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A STUDY ON $N\hat{g}^*s-$ CLOSED SETS IN NANO TOPOLOGICAL SPACES

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Abstract: In this paper, we define and study about a new type of Nano generalized closed set called $N\hat{g}^*s-$closed sets in nano topological space. The relationship of $N\hat{g}^*s-$closed sets with other known Nano generalized closed sets and the characteristics of $N\hat{g}^*s-$interior, $N\hat{g}^*s-$exterior, $N\hat{g}^*s-$closure, $N\hat{g}^*s-$boundary and $N\hat{g}^*s-$border are studied.

Keywords and Phrases: $N\hat{g}^*s-$closed sets, $N\hat{g}^*s-$interior, $N\hat{g}^*s-$exterior, $N\hat{g}^*s-$clouser, $N\hat{g}^*s-$boundary and $N\hat{g}^*s-$border.

2020 Mathematics Subject Classification: 54A05, 54A10.

1. Introduction

N. Levine [8] introduced the concept of generalized closed sets in 1970. In 2013, M. Lellis Thivagar [7] has introduced nano topological space with respect to a subset X of universe U, which is defined in terms of lower and upper approximation of X. The elements of a nano topological space are called the nano-open sets. After studying nano-interior and nano-closure of a set. He has also introduced, among other, some certain weak from of nano open sets such as nano $\alpha-$open sets, nano semi-open sets and nano pre open sets. K. Bhuvaneswari and K. Mythili Gnanapriya [3] was introduced nano generalized closed set [2014] in nano topological space. The concept of $\hat{g}^*s-$closed sets was introduced by S. Pious Missier and M. Anto [9] in 2014. The aim of this paper is to introduce a new class of sets on nano topological spaces called $N\hat{g}^*s-$closed sets. Further, we investigate and discuss the relation of this new set with existing ones.

2. Preliminaries

Definition 2.1. [7] *Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Then U is divided into disjoint equivalence classes. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let $X \subseteq U$.*

- *The lower approximation of X with respect to R is the set of all objects, which can be classified as X with respect to R and it is denoted by $L_R(X)$. That is $L_R(X) = U_{x\in U}\left\{R(x) \subseteq X\right\}$, where $R(X)$ denotes the equivalence class determined by $X \in U$.*

- *The upper approximation of X with respect to R is the set of all objects, which can be classified as X with respect to R and it is denoted by $U_R(X)$. That is $U_R(X) = U_{x\in U}\left\{R(x) \cap X \neq \phi\right\}$.*

- *The boundary of the region of X with respect to R is the set of all objects, which can be classified neither as X nor as not X with respect to R and it is denoted by $B_R(X) = U_R(X) - L_R(X)$.*

Definition 2.2. [7] *If (U, R) is an approximation space and $X, Y \subseteq U$; then*

1. $L_R(X) \subseteq X \subseteq U_R(X)\phi$;

2. $L_R(\phi) = U_R(\phi) = \phi$ *and* $L_R(U) = U_R(U) = U$;

3. $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$;

4. $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$;

5. $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$;

6. $L_R(X \cap Y) \subseteq L_R(X) \cap L_R(Y)$;

7. $L_R(X) \subseteq L_R(Y)$ *and* $U_R(X) \subseteq U_R(Y)$ *whenever* $X \subseteq Y$;

8. $U_R(X^c) = [L_R(X)]^c$ *and* $L_R(X^c) = [U_R(X)]^c$;

9. $U_R(U_R(X)) = L_R(U_R(X)) = U_R(X)$;

10. $L_R(L_R(X)) = U_R(L_R(X)) = L_R(X)$;

Definition 2.3. [7] *Let U be the universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq U$. $\tau_R(X)$ satisfies the following axioms:*

(i) U *and* $\phi \in \tau_R(X)$

(ii) The union of elements of any sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

(iii) The intersection of the element of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.

That is, $\tau_R(X)$ forms a topology on U is called the nano topology on U with respect to X. We call $(U, \tau_R(X))$ is called the nano topological space.

