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UNIFORMLY CONVEX AND STARLIKE PROBABILITY DISTRIBUTION

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Abstract: The purpose of the present paper is to introduce $k-$ uniformly convex and $k-$ uniformly starlike discrete probability distributions and obtain some results regarding moments, factorial moments and moment generating functions for these distributions.

Keywords and Phrases: Probability Distribution, $k-$ uniformly convex function, $k-$ uniformly starlike functions.

2020 Mathematics Subject Classification: 97K50, 30C45.

1. Introduction and Preliminaries

The probability distribution is one of the most important topics in statistics. A theoretical probability distribution is a law according to which distinct values of the random variables are distributed with specified probabilities according to certain rule which can be expressed mathematically. These law can be generate on the basis of given conditions or on the basis of the experimental observations. If a random variable X takes a finite number or countably infinite number of values, then X is called discrete random variable which takes values $x_1, x_2, x_3, \ldots$ with probabilities $p_1, p_2, p_3, \ldots$ and let $p(X = x_i) = p_i$. Then p_i is called the probability mass function if it satisfies the following conditions

(i) $p_i \geq 0$

(ii) $\sum_i p_i = 1$.

Some important examples of the discrete probability distribution are discrete uniform distribution, Bernoulli distribution, Binomial distribution, Poisson distribution, Negative Binomial distribution, Geometric distribution, Hypergeometric distribution, Beta-Binomial distribution and Zeta distribution etc. For detailed study of these probability distributions, one may refer to the following excellent text book by Gupta and Kapoor [6].

Recently, Porwal [10] introduce starlike and convex type probability distribution by using the definition of starlike and convex functions and obtain results regarding moments, factorial moments, mean, variance and moment generating functions. Further, these results were generalized by Porwal and Magesh [11]. These papers establish a co-relation between probability distribution and Geometric Function Theory and opens up a new direction of research in the field of univalent functions. In [10], first author of this paper introduced Starlike and Convex type probability distribution in the following way.

Let $\mathcal{A}$ represent the class of functions f of the form

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n, \tag{1.1}$$

which are analytic in the open unit disk $\mathbb{U} = \{z : z \in \mathbb{C} \text{ and } |z| < 1\}$ and satisfy the normalization condition $f(0) = f'(0) - 1 = 0$. Further, we denote by $\mathcal{S}$ the subclass of $\mathcal{A}$ consisting of functions f of the form (1.1) which are also univalent in $\mathbb{U}$.

A function $f(z)$ of the form (1.1) is said to be starlike of order $\alpha, (0 \leq \alpha < 1)$, if it satisfy the following condition

$$\Re\left\{\frac{zf'(z)}{f(z)}\right\} > \alpha, \qquad z \in \mathbb{U}.$$

Also, a function $f(z)$ of the form (1.1) is said to be convex of order $\alpha, (0 \leq \alpha < 1)$, if it satisfy the following condition

$$\Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} > \alpha, \qquad z \in \mathbb{U}.$$

The classes of all starlike and convex functions of order α are denoted by $S^*(\alpha)$ and $C(\alpha)$, respectively.

For $\alpha = 0$, the classes of $S^*(\alpha)$ and $C(\alpha)$ reduce to the classes of starlike functions S^* and convex functions C, respectively.

The classes of $S^*(\alpha)$, $C(\alpha)$, S^* and C were studied earlier by Robertson [12] and Silverman [14], (see also [3]). Bharti *et al.* [1] introduced the subclasses of k-uniformly convex functions of order α and corresponding class of starlike functions as follows:

A function $f \in \mathcal{A}$ of the form (2) is in $k - UCV(\alpha)$, if and only if

$$Re\left\{1 + \frac{zf''(z)}{f'(z)}\right\} \geq k\left|\frac{zf''(z)}{f'(z)}\right| + \alpha, \tag{1.2}$$

where $0 \leq k < \infty$, $0 \leq \alpha < 1$ and $z \in U$. Using the Alexander transform we can obtain the class $k - S_p(\alpha)$ in the following way $f \in k - UCV(\alpha) \Leftrightarrow zf' \in k - S_p(\alpha)$.

