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CONCIRCULAR CURVATURE TENSOR ON ALMOST KENMOTSU MANIFOLDS ADMITTING THE NULLITY DISTRIBUTIONS

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Abstract: The object of the present paper is to study almost Kenmotsu manifolds with characteristic vector field ξ belonging to some nullity distributions, considering concircular curvature tensor. Locally ϕ-concircularly symmetric almost Kenmotsu manifolds, concircularly ϕ-recurrent almost Kenmotsu manifolds and locally concircularly ϕ-recurrent three-dimensional almost Kenmotsu manifolds are studied. And we have obtained some interesting results.

Keywords and Phrases: Almost Kenmotsu manifolds, Nullity distributions, Concircular curvature tensor.

2020 Mathematics Subject Classification: 53C25, 53D15.

1. Introduction

Nowadays the study of nullity distributions occupies an important position in differential geometry. In 1966, Gray [10] introduced the notion of k-nullity distribution and later studied by Tanno [18] on a Riemannian manifold (M, g), and is defined for any $p \in M$ and $k \in \Re$ as follows:

$$N_p(k) = \{Z \in T_pM : R(X,Y)Z = k[g(Y,Z)X - g(X,Z)Y]\}, \tag{1.1}$$

for any $X, Y \in T_pM$, where T_pM denotes the tangent vector space of M at any point $p \in M$ and R denotes the Riemannian curvature tensor of type $(1,3)$. Moreover, if k is a smooth function then the distribution is called generalized k-nullity distribution.

Later, Blair, Koufogiorgos and Papantoniou [4] introduced a generalized notion of k-nullity distribution called (k,μ)-nullity distribution on a contact metric manifold M^{2n+1}, and is defined for any $p \in M^{2n+1}$ and $(k,\mu) \in \Re^2$ as follows:

$$\begin{aligned} N_p(k,\mu) = \{Z \in T_pM^{2n+1} : R(X,Y)Z = k[g(Y,Z)X - g(X,Z)Y] \\ + \mu[g(Y,Z)hX - g(X,Z)hY]\}, \end{aligned} \tag{1.2}$$

where $h = \frac{1}{2}\pounds_\xi \phi$ and $\pounds$ denotes the Lie differentiation.

A new class of almost contact metric manifolds, called Kenmotsu manifolds, have been introduced and studied by Kenmotsu in 1972 [11]. An almost contact metric manifold M^{2n+1} with 1-form η and fundamental 2-form Φ defined by $\Phi(X,Y) = g(X,\phi Y)$, where ϕ is a $(1,1)$ tensor field such that $d\eta = 0$ and $d\Phi = 2\eta \wedge \Phi$ is called almost Kenmotsu manifold. The normality of an almost contact metric manifold is given by vanishing the $(1,2)$-type torsion tensor $N = [\phi,\phi] + 2d\eta \otimes \xi$, where $[\phi,\phi]$ is the Nijenhuis torsion of ϕ [3]. According to [11], the normality of an almost Kenmotsu manifold is given by

$$(\nabla_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X, \tag{1.3}$$

for any vector fields X, Y on M^{2n+1}.

Dileo and Pastore [8] introduced another generalized notion of the k-nullity distribution called $(k,\mu)'$-nullity distribution on an almost Kenmotsu manifold M^{2n+1} and is defined for any $p \in M^{2n+1}$ and $(k,\mu) \in \Re^2$ as follows:

$$\begin{aligned} N_p(k,\mu)' = \{Z \in T_pM^{2n+1} : R(X,Y)Z = k[g(Y,Z)X - g(X,Z)Y] \\ + \mu[g(Y,Z)h'X - g(X,Z)h'Y]\}, \end{aligned} \tag{1.4}$$

where $h' = h \circ \phi$. Recently, Dileo et al. ([8], [9]), Wang et al. ([21], [22], [23], [24]) and De et al. [12] obtained some important results on almost Kenmotsu manifolds with characteristic vector field ξ belonging to some nullity distributions. In [6], De and Mandal proved some interesting results on locally ϕ-conformally symmetric almost Kenmotsu manifolds. In this paper, we study almost Kenmotsu manifolds with concircular curvature tensor C given by [25]

$$C(X,Y)Z = R(X,Y)Z - \frac{r}{2n(2n+1)}\Big[g(Y,Z)X - g(X,Z)Y\Big], \tag{1.5}$$

where X, Y, Z are any vector fields and r is the scalar curvature.