Definition 2.4. [7] *If $(U, \tau_R(X))$ is a nano topological space with respect to X where, $X \subseteq U$ and if $A \subseteq U$, then the nano interior of A is defined as the union of all nano open subsets of A and it is denoted by $Nint(A)$.That is,$Nint(A)$ is the largest nano open subsets of A. The nano closure of A is defined as the intersection of all nano closed sets containing A and it is denoted by $Ncl(A)$ or $Ncl(A)$ is the smallest nano closed set containing A.*

Definition 2.5. [7] *A subset A of a nano topological space $(U, \tau_R(X))$ is called*

(i) nano semi open if $A \subseteq Ncl(Nint(A))$ and nano semi closed if $Nint(Ncl(A)) \subseteq A$.

(ii) nano $\alpha-$open if $A \subseteq Nint(Ncl(Nint(A)))$ and nano $\alpha-$closed if $Ncl(Nint(Ncl(A))) \subseteq A$.

(iii) nano regular open if $A = Nint(Ncl(A))$ and nano regular if $Ncl(Nint(A)) = A$.

The nano semi closure (respectively, nano $\alpha-$closure and nano regular closure) of a subset A of a space $(U, \tau_R(X))$ is the intersection of all nano semi closed sets (respectively, nano $\alpha-$closed and nano regular closed) sets containing A and its denoted by $Nscl(A)$ respectively, $N\alpha cl(A)$ and $Nrcl(A)$.

The nano semi interior of a subset A of a space $(U, \tau_R(X))$ is the union of all nano semi open set contained in A and is denoted by $Nsint(A)$.

Definition 2.6. [6] *A subset A of a nano topological space $(U, \tau_R(X))$ is called $N\hat{g}-$closed if $Ncl(A) \subseteq V$ whenever $A \subseteq V$ and V is $Ns-$open in $(U, \tau_R(X))$.*

Definition 2.7. [3] *A subset A of a nano topological space* $(U, \tau_R(X))$ *is called* $Ng-$*closed if* $Ncl(A) \subseteq V$ *whenever* $A \subseteq V$ *and* V *is* $N-$*open in* $(U, \tau_R(X))$.

Definition 2.8. [2] *A subset A of a nano topological space* $(U, \tau_R(X))$ *is called* $Nsg-$*closed if* $Nscl(A) \subseteq V$ *whenever* $A \subseteq V$ *and* V *is* $Ns-$*open in* $(U, \tau_R(X))$.

Definition 2.9. [13] *A subset A of a nano topological space* $(U, \tau_R(X))$ *is called* Ng^*-*closed if* $Ncl(A) \subseteq V$ *whenever* $A \subseteq V$ *and* V *is* $Ng-$*open in* $(U, \tau_R(X))$.

Definition 2.10. [12] *A subset A of a nano topological space* $(U, \tau_R(X))$ *is called* Ng^*s-*closed if* $Nscl(A) \subseteq V$ *whenever* $A \subseteq V$ *and* V *is* $Ng-$*open in* $(U, \tau_R(X))$.

Definition 2.11. [2] *A subset A of a nano topological space* $(U, \tau_R(X))$ *is called* $Ngs-$*closed if* $Nscl(A) \subseteq V$ *whenever* $A \subseteq V$ *and* V *is* $N-$*open in* $(U, \tau_R(X))$.

Definition 2.12. [19] *A subset A of a nano topological space* $(U, \tau_R(X))$ *is called* $Ng\alpha-$*closed if* $N\alpha cl(A) \subseteq V$ *whenever* $A \subseteq V$ *and* V *is* $N\alpha-$*open in* $(U, \tau_R(X))$.

Definition 2.13. [17] *A subset A of a nano topological space* $(U, \tau_R(X))$ *is called* $N\ddot{g}-$*closed if* $Ncl(A) \subseteq V$ *whenever* $A \subseteq V$ *and* V *is* $Nsg-$*open in* $(U, \tau_R(X))$.

Definition 2.14. [17] *A subset A of a nano topological space* $(U, \tau_R(X))$ *is called* $N\ddot{g}\alpha-$*closed if* $N\alpha cl(A) \subseteq V$ *whenever* $A \subseteq V$ *and* V *is* $Nsg-$*open in* $(U, \tau_R(X))$.