It is worthy to note that for $\alpha = 0$, the classes $k - UCV(\alpha)$ and $k - S_p(\alpha)$ reduce to the classes $k - UCV$ and $k - S_p$ of $\mathcal{S}$ consisting, respectively, of functions which are k-uniformly convex and k-starlike in $\mathbb{U}$. The class $k - UCV$ was introduced by Kanas and Wisniowska [7], where its geometric definition and connections with the conic domains were considered. The class $k - S_p$ was investigated in [8]. In particular, when $k = 1$, we obtain $1 - UCV \equiv UCV$ and $1 - S_p \equiv S_p$, where UCV and S_p are familiar classes of uniformly convex functions and parabolic starlike functions in U, respectively (see, for detailed study Goodman [4], [5], Ma and Minda [9], Rønning [13] and Subramanian et al. [15]), (see also [2]).

To define $k-$ uniformly convex and $k-$ uniformly starlike probability distribution, we need the following lemmas:

Lemma 1.1. [1] *A function $f \in \mathcal{A}$ is in $k - UCV(\alpha)$, if it satisfies the following condition*

$$\sum_{n=2}^{\infty} n[n(1 + k) - (k + \alpha)]|a_n| \leq 1 - \alpha. \tag{1.3}$$

Lemma 1.2. [1] *A function $f \in \mathcal{A}$ is in $k - S_p(\alpha)$, if it satisfies the following inequality*

$$\sum_{n=2}^{\infty}[n(1 + k) - (k + \alpha)]|a_n| \leq 1 - \alpha. \tag{1.4}$$

In 2019, Porwal [10] introduced the starlike and convex type probability distribution by using the definition of starlike and convex function in the following way:

The starlike distribution of order α is defined for a function $f(z)$ of the form (1.1) with condition that $a_n \geq 0$ and satisfy the condition that

$$\sum_{n=2}^{\infty}\left(\frac{n-\alpha}{1-\alpha}\right)a_n = 1. \tag{1.5}$$

The probability mass function of a starlike distribution of order α is as follows

$$p(n) = \begin{cases} 0, & \text{if n} = 0,1 \\ \frac{n-\alpha}{1-\alpha}a_n, & \text{if n} \geqslant 2. \end{cases} \tag{1.6}$$

Since $p(n) \geq 0$ and $\sum_{n=0}^{\infty} p(n) = 1$.

Similarly, the convex distribution of order α associated with the function $f(z)$ of the form (1.1) with condition that $a_n \geq 0$ and satisfy the condition that

$$\sum_{n=2}^{\infty}\left(\frac{n(n-\alpha)}{1-\alpha}\right)a_n = 1. \tag{1.7}$$

The probability mass function of a Convex distribution of order α is as follows:

$$p(n) = \begin{cases} 0, & \text{if n} = 0,1 \\ \frac{n(n-\alpha)}{1-\alpha}a_n, & \text{if n} \geqslant 2. \end{cases} \tag{1.8}$$

Since $p(n) \geq 0$ and $\sum_{n=0}^{\infty} p(n) = 1$.

For $\alpha = 0$, these distributions are called starlike and convex distributions. These distributions are defined for the function $f(z)$ of the form (1.1) satisfying the condition (1.6) and (1.7) with $\alpha = 0$. Therefore, it is natural to ask whether these distributions are defined for the functions satisfying the following coefficient inequalities $\sum_{n=2}^{\infty}\left(\frac{n-\alpha}{1-\alpha}\right)a_n < 1$, and $\sum_{n=2}^{\infty} n\left(\frac{n-\alpha}{1-\alpha}\right)a_n < 1$. In the present paper we attempt to fill this gap, by introducing $k-$uniformly convex type probability distribution of order α associated with the function $f(z)$ of the form (1.1) with condition that $a_n \geq 0$ and satisfy the condition (1.3). The condition (1.3) can be re-written in the following form

$$\sum_{n=2}^{\infty}\left(\frac{n(n(1+k)-(k+\alpha))}{1-\alpha}\right)a_n = 1-\epsilon, \qquad 0 \leq \epsilon < 1. \tag{1.9}$$

The probability mass function of $k-$uniformly convex distribution is defined as

$$p\left(n\right)=\begin{cases} 0, \quad \text{if n} = 0{,}1 \\ \frac{n(n(1+k)-(k+\alpha))}{(1-\epsilon)(1-\alpha)}a_n, \quad \text{if n} \geqslant 2. \end{cases} \tag{1.10}$$

Since $p(n) \geq 0$ and $\sum_{n=0}^{\infty} p(n) = 1$.

Here $p(n)$ is a probability mass function.