The concircular curvature tensor has a lot of importance in differential geometry. Several researchers have made a remarkable contribution to its study. From the Riemannian viewpoint, the concircular curvature tensor is the most significant curvature tensor of type $(1,3)$. Blair et al. [5] have studied the concircular curvature tensor on $N(\kappa)$-contact metric manifolds. Singh and Kothari [16] have studied the Tachibana concircular curvature tensor equivalent to the concircular curvature tensor of the Riemannian space. In [19], classification of (κ,μ)-manifolds is discussed, and they [19] considered that concircular curvature tensor Z satisfies the equation $Z(\xi,X)\cdot S = 0$, where S denotes the Ricci tensor. Generalized Sasakian space forms are studied in [2] considering certain conditions on the concircular curvature tensor. Özgür and Tripathi [13] have discussed the concircular curvature tensor on $N(k)$-quasi Einstein manifolds and obtained a necessary and sufficient condition for an $N(k)$-quasi Einstein manifold to satisfy the condition $R(\xi,X)\cdot Z = 0$, where R and Z denote, respectively, Riemannian curvature tensor and concircular curvature tensor. The classification of P-Sasakian manifolds is studied in [14] based on certain conditions satisfied by the concircular curvature tensor. In [1], perfect fluid space-times are studied considering vanishing concircular curvature tensor. The present paper deals with the concircular curvature tensor on almost Kenmotsu manifolds.

The paper is organized as follows: In Section 2, we give some basic formulas and properties of almost Kenmotsu manifolds. In Section 3, we study locally ϕ-concircularly symmetric almost Kenmotsu manifolds with ξ belonging to some nullity distributions. Section 4 is concerned with the study of concircularly ϕ-recurrent almost Kenmotsu manifolds. Section 5 deals with the locally concircularly ϕ-recurrent three-dimensional almost Kenmotsu manifolds. Some important conclusions are summarized in Section 6.

2. Almost Kenmotsu manifolds and nullity distributions

Let M^{2n+1} be an almost Kenmotsu manifold with structure (ϕ,ξ,η,g), where ϕ is a $(1,1)$ tensor field, ξ a characteristic vector field, η a 1-form and g a Riemannian metric such that [3]

$$\phi^2(X) = -X + \eta(X)\xi, \quad \eta(\xi) = 1, \quad \eta\circ\phi = 0, \quad \phi\xi = 0, \quad g(X,\xi) = \eta(X), \tag{2.1}$$

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y), \tag{2.2}$$

for all vector fields X, Y on M^{2n+1}. Let D be the distribution orthogonal to ξ and defined by $D = \mathrm{Ker}(\eta) = \mathrm{Im}(\phi)$. The two tensor fields $h = \frac{1}{2}\pounds_\xi \phi$ and $l = R(\cdot, \xi)\xi$ on an almost Kenmotsu manifold M^{2n+1} are symmetric and satisfy the following relations [15]

$$h\xi = 0, \; l\xi = 0, \; \mathrm{tr}(h\phi) = 0, \; h\phi + \phi h = 0, \tag{2.3}$$

$$\nabla_X \xi = -\phi^2 X - \phi h X, \tag{2.4}$$

$$\phi l \phi - l = 2(h^2 - \phi^2), \tag{2.5}$$

$$\mathrm{tr}(l) = S(\xi, \xi) = g(Q\xi, \xi) = -2n - \mathrm{tr}\, h^2, \tag{2.6}$$

$$R(X, Y)\xi = \eta(X)(Y - \phi h Y) - \eta(Y)(X - \phi h X) + (\nabla_Y \phi h)X - (\nabla_X \phi h)Y, \tag{2.7}$$

for all vector fields X, Y on M^{2n+1}.

