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A STUDY ON MINIMAL PRE-γ-OPEN SETS AND FINITE PRE-γ-OPEN SETS

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Abstract: The aim of this paper is to investigate some new characterizations of minimal pre-γ-open sets in a topological space. We have introduced and investigated finite pre-γ-open sets and pre-γ-locally finite spaces. Moreover relationships between finite and minimal pre-γ-open sets are obtained.

Keywords and Phrases: Pre-γ-open sets, minimal pre-γ-open sets, finite pre-γ-open sets, pre-γ-locally finite space, pre-γ-regular.

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1. Introduction and Preliminaries

The notion of an operation γ was initiated by S. Kasahara [4] in 1979. Using this operation, H. Ogata [5] introduced the concept of γ-open sets. Hariwan Z. Ibrahim [1, 2] studied the pre-γ-open sets in topological spaces. Later, Vadivel and Sivashanmugaraja investigated the concept of pre-γ-open sets in their papers [7, 8, 9]. The purpose of this paper is to study some characterizations of minimal pre-γ-open sets, finite pre-γ-open sets and pre-γ-locally finite space. Throughout the present paper, the space (X, τ) or X always mean topological space.

Definition 1.1. [5] *Let X be a space. An operation γ on τ is a mapping $\gamma : \tau \to P(X)$ such that $V \subseteq \gamma(U)$ $\forall$ $U \in \tau$, where $\gamma(U)$ means the value of γ at U.*

Definition 1.2. [5] *A subset λ of X is said to be γ-open if $\forall$ $x \in \lambda$, $\exists$ an open set U such that $x \in U$ and $\gamma(U) \subseteq \lambda$. The set of all γ-open sets in X is denoted by τ_γ.*

Definition 1.3. *A subset λ of X is said to be*

(i) pre-γ-open [1] *(in short, p_γ-open) if $\lambda \subseteq \tau_\gamma$-$int(cl(\lambda))$.*

(ii) pre-γ-closed [2] *(in short, p_γ-closed) iff $X - \lambda$ is p_γ-open.*

Moreover, p_γ-$O(X)$ denotes the collection of all pre-γ-open sets of (X, τ). p_γ-$C(X)$ denotes the collection of all pre-γ-closed sets of (X, τ).

Definition 1.4. [2] *A subset λ of X. Then*

(i) The intersection of p_γ-$C(X)$ containing λ is said to be pre-γ-closure of λ and is mentioned by $p_\gamma cl(\lambda)$.

(ii) The union of p_γ-$O(X)$ is contained in λ is said to be pre-γ-interior of λ and is mentioned by $p_\gamma int(\lambda)$.

Definition 1.5. [2] *An $N \subseteq X$ is said to be pre-γ-neighborhood (in short, pre-γ-nbd) of a point $p \in X$ if $\exists$ a p_γ-open set W such that $p \in W \subseteq N$.*

Definition 1.6. [6] *Let X be a space and γ be an operation on τ. A subset λ of X is said to be:*

(1) γ-pre-open (in short, γ-po) if $\lambda \subseteq \tau_\gamma$-$int(\tau_\gamma$-$cl(\lambda))$.

(2) γ-pre-closed (in short, γ-pc) if τ_γ-$cl(\tau_\gamma$-$int(\lambda)) \subseteq \lambda$.

Moreover, τ_γ-POX denotes the collection of all γ-pre-open sets of (X, τ) and τ_γ-PCX denotes the collection of all γ-pre-closed sets of (X, τ).

Definition 1.7. [1] *A space X is said to be pre-γ-T_2-space if $\forall$ two distinct points x, y of X, $\exists$ two disjoint p_γ-open sets P, Q such that $x \in P$ and $y \in Q$.*

Definition 1.8. [3] *Let X be a space and $\lambda \subseteq X$ a γ-open set, then λ is said to be a minimal γ-open set (in short, m_γ-open) if only $\emptyset, \lambda \subseteq \tau_\gamma$.*

2. Minimal Pre-γ-open Sets

In this part, we introduced the concept of pre-γ-regular and minimal pre-γ-open sets. Further characteristics of minimal pre-γ-open sets are studied.