Definition 2.15. [18] *A subset A of a nano topological space* $(U, \tau_R(X))$ *is called* N^*g-*closed if* $Ncl(A) \subseteq V$ *whenever* $A \subseteq V$ *and* V *is* $N\hat{g}-$*open in* $(U, \tau_R(X))$.

Definition 2.16. [14] *A subset A of a nano topological space* $(U, \tau_R(X))$ *is called* $N\hat{g}\alpha-$*closed if* $N\alpha cl(A) \subseteq V$ *whenever* $A \subseteq V$ *and* V *is* $N\hat{g}-$*open in* $(U, \tau_R(X))$.

Definition 2.17. [5] *A subset A of a nano topological space* $(U, \tau_R(X))$ *is called* $Ng^{\#}s-$*closed if* $Nscl(A) \subseteq V$ *whenever* $A \subseteq V$ *and* V *is* $N\alpha g-$*open in* $(U, \tau_R(X))$.

Definition 2.18. [4] *A subset A of a nano topological space* $(U, \tau_R(X))$ *is called* $N\psi-$*closed if* $Ncl(A) \subseteq V$ *whenever* $A \subseteq V$ *and* V *is* $Ngs-$*open in* $(U, \tau_R(X))$.

Definition 2.19. [11] *A subset A of a nano topological space* $(U, \tau_R(X))$ *is called* $Ng\alpha g-$*closed if* $Ncl(A) \subseteq V$ *whenever* $A \subseteq V$ *and* V *is* $N\alpha g-$*open in* $(U, \tau_R(X))$.

Definition 2.20. [5] *A subset A of a nano topological space* $(U, \tau_R(X))$ *is called* $Ng^{\#}\alpha-$*closed if* $N\alpha cl(A) \subseteq V$ *whenever* $A \subseteq V$ *and* V *is* $Ng-$*open in* $(U, \tau_R(X))$.

Definition 2.21. [5] *A subset A of a nano topological space* $(U, \tau_R(X))$ *is called* $N\alpha g^*s-$*closed if* $N\alpha cl(A) \subseteq V$ *whenever* $A \subseteq V$ *and* V *is* $Ngs-$*open in* $(U, \tau_R(X))$.

Remark 2.22.

1. $Ngsg-$*closed* [10] *sets defined by Hatem Imran (2016),* $Ns\hat{g}-$*closed* [15] *sets*

defined by Rajendren (2019), and $N\ddot{g}-$closed sets defined by S.M. Sandhya(2020) represents one and the same sets.

2. *$N\hat{g}\alpha-$closed* [14] *sets defined by Rajendren (2019) and $N\hat{g}\alpha-$closed[1] sets defined by M.Davamani (2020)represents one and the same sets.*

3. *Sathyapriya* [17] *defines a set A to be a $N\alpha g^*s-$closed set if $Nscl(A) \subseteq V$ whenever $A \subseteq V$ and V is $N\alpha g-$open in $(U, \tau_R(X))$.*

4. *Dharani* [5] *defines a set A to be a $N\alpha g^*s-$closed set if $N\alpha cl(A) \subseteq V$ whenever $A \subseteq V$ and V is $Ngs-$open.*

5. *Unknown author* [5] *defines a set A to be a $Ng^{\#}s-$closed set if $Nscl(A) \subseteq V$ whenever $A \subseteq V$ and V is $N\alpha g-$open in $(U, \tau_R(X))$.*
It may be noted that $N\alpha g^*s-$ closed set as defined by Sathyapriya and $Ng^{\#}s-$closed set as defined by the unknown author are one and the same.

