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## CONJUGACY CLASS GRAPHS OF SOME K-METACYCLIC GROUPS

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Abstract: A class of K-metacyclic group of order p(p-1) denoted by G, has group presentation  $x^p = y^{p-1} = 1$ ;  $y^{-1}xy = x^r$ ; (r-1,p) = 1 where p is an odd prime and r is a primitive root modulo p. To this group, we attach a simple undirected graph  $\Gamma_G^{cc}$  whose vertices are the conjugacy classes of G and two distinct vertices x and y are connected by an edge if the gcd of the class size of x and y is greater than 1. In this paper,  $\Gamma_G^{cc}$  and  $\Gamma_{G\times G}^{cc}$  are obtained and then different graph theoretic properties like planarity, clique number, chromatic number, independence number, clique polynomial, independence polynomial, dominating number, spectrum and energy of these graphs are studied. The line graph of  $\Gamma_G^{cc}$  is found to be a regular graph and the complement graph of  $\Gamma_G^{cc}$  is found to be a star graph. Various aspects of the line graph and the complement graph are also determined in this paper.

**Keywords and Phrases:** Conjugacy class graph, K-metacyclic group, line graph, complement graph.

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## 1. Introduction

In the study of group theory, various types of graphs such as Cayley graphs, power graphs, commuting graphs etc have been used to investigate the structural properties of groups. Recently, constructing graphs based on the conjugacy classes of groups has become a significant area of research. Among these are the conjugate