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ON A MODULAR EQUATION OF DEGREE 23

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Abstract: On page 249 of his second notebook, Ramanujan incorrectly recorded and crossed out a modular equation of degree 23. B. C. Berndt has corrected and proved the same using the theory of modular forms. The main objective of this article is to prove the modular equation of degree 23 corrected by Berndt by employing the method of parametrization. Using a similar technique, we also prove a modular equation of degree 11 due to Ramanujan.

Keywords and Phrases: Modular equations, hypergeometric series.

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1. Introduction

The Gauss series or the ordinary hypergeometric series ${}_{2}F_{1}(a,b;c;z)$ is defined by

$$_{2}F_{1}(a,b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_{n}(b)_{n}}{(c)_{n}n!} z^{n}, \quad |z| < 1$$

where

$$(a)_n = a(a+1)(a+2)\cdots(a+n-1), \quad n \ge 1$$