3. $N\hat{g}^*s-$closed sets

Definition 3.1. *A subset A of a nano topological space $(U, \tau_R(X))$ is called $N\hat{g}^*s-$closed if $Nscl(A) \subseteq V$ whenever $A \subseteq V$ and V is $N\hat{g}-$open in $(U, \tau_R(X))$.*

Example 3.2. Let $(U, \tau_R(X))$ be a nano topological space where $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{c\}, \{bd\}\}$ and $X = \{b, c\}$. Then the nano topology $\tau_R(X) = \{\phi, U, \{c\}, \{b, c, d\}, \{b, d\}\}$. Then $N\hat{g}^*sC(U, \tau_R(X)) = \{\phi, U, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$.

Remark 3.3. *The following example shows that the intersection of two $N\hat{g}^*s-$closed set need not be a $N\hat{g}^*s-$ closed set.*

Example 3.4. In the above example (3.2), let $A = \{a, d\}$ and $B = \{b, d\}$ are $N\hat{g}^*s-$closed sets. But $A \cap B = \{d\}$ is not a $N\hat{g}^*s-$closed.

Remark 3.5. *The following example shows that the union of two $N\hat{g}^*s-$closed set need not be a $N\hat{g}^*s-$closed sets.*

Example 3.6. In the above example (3.2), let $\{c\}$ and $\{b, d\}$ are $N\hat{g}^*s-$closed sets whose union $\{b, c, d\}$ is not in $N\hat{g}^*s-$closed sets.

Proposition 3.7. *Every $Nr-$closed set in nano topological space $(U, \tau_R(X))$ is $N\hat{g}^*s-$closed.*

Remark 3.8. *The following example shows that the converse of the above proposition* (3.7) *is need not be true.*

Example 3.9. By example (3.2), $\{a, b\}$ is $N\hat{g}^*s-$closed but not $Nr-$closed.

Proposition 3.10. *Every $N-$closed set in a nano topological space $(U, \tau_R(X))$ is $N\hat{g}^*s-$closed.*

Remark 3.11. *The following example shows that the converse of the above proposition* (3.10) *is not true.*

Example 3.12. In the above example (3.2), $\{a, b\}$ is $N\hat{g}^*s-$closed but not $N-$closed.

Proposition 3.13. *Every $Ns-$closed set in a nano topological space $(U, \tau_R(X))$ is $N\hat{g}^*s-$closed.*

Remark 3.14. *The following example shows that the converse of the above proposition* (3.13) *is not true.*

Example 3.15. In the above example (3.2), $\{a, b\}$ is $N\hat{g}^*s-$closed but not $Ns-$closed.

Proposition 3.16. *Every $N\psi-$closed set in a nano topological space $(U, \tau_R(X))$ is $N\hat{g}^*s-$closed.*

Remark 3.17. *The following example shows that the converse of the above proposition* (3.16) *is not true.*

Example 3.18. In the above example (3.2), $\{a, b\}$ is $N\hat{g}^*s-$closed but not $N_\psi-$closed.

Proposition 3.19. *Every $Ng\alpha g-$closed set in a nano topological space $(U, \tau_R(X))$ is $N\hat{g}^*s-$closed.*

Remark 3.20. *The following example shows that the converse of the above proposition* (3.19) *is not true.*

Example 3.21. In the above example (3.2), $\{c\}$ is $N\hat{g}^*s-$ closed but not $Ng\alpha g-$ closed.

Proposition 3.22. *Every $N\ddot{g}-$closed set in a nano topological space $(U, \tau_R(X))$ is $N\hat{g}^*s-$closed.*

Remark 3.23. *The following example shows that the converse of the above proposition is not true.*

Example 3.24. In the above example (3.2), $\{a, d\}$ is $N\hat{g}^*s-$closed but not $N\ddot{g}-$closed.

Proposition 3.25. *Every $Ng^{\#}-$closed set in a nano topological space $(U, \tau_R(X))$ is $N\hat{g}^*s-$closed.*

Remark 3.26. *The following example shows that the converse of the above proposition* (3.25) *is not true.*

Example 3.27. Let $(U, \tau_R(X))$ be a nano topological space where $U = \{a, b, c, d\}$ with $U/R = \{\{a\}, \{b\}, \{c, d\}\}$ and $X = \{a, b\}.\tau_R(X) = \{\phi, U, \{a, b\}\}$. Here $\{a, c\}$ is $N\hat{g}^*s-$closed but not $Ng^{\#}s-$closed.