Definition 2.1. *Let X be a space and λ be a p_γ-open set in X. Then λ is called minimal pre-γ-open set (in short, mp_γ-open) if λ and $\emptyset$ are the only p_γ-open subsets of λ. The family of all mp_γ-open sets in X is denoted by $mp_\gamma O(X)$.*

Example 2.2. Let $X = \{\lambda, \mu, \nu\}$ with $\tau = \{X,\ \phi,\ \{\lambda\},\ \{\lambda,\ \nu\}\}$. Define an operation γ on τ by

$$\gamma(C) = \begin{cases} int(cl(C)), & \text{if } C \neq \{\lambda\} \\ C, & \text{if } C = \{\lambda\}. \end{cases}$$

Here, the subset $\{\lambda\}$ is a m_γ-open and also a mp_γ-open.

Example 2.3. Let $X = \{\lambda, \mu, \nu\}$ with $\tau = \{X,\ \phi,\ \{\lambda,\ \mu\}\}$. Define $\gamma(C) = C$. The subset $\{\lambda,\ \mu\}$ is m_γ-open but not mp_γ-open.

Example 2.4. In the above example 2.3, the subset $\{\lambda\}$ is mp_γ-open but not m_γ-open set.

From the above examples 2.2, 2.3 and 2.4 proves that m_γ-open and mp_γ-open sets are independent.

Definition 2.5. *An operation γ on p_γ-$O(X)$ is called pre-γ-regular (in short, p_γ-regular), if $\forall$ p_γ-open sets λ and μ of each $x \in X$, $\exists$ a p_γ-open set ν of x such that $\gamma(\lambda) \cap \gamma(\mu) \supseteq \gamma(\nu)$.*

Proposition 2.6. *In (X, τ),*

(1) If $\lambda \subseteq mp_\gamma$-$O(X)$ and μ a p_γ-open set, then $\lambda \cap \mu = \emptyset$ or $\lambda \subseteq \mu$, where γ is p_γ-regular;

(2) If μ and ν be two mp_γ-open sets, then $\mu \cap \nu = \emptyset$ or $\mu = \nu$, where γ is p_γ-regular.

Lemma 2.7. *Let $\lambda \subseteq mp_\gamma$-$O(X)$. If $x \in \lambda$, then for any p_γ-open nbd μ of x, $\lambda \subseteq \mu$, where γ is p_γ-regular.*

Proof. Assume on the contrary that μ is a p_γ-open nbd of $x \in \lambda$ such that $\lambda \nsubseteq \mu$. Since γ is a p_γ-regular, $\lambda \cap \mu$ is a p_γ-open set with $\lambda \cap \mu \subseteq \lambda$ and $\lambda \cap \mu \neq \emptyset$. This shows a contradiction to our hypothesis that λ is a mp_γ-open set.

Proposition 2.8. *Let $\lambda \subseteq mp_\gamma$-$O(X)$. Then for any $x \in \lambda$, $\lambda = \cap\{\mu : \mu$ is p_γ-open nbd of $x\}$, where γ is p_γ-regular.*

Proposition 2.9. *Let $\lambda \subseteq mp_\gamma$-$O(X)$ such that $x \notin \lambda$. Then $\nu \cap \lambda = \emptyset$ or $\lambda \subseteq \nu$, for any p_γ-open nbd ν of x.*

Corollary 2.10. *Let $\lambda \subseteq mp_\gamma$-$O(X)$ such that $x \notin \lambda$. If $\lambda_x = \cap\{\mu : \mu$ is p_γ-open nbd of $x\}$. Then $\lambda_x \cap \lambda = \emptyset$ or $\lambda \subseteq \lambda_x$, where γ is p_γ-regular.*

Corollary 2.11. *Let $\Gamma(X)$ be the class of monotone operators of X such that $\gamma \in \Gamma(X)$. If $\lambda(\neq \emptyset) \subseteq mp_\gamma$-$O(X)$, then for a subset $\nu(\neq \emptyset)$ of λ, $\lambda \subseteq pcl_\gamma(\nu)$, where γ is p_γ-regular.*

