^{(C-\{PCF\})}(X) = \{U, \{P_8\}, \{P_4, P_5, P_6\}, \{P_1, P_3\}\} \neq \beta_M^C(X).$$

From (i) and (ii) we get,  $\beta_M^N(X) = \beta_M^C(X)$  and  $\beta_M^{(N-\{x\})}(X) \neq \beta_M^C(X)$  for every  $x$  in  $N$ . Since  $CORE(A) = \{ \text{Fruits and Vegetables, Properly Cooked Foods} \}$ .

**Result.** From the core we conclude that the essential key factors for a healthy life is Fruits and Vegetables and Properly Cooked Foods.

### 6. Conclusion and Future work

One of the recent successful tool to handle uncertainty problems was rough set. It was proposed with the goal of the induction of approximations of concepts, it offers mathematical tools to discover patterns hidden in data. This manuscript had been written to contribute to this field by introducing some kind of approximation spaces generated by “On milli continuous functions and its application”.

Firstly, we define the notion of milli continuous functions with their main properties. Then we compare milli continuous with nano continuous functions. Also, the notion of milli contra continuous, milli perfectly continuous and milli strongly continuous functions were introduced and their properties were investigated. Then the milli contra continuous is compared with nano contra continuous functions and milli perfectly continuous, milli strongly continuous functions are compared with other milli continuous functions. Finally, we define real-life application that were produced by using the concepts of milli basis in milli topological spaces. The essential food habits for a healthy lifestyle were identified. In the future, this work will be extended with some real-life applications by using the corresponding set of milli open in milli topological spaces.

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