

## SOME ASPECTS OF ROUGHLY PRIME SUBMODULES

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**Abstract:** Let  $R$  be a commutative ring with unity and  $M$  be an  $R$ -module. The aim of this article is to introduce and investigate certain properties of a new notion of prime submodules of a module, namely roughly prime submodules. A submodule  $N$  of  $M$  is said to be a roughly prime submodule of  $M$  if for  $rm \in N$ ,  $r \in R$  and  $m \in M$ , implies that either  $m \in N + Z(M)$  or  $r \in (N + Z(M) : M)$  where  $Z(M)$  is the singular submodule of  $M$ . The interaction of this submodule with other classes of modules as well as its characteristics in terms of direct sum, intersection and homomorphic image are studied along with the exploration of its behaviour in quotient modules.

**Keywords and Phrases:** Prime submodules, Singular submodules, Quotient Submodules.

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### 1. Introduction

Let  $M$  be an  $R$ -module. Prime submodules, which came as a generalization of prime ideals of rings was first introduced by Dauns in 1978 [7]. A proper submodule  $N$  of  $M$  is said to be prime if for any  $r \in R$  and  $m \in M$  with  $rm \in N$ , we have  $m \in N$  or  $r \in (N :_R M)$  where  $(N :_R M) = \{x \in R: xM \subseteq N\}$ . This has garnered immense attention from researchers which has led to the introduction of several related concepts like fully prime submodules, approximately prime submodules, almost prime submodules etc. A proper submodule  $N$  of  $M$  is said to be semiprime