Proof. Let $\nu(\neq \emptyset) \subseteq \lambda$. Let $x \in \lambda$ and μ be any p_γ-nbd μ of x. By Proposition 2.8, we get $\lambda \subseteq \mu$. Since γ is monotone, $\nu = \gamma(\lambda) \cap \nu \subseteq \gamma(\mu) \cap \nu$. So, we get $\gamma(\mu) \cap \nu \neq \emptyset$ and therefore $x \in pcl_\gamma(\nu)$. This gives that $\lambda \subseteq pcl_\gamma(\nu)$.

Proposition 2.12. *Let $\lambda(\neq \emptyset) \subseteq p_\gamma$-$O(X)$. If $\lambda \subseteq pcl_\gamma(\nu)$, then $pcl_\gamma(\lambda) = pcl_\gamma(\nu)$, for any subset $\nu(\neq \emptyset)$ of λ.*

Proof. Since for any $\nu(\neq \emptyset) \subseteq \lambda$, $pcl_\gamma(\nu) \subseteq pcl_\gamma(\lambda)$. On the other hand, by assumption we get, $pcl_\gamma(\lambda) \subseteq pcl_\gamma(pcl_\gamma(\nu)) = pcl_\gamma(\nu)$ implies $pcl_\gamma(\lambda) \subseteq pcl_\gamma(\nu)$.

Proposition 2.13. *Let $\lambda(\neq \emptyset) \subseteq p_\gamma$-$O(X)$. If $pcl_\gamma(\lambda) = pcl_\gamma(\nu)$, for any subset $\nu(\neq \emptyset)$ of λ, then λ is a mp_γ-open set.*

Proof. We assume on the contrary that λ is not a mp_γ-open set. Then $\exists$ a p_γ-open set $\eta(\neq \emptyset)$ such that $\lambda \supseteq \eta$ and therefore $\exists$ $x \in \lambda$ such that $x \notin \eta$. Thus we get $pcl_\gamma(\{x\}) \subseteq X \setminus \eta$ gives that $pcl_\gamma(\{x\}) \neq pcl_\gamma(\lambda)$. Hence the proof.

From the Propositions 2.9, 2.12 and 2.13, we get:

Theorem 2.14. *Let $\lambda(\neq \emptyset)$ be any p_γ-open set of X and $\gamma \in \Gamma(X)$. Then the below statements are equivalent:*

(1) λ is mp_γ-open set, where γ is p_γ-regular;

(2) $\lambda \subseteq pcl_\gamma(\lambda)$, for any $\nu(\neq \emptyset) \subseteq \lambda$;

(3) $pcl_\gamma(\lambda) = pcl_\gamma(\nu)$, for any subset $\nu(\neq \emptyset) \subseteq \lambda$.

Lemma 2.15. *If $\lambda(\subseteq X)$ is a mp_γ-$O(X)$, where $\gamma \in \Gamma(X)$, then any $\nu(\neq \emptyset) \subseteq \lambda$ is a γ-po-set, where γ is p_γ-regular.*

Proof. Let λ be a mp_γ-open set and $\lambda \supseteq \nu \neq \emptyset$. By Proposition 2.13, we get $\lambda \subseteq pcl_\gamma(\nu)$ implies $pint_\gamma(\lambda) \subseteq pint_\gamma(pcl_\gamma(\nu))$. By hypothesis λ is a p_γ-open set, we get $\nu \subseteq \lambda = pint_\gamma(\lambda) \subseteq pint_\gamma(pcl_\gamma(\nu))$ or $\nu \subseteq pint_\gamma(pcl_\gamma(\nu))$, that is, ν is γ-po-set.