Similarly, the probability mass function of $k-$ uniformly starlike distribution is defined as

$$p\left(n\right)=\begin{cases} 0, \quad \text{if n} = 0{,}1 \\ \frac{n(1+k)-(k+\alpha)}{(1-\epsilon)(1-\alpha)}a_n, \quad \text{if n} \geqslant 2. \end{cases} \tag{1.11}$$

By specializing the parameters in these distributions, we have

1. For $k = 0, \epsilon = 0$, then these distributions reduce to the convex and starlike type probability distributions of order α studied by Porwal [10].

2. For $k = 0, \alpha = 0$ and $\epsilon = 0$ then these distributions reduce to the convex and starlike type probability distributions studied by Porwal [10].

In the present paper, we obtain the results regarding moments, factorial moments and moment generating functions. We improve and generalize the results of [10].

Definition 1.1. *If X is a discrete random variable which can take the values x_1, x_2, x_3, ... with respective probabilities p_1, p_2, p_3, ... then expectation of X, denoted by $E(X)$, is defined as*

$$E(X) = \sum_{n=1}^{\infty} p_n x_n. \tag{1.12}$$

Definition 1.2. *The r^{th} moment of a discrete probability distribution about $X = 0$ is defined by*

$$\mu_r^{'} = E(X^r).$$

Definition 1.3. *The r^{th} factorial moment of the discrete probability distribution is defined as*

$$\mu_{(r)}^{'} = \sum_{n=0}^{\infty} n(n-1)\dots(n-r+1)p(n) = \sum_{n=0}^{\infty} n^{(r)}p(n).$$

Definition 1.4. *The moment generating function (m.g.f.) of a random variable X is denoted by $M_X(t)$ and defined by*

$$M_X(t) = E(e^{tX}). \tag{1.13}$$

2. Main Results

Our first theorem gives the first four moments of $k-$ uniformly convex probability distribution about the origin.

Theorem 2.1. *The first four moments $\mu_1^{'}$, $\mu_2^{'}$, $\mu_3^{'}$ and $\mu_4^{'}$ of the $k-$ uniformly convex probability distribution are defined by the relation*

1. $\mu_1^{'} = \frac{1}{(1-\epsilon)(1-\alpha)}\left[(k+1)f^{'''}(1) + (2k+3-\alpha)f^{''}(1) + (1-\alpha)(f'(1)-1)\right].$

2. $\mu_2^{'} = \frac{1}{(1-\epsilon)(1-\alpha)}\left[(k+1)f^{iv}(1) + (5k+6-\alpha)f^{'''}(1) + (4k+7-3\alpha)f^{''}(1) + (1-\alpha)(f'(1)-1)\right].$

3. $\mu_3^{'} = \frac{1}{(1-\epsilon)(1-\alpha)}\left[(k+1)f^{v}(1) + (9k+10-\alpha)f^{iv}(1) + (19k+25-6\alpha)f^{'''}(1) + (8k+15-7\alpha)f^{''}(1) + (1-\alpha)(f'(1)-1)\right].$

4. $\mu_4^{'} = \frac{1}{(1-\epsilon)(1-\alpha)}\left[(k+1)f^{vi}(1) + (14k+15-\alpha)f^{v}(1) + (55k+65-10\alpha)f^{iv}(1) + (65k+90-25\alpha)f^{'''}(1) + (16k+31-15\alpha)f^{''}(1) + (1-\alpha)(f'(1)-1)\right].$

Proof. By using the Definition 1.2, we have

1.

$$\begin{aligned}
\mu_1^{'} &= \sum_{n=0}^{\infty} np(n) \\
&= \sum_{n=2}^{\infty} n.\left(\frac{n(n(1+k)-(k+\alpha))}{(1-\epsilon)(1-\alpha)}\right) a_n \\
&= \frac{1}{(1-\epsilon)(1-\alpha)}\left[(k+1)\sum_{n=2}^{\infty} n(n-1)(n-2)a_n + \right. \\
&\left. (2k+3-\alpha)\sum_{n=2}^{\infty} n(n-1)a_n + (1-\alpha)\sum_{n=2}^{\infty} na_n\right] \\
&= \frac{1}{(1-\epsilon)(1-\alpha)}\left[(k+1)f'''(1) + (2k+3-\alpha)f''(1) + (1-\alpha)\left(f'(1)-1\right)\right].
\end{aligned}$$

2.