Now we give some basic properties on almost Kenmotsu manifolds with ξ belonging to the $(k, \mu)'$-nullity distribution. The $(1, 1)$-type tensor field h' satisfies $h'\phi + \phi h' = 0$ and $h'\xi = 0$. Also it is known that

$$h = 0 \Leftrightarrow h' = 0, \quad h'^2 = (k+1)\phi^2 \;\; (\Leftrightarrow h^2 = (k+1)\phi^2). \tag{2.8}$$

For an almost Kenmotsu manifold, we have from (1.4)

$$R(X, Y)\xi = k[\eta(Y)X - \eta(X)Y] + \mu[\eta(Y)h'X - \eta(X)h'Y], \tag{2.9}$$

$$R(\xi, X)Y = k[g(X, Y)\xi - \eta(Y)X] + \mu[g(h'X, Y)\xi - \eta(Y)h'X], \tag{2.10}$$

where $k, \mu \in \Re$. Contracting Y in (2.10), we get

$$S(X, \xi) = 2k\eta(X). \tag{2.11}$$

Let $X \in D$ be the eigenvector of h' corresponding to the eigenvalue λ and orthogonal to ξ. It follows from (2.8) that $\lambda^2 = -(k+1)$, a constant. Therefore, $k \leq -1$ and $\lambda = \pm\sqrt{-k-1}$. We denote $[\lambda]'$ and $[-\lambda]'$ as the corresponding eigenspaces associated with h' corresponding to the non-zero eigenvalues λ and $-\lambda$ respectively. We have the following lemmas.

Lemma 2.1. ([[8], Proposition 4.1]) *Let $(M^{2n+1}, \phi, \xi, \eta, g)$ be an almost Kenmotsu manifold such that ξ belongs to the $(k, \mu)'$-nullity distribution and $h' \neq 0$. Then $k < -1, \mu = -2$ and Spec$(h') = \{0, \lambda, -\lambda\}$ with 0 as simple eigenvalue and $\lambda = \sqrt{-k-1}$. The distributions $[\xi] \oplus [\lambda]'$ and $[\xi] \oplus [-\lambda]'$ are integrable with totally geodesic leaves. The distributions $[\lambda]'$ and $[-\lambda]'$ are integrable with totally umbilical* *leaves.*

Lemma 2.2. ([[8], Lemma 4.1]) *Let $(M^{2n+1}, \phi, \xi, \eta, g)$ be an almost Kenmotsu manifold with $h' \neq 0$ and ξ belongs to the $(k, -2)'$-nullity distribution. Then for every $X, Y \in T_pM$,*

$$(\nabla_X h')Y = -g(h'X + h'^2X, Y)\xi - \eta(Y)(h'X + h'^2X). \tag{2.12}$$

According to Takahashi [17] and De et al. [7], we have the following definitions:

Definition 2.1. *An almost Kenmotsu manifold is said to be ϕ-symmetric if it satisfies*

$$\phi^2((\nabla_W R)(X, Y)Z) = 0, \tag{2.13}$$

for all vector fields $W, X, Y, Z \in T_pM$. In addition, if the vector fields W, X, Y, Z are orthogonal to ξ, then the manifold is called locally ϕ-symmetric.

Definition 2.2. *An almost Kenmotsu manifold is said to be ϕ-recurrent if it satisfies*

$$\phi^2((\nabla_W R)(X, Y)Z) = A(W)R(X, Y)Z, \tag{2.14}$$

for all vector fields $W, X, Y, Z \in T_pM$. In (2.14), if the vector fields W, X, Y, Z are orthogonal to ξ, then the manifold is called locally ϕ-recurrent.

3. Locally ϕ-concircularly symmetric almost Kenmotsu manifolds

Consider a locally ϕ-concircularly symmetric almost Kenmotsu manifold with ξ belongs to $(k, \mu)'$-nullity distribution. Then we have

$$\phi^2((\nabla_W C)(X, Y)Z) = 0, \tag{3.1}$$

for all vector fields X, Y, Z, W orthogonal to ξ.
Putting $Z = \xi$ in (3.1), we get

$$\phi^2((\nabla_W C)(X, Y)\xi) = 0. \tag{3.2}$$

By the influence of (1.5) and (2.9) in (3.2), we obtain

$$\begin{aligned} 0 = \frac{(k+1)2n}{2n+1}&[(\nabla_W \eta)(Y)\phi^2 X - (\nabla_W \eta)(X)\phi^2 Y] + \mu[(\nabla_W \eta)(Y)\phi^2(h'X) \\ &+ \eta(Y)\phi^2((\nabla_W h')X) - (\nabla_W \eta)X\phi^2(h'Y) - \eta(X)\phi^2((\nabla_W h')Y)]. \end{aligned} \tag{3.3}$$