By using Theorem 2.14 (3), we show the following:

Lemma 2.16. *Let $\mu(\neq \emptyset) \subseteq X$. Let λ be a mp_γ-open set and $\gamma \in \Gamma(X)$. If $\exists$ a p_γ-open set ν containing μ such that $\nu \subseteq pcl_\gamma(\mu \cup \lambda)$, then for any subset $\eta(\neq \emptyset)$*

of λ, $\mu \cup \eta$ is a γ-po-set, where γ is p_γ-regular.

Proof. Let λ is a mp_γ-open set. By hypothesis γ is p_γ-regular, so for any subset $\eta(\neq \emptyset)$ of λ, we have $pcl_\gamma(\mu \cup \eta) = pcl_\gamma(\mu) \cup pcl_\gamma(\eta) = pcl_\gamma(\mu) \cup pcl_\gamma(\lambda) = pcl_\gamma(\mu \cup \lambda)$. Since λ is a mp_γ-open set, we get $\nu \subseteq pcl_\gamma(\mu \cup \lambda) = pcl_\gamma(\mu \cup \eta)$ implies $pint_\gamma(\nu) \subseteq pint_\gamma(pcl_\gamma(\mu \cup \eta))$. Since ν is a p_γ-open set such that $\mu \subseteq \nu$, we get $\mu \subseteq \nu = pint_\gamma(\nu) \subseteq pint_\gamma(pcl_\gamma(\mu \cup \eta))$ or

$$\mu \subseteq pint_\gamma(pcl_\gamma(\mu \cup \eta)) \tag{2.1}$$

and $pint_\gamma(\lambda) = \lambda \subseteq pcl_\gamma(\lambda) \subseteq pcl_\gamma(\mu) \cup pcl_\gamma(\lambda) = pcl_\gamma(\mu \cup \lambda)$ implies

$$pint_\gamma(\lambda) \subseteq pint_\gamma(pcl_\gamma(\mu \cup \lambda)) \tag{2.2}$$

Since λ is a p_γ-open set, we have

$$\eta \subseteq \lambda = pint_\gamma(\lambda) \subseteq pint_\gamma(pcl_\gamma(\mu \cup \lambda)) \subseteq pint_\gamma(pcl_\gamma(\mu \cup \eta)) \tag{2.3}$$

From (2.1) and (2.3), $\mu \cup \eta \subseteq pint_\gamma(pcl_\gamma(\mu \cup \eta))$ gives $\mu \cup \eta$ is a γ-po- set.

Corollary 2.17. *Let $\mu(\neq \emptyset) \subseteq X$ and λ be a mp_γ-open set and $\gamma \in \Gamma(X)$. If $\exists$ p_γ-open set ν containing μ such that $\nu \subseteq pcl_\gamma(\mu \cup \lambda)$, then for any subset $\eta(\neq \emptyset)$ of λ, $\mu \cup \eta$ is a γ-po-set, where γ is p_γ-regular.*

Proof. Let λ be a mp_γ-open set and $\mu \subseteq X$. Suppose $\exists$ a p_γ-open set ν containing μ such that $\nu \subseteq pcl_\gamma(\lambda)$. So we get $\nu \subseteq pcl_\gamma(\mu) \cup pcl_\gamma(\lambda) = pcl_\gamma(\lambda \cup \mu)$. By Theorem 2.16, it follows that for any $\eta(\neq \emptyset) \subseteq \lambda$, $\mu \cup \eta$ is a γ-po- set.

3. Finite Pre-γ-open Sets

Proposition 3.1. *Let μ $(\neq \emptyset)$ be a finite p_γ-open (in short, fp_γ-open set), $\exists$ at least one (finite) mp_γ-open set λ such that $\lambda \subseteq \mu$.*

Proof. Assume that μ is a fp_γ-open set. Then we have two possibilities:
(i) μ is a mp_γ-open set;
(ii) μ is not a mp_γ-open set.
In (i), if we take $\mu = \lambda$, then it is true. If (ii) is true, then $\exists$ a fp_γ-open set $\mu_1(\neq \emptyset) \subset \mu$. If μ_1 is mp_γ-open set, we take $\lambda = \mu_1$. If μ_1 is not a mp_γ -open set, then $\exists$ a fp_γ-open set $\mu_2(\neq \emptyset)$ such that $\mu_2 \subseteq \mu_1 \subseteq \mu$, continuing this process and have a sequence of p_γ-open set of X ... $\subseteq \mu_m \subseteq ... \subseteq \mu_2 \subseteq \mu_1 \subseteq \mu$. By hypothesis μ is a finite, that is, for some $k \in N$, we get a mp_γ-open set, namely μ_k such that $\mu_k = \lambda$.