$$\mu_2' = \sum_{n=0}^{\infty} n^2 p(n)$$

$$\sum_{n=2}^{\infty} n^2 \left(\frac{n(n(1+k)-(k+\alpha))}{(1-\epsilon)(1-\alpha)} \right) a_n$$

$$= \frac{1}{(1-\epsilon)(1-\alpha)} \left[(k+1) \sum_{n=2}^{\infty} n(n-1)(n-2)(n-3)a_n + \right.$$

$$(5k+6-\alpha) \sum_{n=2}^{\infty} n(n-1)(n-2)a_n +$$

$$\left. (4k+7-3\alpha) \sum_{n=2}^{\infty} n(n-1)a_n + (1-\alpha) \sum_{n=2}^{\infty} na_n \right]$$

$$= \frac{1}{(1-\epsilon)(1-\alpha)} \left[(k+1)f^{iv}(1) + (5k+6-\alpha)f'''(1) + \right.$$

$$\left. (4k+7-3\alpha)f''(1) + (1-\alpha)\left(f'(1)-1\right) \right].$$

3.

$$\mu_3' = \sum_{n=0}^{\infty} n^3 p(n)$$

$$\sum_{n=2}^{\infty} n^3 \left(\frac{n(n(1+k)-(k+\alpha))}{(1-\epsilon)(1-\alpha)} \right) a_n$$

$$= \frac{1}{(1-\epsilon)(1-\alpha)} \left[(k+1) \sum_{n=2}^{\infty} n(n-1)(n-2)(n-3)(n-4)a_n \right.$$

$$+ (9k+10-\alpha) \sum_{n=2}^{\infty} n(n-1)(n-2)(n-3)a_n + (19k+25-6\alpha)$$

$$\left. \sum_{n=2}^{\infty} n(n-1)(n-2)a_n + (8k+15-7\alpha) \sum_{n=2}^{\infty} n(n-1)a_n + (1-\alpha) \sum_{n=2}^{\infty} na_n \right]$$

$$= \frac{1}{(1-\epsilon)(1-\alpha)} \left[(k+1)f^{v}(1) + (9k+10-\alpha)f^{iv}(1) \right.$$

$$\left. + (19k+25-6\alpha)f'''(1) + (8k+15-7\alpha)f''(1) + (1-\alpha)\left(f'(1)-1\right) \right].$$

4.

$$\mu_4^{'} = \sum_{n=0}^{\infty} n^4 p(n)$$

$$\sum_{n=2}^{\infty} n^4 \left(\frac{n(n(1+k)-(k+\alpha))}{(1-\epsilon)(1-\alpha)} \right) a_n$$

$$= \frac{1}{(1-\epsilon)(1-\alpha)} \left[(k+1) \sum_{n=2}^{\infty} n(n-1)(n-2)(n-3)(n-4)(n-5)a_n + \right.$$

$$(14k+15-\alpha) \sum_{n=2}^{\infty} n(n-1)(n-2)(n-3)(n-4)a_n +$$

$$(55k+65-10\alpha) \sum_{n=2}^{\infty} n(n-1)(n-2)(n-3)a_n +$$

$$(65k+90-25\alpha) \sum_{n=2}^{\infty} n(n-1)(n-2)a_n +$$

$$\left. (16k+31-15\alpha) \sum_{n=2}^{\infty} n(n-1)a_n + (1-\alpha) \sum_{n=2}^{\infty} na_n \right]$$

$$= \frac{1}{(1-\epsilon)(1-\alpha)} \left[(k+1)f^{vi}(1) + (14k+15-\alpha)f^{v}(1) \right.$$
$$+ (55k+64-10\alpha)f^{iv}(1) + (65k+90-25\alpha)f'''(1)$$
$$\left. +(16k+31-15\alpha)f''(1) + (1-\alpha)\left(f'(1)-1\right) \right].$$

Example 2.1. The first four moments of the $k-$uniformly convex distribution of order α associated with the function $f(z) = z + \frac{(1-\epsilon)(1-\alpha)}{n[n(k+1)-(k+\alpha)]} z^n$ are

$$\mu_1^{'} = n.$$

$$\mu_2^{'} = n^2.$$

$$\mu_3^{'} = n^3.$$

$$\mu_4^{'} = n^4.$$

Remark 2.1. *If we put $\epsilon = 0, k = 0$ in Theorem 2.1, then we obtain the first four moments of convex distribution of order α studied by Porwal* [10].

Remark 2.2. *If we put $\epsilon = 0, k = 0, \alpha = 0$ in Theorem 2.1, then we obtain the* *first four moments of convex distribution studied by Porwal* [10].

Remark 2.3. *If we put $k = 1, \alpha = 0$ in Theorem 2.1, then we obtain the first four moments for the class uniformly convex functions studied by Subramanian et al.* [15].