Using (2.1), (2.4) and (2.12) in (3.3), we get

$$\begin{aligned} 0 = \frac{(k+1)2n}{2n+1}&[\{g(Y, W) + g(h'W, Y)\}(-X) + \{g(X, W) + g(h'W, X)\}Y] \\ &+ \mu[(-h'X)\{g(Y, W) + g(h'W, Y)\} + h'Y\{g(X, W) + g(h'W, X)\}]. \end{aligned} \tag{3.4}$$

Making use of proposition (4.1) from [10], we get

$$0 = \frac{(k+1)2n}{2n+1}[\{g(Y,W) + g(h'W,Y)\}(-X) + \{g(X,W) + g(h'W,X)\}Y] \\ - 2[(-h'X)\{g(Y,W) + g(h'W,Y)\} + h'Y\{g(X,W) + g(h'W,X)\}]. \quad (3.5)$$

Letting $X, Y, W \in [-\lambda]'$ in (3.5), we have

$$[\frac{(k+1)2n}{2n+1} + 2\lambda](1-\lambda)[g(X,W)Y - g(Y,W)X] = 0. \quad (3.6)$$

Again from proposition (4.1) of [10], we have $k < -1$ and hence $\lambda > 0$. Therefore from (3.6), we get $\lambda = 1$ and $k = -2$. Hence from theorem (4.1) of [6], one can state the following:

Theorem 3.1. *A locally ϕ-concircularly symmetric $(2n+1)$-dimensional almost Kenmotsu manifold $(M^{2n+1}, \phi, \xi, \eta, g)(n > 1)$ with the characteristic vector field ξ belonging to the $(k,\mu)'$-nullity distribution and $h' \neq 0$ is locally isometric to the Riemannian product of an $(n+1)$-dimensional manifold of constant sectional curvature -4 and a flat n-dimensional manifold.*

Now we consider locally ϕ-concircularly symmetric $(2n+1)$-dimensional almost Kenmotsu manifold $(M^{2n+1}, \phi, \xi, \eta, g)$ $(n > 1)$ with the characteristic vector field ξ belonging to the (k,μ)-nullity distribution. Then we have

$$\phi^2((\nabla_W C)(X,Y)Z) = 0, \quad (3.7)$$

for any vector fields X, Y, Z, W orthogonal to ξ.
Putting $X = \xi$ in (3.7), we get

$$\phi^2((\nabla_W C)(\xi,Y)Z) = 0. \quad (3.8)$$

Using (1.5), (2.4) and (2.1) in (3.8), we obtain

$$0 = \frac{dr(W)}{2n(2n+1)}[\{-Y + \eta(Y)\xi\}]\eta(Z) + [1 + \frac{r}{2n(2n+1)}][\{g(Z,W) \\ - \eta(Z)\eta(W)\}\{-Y + \eta(Y)\xi\} - g(Y,Z)\{-W + \eta(W)\xi\}]. \quad (3.9)$$

Since Y, Z, W are orthogonal to ξ, we have from (3.9) that

$$[1 + \frac{r}{2n(2n+1)}][g(Y,Z)W - g(Z,W)Y] = 0. \quad (3.10)$$

This implies that

$$r = -2n(2n+1). \tag{3.11}$$

Thus we can state the following:

Theorem 3.2. *Let $M^{2n+1}(n>1)$ be a locally ϕ-concircularly symmetric almost Kenmotsu manifold with the characteristic vector field ξ belonging to the (k,μ)-nullity distribution. Then the scalar curvature of M^{2n+1} is $-2n(2n+1)$.*

4. Concircularly ϕ-recurrent almost Kenmotsu manifolds

Definition 4.1. *An almost Kenmotsu manifold is concircularly ϕ-recurrent if there exists a non-zero 1-form A such that*

$$\phi^2((\nabla_W C)(X,Y)Z) = A(W)C(X,Y)Z, \tag{4.1}$$

for all vector fields X, Y, Z, W.