Moreover, fp_γ-$O(X)$ denotes the family of all finite p_γ-open sets in X.

Definition 3.2. *A space X is called pre-γ-locally finite space (in short, p_γ-lfsp),* *if* $\forall\ x \in X\ \exists$ *a* fp_γ*-open set* λ *in* X *such that* $x \in \lambda$.

Lemma 3.3. *Let* μ *a nonempty* p_γ*-open set in a* p_γ*-lfsp* X, $\exists$ *at least one* mp_γ*-open set* $\lambda \subseteq \mu$, *where pre-*γ *is regular.*

Proof. Suppose $x \in \mu$. By hypothesis X is a p_γ-lfsp, then $\exists$ a fp_γ-open set μ_y such that $x \in \mu_x$. Since $\mu \cap \mu_x$ is a fp_γ-open set, so by proposition 3.1, $\exists$ a mp_γ-open set λ such that $\lambda \subseteq \mu \cap \mu_x \subseteq \mu$. Hence the proof.

Lemma 3.4. *Suppose* X *be a* p_γ*-lfsp and for any* $\alpha \in I$, μ_α *is a* p_γ*-open set and* $\lambda(\neq \emptyset)$ *is a* fp_γ*-open set. Then* $\lambda \cap (\cap_{\alpha\in I}\mu_\alpha)$ *is a* fp_γ*-open set, where* γ *is* p_γ*-regular.*

Proof. Let X is a p_γ-lfsp. Then $\exists\ m \in Z$ such that $\lambda \cap (\bigcap_{\alpha\in I} \mu_\alpha) = \lambda \cap (\bigcap_{i=1}^{m} \mu_i)$. Since γ is p_γ-regular, $\lambda \cap (\bigcap_{\alpha\in I} \mu_\alpha)$ is a fp_γ-open set.

Theorem 3.5. *Let* μ_α *be a* p_γ*-open set, for any* $\beta \in J$, $\lambda_\beta(\neq \emptyset)$ *a* fp_γ*-open set and for any* $\alpha \in I$. *Then* $(\bigcup_{\beta\in J} \lambda_\beta) \cap (\bigcap_{\alpha\in I} \mu_\alpha)$ *is a* p_γ*-open set, where* γ *is* p_γ*-regular.*

4. Further Results

Let λ be a fin-p_γ-open set. It is obvious, by Proposition 2.6 and Lemma 3.4, that if γ is p_γ-regular, then $\exists$ a $p \in N$ such that $mp_\gamma O(\lambda)$ holding that
(1) For any r, q with $1 \leq l,\ q \leq p$ and $l \neq q$, $\lambda_l \cap \lambda_q = \emptyset$.
(2) If ν is a mp_γ-open set in λ, then $\exists\ r$ with $1 \leq r \leq p$ such that $\nu = \lambda_r$.

Theorem 4.1. *Let* $\lambda(\neq \emptyset)$ *be a* fp_γ*-open set such that which is not a* mp_γ*-open set. Let* $mp_\gamma O(\lambda)$, *and* $y \in \lambda$-$(\bigcup mp_\gamma O(\lambda))$. *If* $\lambda_y = \cap\{B : B$ *is a* p_γ*-open nbd of* $y\}$. *Then* $\exists$ *a natural number* $\zeta \in \{1, 2, ..., p\}$ *such that* $\lambda_\zeta \subset \lambda_y$, *where* γ *is* p_γ*-regular.*