Theorem 2.2. *The r^{th} factorial moment of the $k-$ uniformly convex probability distribution is given by the relation*

$$\mu'_{(r)} = \begin{cases} \mu'_1, & \text{if } \ r = 1 \\ \frac{d^r}{dz^r}\left[(k+1)z^2 f''(z) + (1-\alpha)zf'(z)\right]_{(z=1)}, & \text{if } r \geqslant 2 \end{cases}.$$

Proof. By using the Definition 1.3, we have

$$\mu'_{(1)} = \mu'_1,$$

and

$$\begin{aligned} \mu'_{(r)} &= \sum_{n=0}^{\infty} n^{(r)} p(n) \\ &= \sum_{n=0}^{\infty} n^{(r)} n \left(\frac{n(1+k) - (k+\alpha)}{(1-\epsilon)(1-\alpha)} \right) a_n \\ &= \frac{1}{(1-\epsilon)(1-\alpha)} \sum_{n=2}^{\infty} \left[(k+1)n^2 n^{(r)} a_n - (k+\alpha) n n^{(r)} a_n \right] \\ &= \frac{1}{(1-\epsilon)(1-\alpha)} \frac{d^r}{dz^r} \left[(k+1)z^2 f''(z) + (1-\alpha)zf'(z) \right]. \end{aligned}$$

Example 2.2. The r^{th} factorial moment of the $k-$uniformly convex distribution of order α associated with the function $f(z) = z + \frac{(1-\epsilon)(1-\alpha)}{n[n(k+1)-(k+\alpha)]} z^n$ is

$$\mu'_{(1)} = n,$$

and

$$\mu'_{(r)} = \begin{cases} n(n-1)\ldots(n-r+1) & , \text{if } \ r \le n \\ 0, & \text{if } r > n \end{cases}.$$

Theorem 2.3. *The moment generating function of the $k-$uniformly convex distribution of order α is given by*

$$M_X(t) = \frac{1}{(1-\epsilon)(1-\alpha)} \left[(k+1)e^{2t} f''(e^t) + (1-\alpha)\left(e^t f'(e^t) - e^t \right) \right].$$

Proof. By using the Definition 1.4, we have

$$\begin{aligned} M_X(t) &= E(e^{tX}) \\ &= \sum_{n=0}^{\infty} e^{tn} p(n) \\ &= \sum_{n=0}^{\infty} e^{tn} \left(\frac{n(n(1+k)-(k+\alpha))}{(1-\epsilon)(1-\alpha)} \right) a_n \\ &= \frac{1}{(1-\epsilon)(1-\alpha)} \sum_{n=2}^{\infty} \left[(k+1)n(n-1)e^{tn} a_n + (1-\alpha) n a_n e^{tn} \right] \\ &= \frac{1}{(1-\epsilon)(1-\alpha)} \left[(k+1)e^{2t} f''(e^t) + (1-\alpha)\left(e^t f'(e^t) - e^t \right) \right]. \end{aligned}$$

Example 2.3. The moment generating function of the $k-$uniformly convex distribution of order α associated with the function $f(z) = z + \frac{(1-\epsilon)(1-\alpha)}{n[n(k+1)-(k+\alpha)]} z^n$ is

$$M_X(t) = e^{tn}.$$

Theorem 2.4. *The first four moments $\mu_1^{'}$, $\mu_2^{'}$, $\mu_3^{'}$ and $\mu_4^{'}$ of the $k-$ uniformly starlike probability distribution are defined by the relation*

1. $\mu_1^{'} = \frac{1}{(1-\epsilon)(1-\alpha)} \left[(k+1) f^{''}(1) + (1-\alpha)(f'(1)-1) \right].$

2. $\mu_2^{'} = \frac{1}{(1-\epsilon)(1-\alpha)} \left[(k+1) f^{'''}(1) + (2k+3-\alpha) f^{''}(1) + (1-\alpha)(f'(1)-1) \right].$

3. $\mu_3^{'} = \frac{1}{(1-\epsilon)(1-\alpha)} \left[(k+1) f^{iv}(1) + (5k+6-\alpha) f^{'''}(1) + \right.$
   $\left. (4k+7-3\alpha) f^{''}(1) + (1-\alpha)(f'(1)-1) \right].$

4. $\mu_4^{'} = \frac{1}{(1-\epsilon)(1-\alpha)} [(k+1) f^{v}(1) + (9k+10-\alpha) f^{iv}(1) +$
   $(19k+25-6\alpha) f^{'''}(1) + (8k+15-7\alpha) f^{''}(1) + (1-\alpha)(f'(1)-1)].$

Proof. The proof of above theorem is much akin to that of Theorem 2.1, hence we omit the details involved.