Now consider a concircularly ϕ-recurrent almost Kenmotsu manifold. Then using (2.1) in (4.1), we have

$$-(\nabla_W C)(X,Y)Z + \eta((\nabla_W C)(X,Y)Z)\xi = A(W)C(X,Y)Z. \tag{4.2}$$

Taking innerproduct of (4.2) with U and in view of (1.5), we have

$$\begin{aligned}
&-g((\nabla_W R)(X,Y)Z,U) + \eta((\nabla_W R)(X,Y)Z)\eta(U) + \frac{dr(W)}{2n(2n+1)} \\
&[g(Y,Z)g(X,U) - g(X,Z)g(Y,U) + g(X,Z)\eta(Y)\eta(U) - g(Y,Z)\eta(X)\eta(U)] \\
&\quad = A(W)\{g(R(X,Y)Z,U) - \frac{r}{2n(2n+1)}[g(Y,Z)g(X,U) - g(X,Z)g(Y,U)]\}.
\end{aligned} \tag{4.3}$$

Putting $X = U = e_i$ in (4.3), where e_i, $i = 1, 2, ...2n+1$ is an orthonormal basis of the tangent space at each point of the manifold and taking summation over i, we get

$$\begin{aligned}
&-(\nabla_W S)(Y,Z) + \eta((\nabla_W R)(\xi,Y)Z) + \frac{dr(W)}{2n(2n+1)}[(2n-1)g(Y,Z) + \eta(Y)\eta(Z)] \\
&\quad = A(W)[S(Y,Z) - \frac{r}{2n+1}g(Y,Z)].
\end{aligned} \tag{4.4}$$

Setting $Y = Z = \xi$ in (4.4), we obtain

$$(\nabla_W S)(\xi,\xi) = \frac{dr(W)}{2n+1} - A(W)[S(\xi,\xi) - \frac{r}{2n+1}]. \tag{4.5}$$

Using lemma (3) of [23] in (4.5), we get

$$4n^2(k+1)A(W) = dr(W). \tag{4.6}$$

If r is a constant, then we have $A(W) = 0$ or $k = -1$. Further if $k = -1$, then $h' = 0$ and $h = 0$. This is contradiction to the assumption that $h' \neq 0, h \neq 0$. Therefore we can state the following:

Theorem 4.1. *A concircularly ϕ-recurrent almost Kenmotsu manifold M^{2n+1} with the characteristic vector field ξ belonging to the $(k,\mu)'$-nullity distribution is concircularly ϕ-symmetric provided the scalar curvature r is a constant.*

5. Locally concircularly ϕ-recurrent three-dimensional almost Kenmotsu manifolds

Definition 5.1. *Concircular curvature tensor C on a three-dimensional almost Kenmotsu manifold is given by*

$$C(X,Y)Z = R(X,Y)Z - \frac{r}{6}[g(Y,Z)X - g(X,Z)Y] \tag{5.1}$$

A three-dimensional almost Kenmotsu manifold is said to be locally concircularly ϕ-recurrent if

$$\phi^2((\nabla_W C)(X,Y)Z) = A(W)C(X,Y)Z. \tag{5.2}$$

For a three-dimensional Riemannian manifold, we have [20]

$$\begin{aligned} R(X,Y)Z = {} & S(Y,Z)X - S(X,Z)Y + g(Y,Z)QX - g(X,Z)QY \\ & - \frac{r}{2}[g(Y,Z)X - g(X,Z)Y]. \end{aligned} \tag{5.3}$$

For $\xi \in (k,\mu)'$-nullity distribution, equation (5.3) becomes

$$\begin{aligned} R(X,Y)Z = {} & (\frac{r}{2} - 2k)[g(Y,Z)X - g(X,Z)Y] \\ & - (\frac{r}{2} - 3k)[g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi + \eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y] \\ & - 2g(Y,Z)h'X + 2g(X,Z)h'Y - 2g(h'Y,Z)X + 2g(h'X,Z)Y. \end{aligned} \tag{5.4}$$