Proof. Assume on the contrary that for any natural number $\zeta \in \{1, 2, ..., p\}$, $\lambda_\zeta \not\subseteq \lambda_y$. By Corollary 2.10, for any mp_γ-open set, λ_ζ in λ, $\lambda_\zeta \cap \lambda_y = \emptyset$. By lemma 3.4, $\emptyset \neq \lambda_y$ is a fp_γ-open set. So by Proposition 3.1, $\exists$ a mp_γ-open set ν such that $\nu \subseteq \lambda_y$. Since $\nu \subseteq \lambda_y \subseteq \lambda$, then ν is a mp_γ-open set in λ. By assumption, for any mp_γ-open set λ_ζ, we have $\lambda_\zeta \cap \nu \subseteq \lambda_\zeta \cap \lambda_y = \emptyset$. So $\nu \neq \lambda_\zeta$, for any natural number $\zeta \in \{1, 2, ..., p\}$, which shows a contradiction to our assumption. This completes the proof.

Lemma 4.2. *Let* $\lambda(\neq \emptyset) \subseteq fp_\gamma$*-*$O(X)$ *such that* $\lambda \not\subseteq mp_\gamma$*-*$O(X)$. *Let* $mp_\gamma O(\lambda)$ *and* $x \in \lambda$-$(\bigcup mp_\gamma O(\lambda)$ *and* $x \in \lambda$-$(\bigcup_{i=1}^{p} \lambda_i)$. *Then* $\exists\ \zeta \in \{1, 2, ..., p\}, \zeta \in N$ *such that for any* p_γ*-open nbd* μ_x *of* x, $\lambda_\zeta \subset \mu_x$, *where* γ *is* p_γ*-regular.*

Proof. This proof follows from Theorem 4.1 as $\cap\{\mu : \mu$ is p_γ-open nbd of $x\} \subseteq \mu_x$.

Theorem 4.3. *Let* $\lambda(\neq \emptyset) \subseteq fp_\gamma$*-*$O(X)$ *such that* $\lambda \not\subseteq mp_\gamma$*-*$O(X)$. *Let* $mp_\gamma O(\lambda)$ *and* $y \in \lambda \setminus (\bigcup_{i=1}^{p} \lambda_i)$. *Then* $\exists\ \zeta \in \{1, 2, ..., p\}, \zeta \in N$ *such that* $y \in pcl_\gamma(\lambda_\zeta)$.

Proof. This follows from Proposition 4.1 that $\exists\ \zeta \in \{1, 2, ..., p\}, \zeta \in N$ such that $\lambda_\zeta \subseteq \mu$ for any p_γ-open nbd μ of x. So $\emptyset \neq \lambda_\zeta \cap \lambda_\zeta \subseteq \lambda_\zeta \cap \mu$ gives $y \in pcl_\gamma(\lambda_\zeta)$.

Theorem 4.4. *Let $\lambda(\neq \emptyset) \subseteq fp_\gamma\text{-}O(X)$, $\gamma \in \Gamma(X)$ and $\forall\ \zeta \in \{1, 2, ..., p\}$, λ_ζ a mp_γ-open set in λ. If $mp_\gamma O(\lambda)$ contains all mp_γ-open sets in λ, then for any $\lambda_\zeta \supseteq \mu_\zeta \neq \emptyset$, $pcl_\gamma(\bigcup_{i=1}^{p} \mu_i) \supseteq \lambda$.*

Proof. Let $\lambda(\neq \emptyset) \subseteq fp_\gamma\text{-}O(X)$. We have two cases:

Case 1. If $\lambda \subseteq mp_\gamma\text{-}O(X)$, then this follows from Proposition 2.13

Case 2. If $\lambda \nsubseteq mp_\gamma\text{-}O(X)$, $y \in \lambda \setminus (\bigcup_{i=1}^{p} \lambda_i)$. Thus by Theorem 4.3, it follows that $y \in \bigcup_{i=1}^{p} pcl_\gamma(\lambda_i)$. Therefore by Proposition 2.13, we have $\lambda \subseteq \bigcup_{i=1}^{p} pcl_\gamma(\lambda_i) = \bigcup_{i=1}^{p} pcl_\gamma(\mu_i) = pcl_\gamma(\bigcup_{i=1}^{p} \mu_i)$.