Remark 2.4. *If we put $k = 1, \alpha = 0$ in Theorem 2.1, then we obtain the first four moments for the class uniformly starlike functions studied by Subramanian et al.* [15].

Theorem 2.5. *The r^{th} factorial moment of the $k-$ uniformly starlike probability* *distribution is given by the relation*

$$\mu'_{(r)} = \begin{cases} \mu'_1, & \text{if } r = 1 \\ \frac{d^r}{dz^r}\left[(k+1)zf'(z) - (k+\alpha)f(z)\right]_{(z=1)}, & \text{if } r \geqslant 2 \end{cases}.$$

Proof. The proof of above theorem is much similar to that of Theorem 2.2, hence we omit the details involved.

Theorem 2.6. *The moment generating function of the $k-$ uniformly starlike distribution of order α is given by*

$$M_X(t) = \frac{1}{(1-\epsilon)(1-\alpha)}\left[(k+1)e^t f'(e^t) - (k+\alpha)f(e^t) - (1-\alpha)e^t\right].$$

Proof. The proof of above theorem is much akin to that of Theorem 2.4, therefore we omit the details involved.

Remark 2.5. *If we put $\epsilon = 0, k = 0$ in Theorems 2.2-2.6, then we obtain the corresponding results of Porwal* [10].

Remark 2.6. *If we put $\epsilon = 0, k = 0, \alpha = 0$ in Theorems 2.2-2.6, then we obtain corresponding results of Porwal* [10].

Remark 2.7. *If we put $k = 1, \alpha = 0$ in Theorems 2.2-2.6, then we obtain the corresponding results for the classes uniformly convex and uniformly starlike functions studied by Subramanian et al.* [10].

3. Conclusion

In this paper we introduce uniformly Convex and uniformly starlike discrete probability distribution. We obtain results regarding Moments, Factorial moments and Moment generating functions for these distributions. Since uniformly convex and uniformly starlike functions are play an important role in geometric function theory so we hope that these distributions also play an important role in geometric function theory as well as probability theory. This paper opens up a new and interesting direction of research in univalent functions and probability distributions.

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References

[1] Bharati R., Parvatham R. and Swaminathan A., On subclasses of uniformly convex functions and corresponding class of starlike functions, Tamkang J. Math., 28 (1997), 17-32.

[2] Dixit K. K. and Porwal S., On a certain class of k-uniformly convex functions with negative coefficients, Bull. Cal. Math. Soc., 100 (6) (2008), 639-652.

[3] Duren P. L., Univalent Functions, Grundleherem der Mathematischen Wissenchaften 259, Springer-Verlag, New York, Berlin, Heidelberg, Tokyo, 1983.

[4] Goodman A. W., On uniformly convex functions, Ann. Polon. Math., 56 (1991), 87-92.

[5] Goodman A. W., On uniformly starlike functions, J. Math. Anal. Appl., 155 (1991), 364-370.

[6] Gupta S. C. and Kapoor V. K., Fundamental of Mathematical Statistics, Sultan Chand and Sons, New Delhi, 2006.

[7] Kanas S. and Wisinowaska A., Conic regions and k-uniform convexity, J. Comput. Appl. Math., 105 (1999), 327-336.

[8] Kanas S. and Wisniowska A., Conic regions and k-starlike functions, Rev. Roum. Math. Pure Appl., 45 (2000), 647-657.

[9] Ma W. and Minda D., Uniformly convex functions, Ann. Polon. Math., 57 (1992), 165-175.

[10] Porwal S., Starlike and convex type probability distribution, Afr. Mat., 30 (7-8) (2019), 1049-1066.

[11] Porwal S. and Magesh N., The Salagean-type probability distribution, Surveys Math. Appl., 17 (2022), 277-285.

[12] Robertson M. S., On the theory of univalent functions, Ann. Math., 37 (1936), 374-408.

[13] Rønning F., Uniformly convex functions and a corresponding class of starlike functions, Proc. Amer. Math. Soc., 118 (1) (1993), 189-196.

[14] Silverman H., Univalent functions with negative coefficients, Proc. Amer. Math. Soc., 51 (1975), 109-116.

[15] Subramanian K. G., Murugusundaramoorthy G., Balasubrahmanyam P. and Silverman H., Subclasses of uniformly convex and uniformly starlike functions, Math. Japon., 42 (3) (1995), 517-522.