Taking covariant derivative of (5.4) with respect to W, we get

$$\begin{aligned} (\nabla_W R)(X,Y)Z = {} & \frac{dr(W)}{2}[g(Y,Z)X - g(X,Z)Y - g(Y,Z)\eta(X)\xi - g(X,Z)\eta(Y)\xi \\ & +\eta(Y)\eta(Z)X - \eta(X)\eta(Z)Y] - (\frac{r}{2} - 3k)[g(Y,Z)\{\nabla_W\eta(X)\xi + \eta(X)\nabla_W\xi\} \\ & -g(X,Z)\{(\nabla_W\eta)(Y)\xi + \eta(Y)\nabla_W\xi\} + (\nabla_W\eta)(Y)\eta(Z)X + \eta(Y)(\nabla_W\eta)(Z)Y \\ & -(\nabla_W\eta)(X)\eta(Z)Y - \eta(X)(\nabla_W\eta)(Z)Y] - 2g(Y,Z)(\nabla_W h')(X) \\ & +2g(X,Z)(\nabla_W h')(Y) - 2g((\nabla_W h')Y,Z)X + 2g((\nabla_W h')X,Z)Y. \end{aligned} \tag{5.5}$$

Since X, Y, Z, W are orthogonal to ξ, we have from (5.5) that

$$(\nabla_W R)(X,Y)Z = \frac{dr(W)}{2}[g(Y,Z)X - g(X,Z)Y] - (\frac{r}{2} - 3k)[g(Y,Z)\{g(X,W) + g(X,h'W)\} - g(X,Z)\{g(Y,W) + g(Y,h'W)\}]\xi + 2[g(Y,Z)g(h'W + h'^2W, X) - g(X,Z)g(h'W + h'^2W, Y)]\xi - 2[g((\nabla_W h')Y,Z)X - g((\nabla_W h')X,Z)Y]. \tag{5.6}$$

Applying ϕ^2 on (5.6) and then using (2.1), we get

$$\phi^2((\nabla_W R)(X,Y)Z) = \frac{dr(W)}{2}[g(X,Z)Y - g(Y,Z)X] + 2[g((\nabla_W h')Y,Z)X - g((\nabla_W h')X,Z)Y]. \tag{5.7}$$

Now using (5.1) and (5.7) in (5.2), we obtain

$$A(W)C(X,Y)Z = \frac{dr(W)}{3}[g(X,Z)Y - g(Y,Z)X] + 2[g((\nabla_W h')Y,Z)X - g((\nabla_W h')X,Z)Y]. \tag{5.8}$$

Using lemma (4.1) of [8] in (5.8), we obtain

$$C(X,Y)Z = \frac{dr(W)}{3A(W)}[g(X,Z)Y - g(Y,Z)X]. \tag{5.9}$$

Putting $W = e_i$ in (5.9), where $e_i, i = 1, 2, 3$ is an orthonormal basis of the tangent space at any point of the manifold and taking summation over $i, 1 \leq i \leq 3$, we get

$$C(X,Y)Z = \frac{dr(e_i)}{3A(e_i)}[g(X,Z)Y - g(Y,Z)X]. \tag{5.10}$$

By virtue of (5.10) in (5.1), we have

$$R(X,Y)Z = a[g(X,Z)Y - g(Y,Z)X], \tag{5.11}$$

where $a = \frac{r}{6} - \frac{dr(e_i)}{3A(e_i)}$ is a scalar. By Schur's theorem, a is a constant on M^3. Therefore we can state the following:

Theorem 5.1. *A three-dimensional concircularly ϕ-recurrent almost Kenmotsu manifold with $\xi \in (k, \mu)'$-nullity distribution is of constant sectional curvature.*

6. Conclusions

In this paper, we have studied almost Kenmotsu manifolds considering concircular curvature tensor. We have considered that ξ belongs to two nullity distributions, (k, μ)-nullity distribution and $(k, \mu)'$-nullity distribution. Locally ϕ-concircularly symmetric almost Kenmotsu manifolds, taking ξ belongs to (k, μ) and $(k, \mu)'$-nullity distributions and $(1, 1)$-type tensor field $h' \neq 0$, are discussed. We proved that a concircularly ϕ-recurrent almost Kenmotsu manifold becomes concircularly ϕ-symmetric if r is a constant and ξ belongs to the $(k, \mu)'$-nullity distribution. Further, we have examined three-dimensional concircularly ϕ-recurrent almost Kenmotsu manifolds taking ξ belongs to the $(k, \mu)'$-nullity distribution and obtained an interesting result.

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