Theorem 4.5. *Let $\lambda(\neq \emptyset) \subseteq fp_\gamma\text{-}O(X)$ and λ_ζ a mp_γ-open set in λ, for each $\zeta \in \{1, 2, ..., p\}$. If for any $\emptyset \neq \mu_\zeta \subseteq \lambda_\zeta$, $\lambda \subseteq pcl_\gamma(\bigcup_{i=1}^{p} \mu_i)$ then $pcl_\gamma(\lambda) = pcl_\gamma(\bigcup_{i=1}^{p} \mu_i)$.*

Proof. For any $\emptyset \neq \mu_\zeta \subseteq \lambda_\zeta, \zeta \in \{1, 2, ..., p\}$, we have $pcl_\gamma(\bigcup_{i=1}^{p} \mu_i) \subseteq pcl_\gamma(\lambda)$. Also, we have $pcl_\gamma(\lambda) \subseteq pcl_\gamma(pcl_\gamma(\bigcup_{i=1}^{p} \mu_i) = pcl_\gamma(\bigcup_{i=1}^{p} \mu_i)$. This implies that for any $\emptyset \neq \mu_\zeta \subseteq \lambda_\zeta$, $pcl_\gamma(\lambda) = pcl_\gamma(\bigcup_{i=1}^{p} \mu_i)$.

Theorem 4.6. *Let $\lambda(\neq \emptyset) \subseteq fp_\gamma\text{-}O(X)$ and $\forall\ \zeta \in \{1, 2, ..., p\}$, λ_ζ a mp_γ-open set in λ. If for any $\emptyset \neq \mu_\zeta \subseteq \lambda_\zeta$, $pcl_\gamma(\lambda) = pcl_\gamma(\bigcup_{i=1}^{p} \mu_i)$, then the family $\{\lambda_i\}$ $(i = 1, 2, 3...p)$ contains all mp_γ-open set of X in λ.*

Proof. Assume on the contrary that $\nu \subseteq mp_\gamma\text{-}O(X)$ in λ and for $\zeta \in \{1, 2, ..., p\}$, $\nu \neq \lambda_i$. So, $\forall\ \zeta \in \{1, 2, ..., p\}$, $\nu \cap pcl_\gamma(\lambda_\zeta) = \emptyset$. This gives that any element of ν is not contained in $pcl_\gamma(\bigcup_{i=1}^{p} \lambda_i)$. This is a contradiction to the fact that $\nu \subseteq \lambda \subseteq pcl_\gamma(\lambda) = pcl_\gamma(\cup_{i=1}^{p} \mu_i)$.

Theorem 4.7. *Let $\lambda(\neq \emptyset) \subseteq fp_\gamma\text{-}O(X)$ and $\lambda_\zeta \subseteq mp_\gamma\text{-}O(X)$ in $\lambda, \forall\ \zeta \in \{1, 2, ..., p\}$, Then the below conditions are equivalent:*

(1) The family $\{\lambda_1, \lambda_2, ..., \lambda_p\}$ contains all mp_γ-open sets in λ;

(2) $pcl_\gamma(\lambda) \subseteq pcl_\gamma(\bigcup_{i=1}^{p} \mu_i)$, for any $\emptyset \neq \mu_\zeta \subseteq \lambda_\zeta$;

(3) $pcl_\gamma(\lambda) = pcl_\gamma(\bigcup_{i=1}^{p} \mu_i)$, where γ is p_γ-regular, for any $\emptyset \neq \mu_\zeta \subseteq \lambda_\zeta$.

Remark 4.8. *Let $\emptyset \neq \lambda$ is a fp_γ-open set and $\{\lambda_1, \lambda_2, ..., \lambda_p\}$ is a family of all mp_γ-open sets in λ such that $\forall\ \zeta \in \{1, 2, ..., p\}$, $x_\zeta \in \lambda_\zeta$. Then by Theorem 4.7, it is obvious that $\{x_1, x_2, ..., x_p\}$ is a γ-po-set.*

Theorem 4.9. *Let $\lambda(\neq \emptyset)$ is a fp_γ-open set and $\{\lambda_1, \lambda_2, ..., \lambda_p\}$ is a family of all mp_γ-open sets in λ. If for any subset μ of $\lambda \setminus \{\lambda_1, \lambda_2, ..., \lambda_p\}$ and $\emptyset \neq \mu_\zeta \subseteq \lambda_\zeta$, $\forall\ \zeta \in \{1, 2, ..., p\}$, then $\bigcup_{i=1}^{p} \mu_i$ is a γ-po-set, where γ is p_γ-regular.*

Proof. Let $\lambda(\neq \emptyset) \subseteq fp_\gamma\text{-}O(X)$ and $\{\lambda_1, \lambda_2, ..., \lambda_p\}$ is a family of all mp_γ-open sets in λ. Then by Proposition 4.4, $\lambda \subseteq pcl_\gamma(\bigcup_{i=1}^{p} \mu_i) \subseteq pcl_\gamma(\bigcup_{i=1}^{p} \mu_i)$. Also, λ is p_γ-open implies $\bigcup_{i=1}^{p} \mu_i \subseteq \lambda = pint_\gamma(\lambda) \subseteq pint_\gamma(pcl_\gamma(\bigcup_{i=1}^{p} \mu_i))$. This follows that

$\bigcup_{i=1}^{p} \mu_i$ is a γ-po-set.

Theorem 4.10. *Let X be a p_γ-lf space and $\gamma \in \Gamma(X)$. If a mp_γ-open set $\lambda \subseteq X$ has more than one element, then X is a pre-γ-T_2 space, where γ is p_γ-regular.*

Proof. Let $x, y \in X$ such that $x \neq y$. Since X is pre-γ-lf space, $\exists$ fp_γ-open sets, $x \in S$ and $y \in T$. By Proposition 3.1 gives that $\exists$ a $mp_\gamma O(S)$ and a $mp_\gamma O(T)$, we have three possibilities:

1. Assume $\exists$ $\zeta \in \{1, 2, ..., p\}$ and $\theta \in \{1, 2, ..., q\}$ such that $x \in S_\zeta$ and $y \in T_\theta$. Then lemma 2.15 gives that $\{x\}$ and $\{y\}$ are disjoint γ-po-sets which contains x and y respectively.

2. Assume $\exists$ $\zeta \in \{1, 2, ..., p\}$ and $\theta \in \{1, 2, ..., q\}$ such that $\lambda \in V_\zeta$ and $y \in W_\theta$. Then by assumption, lemma 2.15 and Theorem 4.9, we can find $\forall$ θ, $y_\theta \in T_i$ such that $\{\lambda\}$ and $\{y, y_1, y_2, ..., y_q\}$ are γ-po-sets and $\{\lambda\} \cap \{y, y_1, y_2, ..., y_q\} = \emptyset$.

3. Assume $\exists$ $\zeta \in \{1, 2, ..., p\}$ and $\theta \in \{1, 2, ..., q\}$ such that $\lambda \in S_\zeta$ and $y \in T_\theta$. Then by assumption and Theorem 4.9, for each ζ and θ, we can find elements $\lambda_\zeta \in S_\zeta$ and $y_\theta \in T_\theta$ such that $\{\lambda, \lambda_1, \lambda_2, ..., \lambda_p\}$ and $\{y, y_1, y_2, ..., y_q\}$ are γ-po-sets and $\{\lambda, \lambda_1, \lambda_2, ..., \lambda_p\} \cap \{y, y_1, y_2, ..., y_q\} = \emptyset$. Thus X is a pre-γ-T_2 space.

5. Conclusion

In this paper, we investigated some new characterizations of minimal pre-γ-open sets. We studied new type of finite open sets called finite pre-γ-open sets in a topological space. This study is also on development of the theory of topological spaces. This study is expected to generate and add new concepts in terms of minimal and finite sets and the ways and means to meet them in a practical way.

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