Proposition 3.28. *Every $N\ddot{g}\alpha-$closed set in a nano topological space $(U, \tau_R(X))$ is $N\hat{g}^*s-$closed.*

Remark 3.29. *The following example shows that the converse of the above proposition* (3.28) *is not true.*

Example 3.30. In the above example (3.2), $\{a, d\}$ is $N\hat{g}^*s-$closed but not $N\ddot{g}\alpha-$closed.

Proposition 3.31. *Every N^*g-closed set in a nano topological space $(U, \tau_R(X))$ is $N\hat{g}^*s-$closed.*

Remark 3.32. *The following example shows that the converse of the above proposition* (3.31) *is not true.*

Example 3.33. In the above example (3.2), $\{b, d\}$ is $N\hat{g}^*s-$ closed but not N^*g- closed.

Proposition 3.34. *Every $N\hat{g}\alpha-$closed set in a nano topological space $(U, \tau_R(X))$ is $N\hat{g}^*s-$closed.*

Remark 3.35. *The following example shows that the converse of the above proposition* (3.34) *is not true.*

Example 3.36. In the above example (3.2), $\{a, c\}$ is $N\hat{g}^*s-$closed but not $N\hat{g}\alpha-$closed.

Proposition 3.37. *Every Ng^*-closed Set in a nano topological space $(U, \tau_R(x))$ is $N\hat{g}^*s-$closed.*

Remark 3.38. *The following example shows that the converse of the above proposition* (3.37) *is not true.*

Example 3.39. In the above example (3.2), $\{c\}$ is $N\hat{g}^*s-$ closed but not Ng^*-closed.

Proposition 3.40. *Every Ng^*s-closed Set in a nano topological space $(U, \tau_R(x))$ is $N\hat{g}^*s-$closed.*

Remark 3.41. *The following example shows that the converse of the above proposition* (3.40) *is not true.*

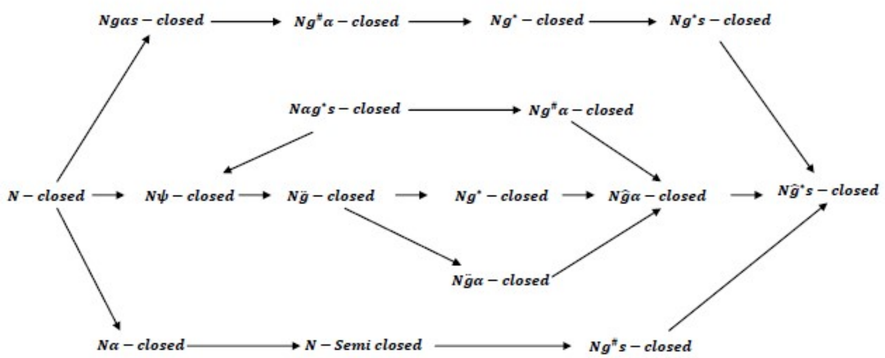
Example 3.42. In the above example (3.27), $\{a, d\}$ is $N\hat{g}^*s-$closed but not Ng^*s-closed.

Proposition 3.43. *Every $Ng^{\#}\alpha-$closed set in a nano topological space $(U, \tau_R(X))$ is $N\hat{g}^*s-$closed.*

Remark 3.44. *The following example shows that the converse of the above proposition* (3.43) *is not true.*

Example 3.45. In the above example (3.2), $\{c\}$ is $N\hat{g}^*s-$ closed but not $Ng^{\#}\alpha-$closed.

Remark 3.46. *From the above Propositions and Remarks, we obtain the following Figure.*



4. $N\hat{g}^*s-$closure and $N\hat{g}^*s-$interior

Definition 4.1. *The intersection of all $N\hat{g}^*s-$closed sets in $(U, \tau_R(X))$ containing A is called $N\hat{g}^*s-$closure of A and is denoted by $N\hat{g}^*s - cl(A)$.*

Definition 4.2. *The union of all $N\hat{g}^*s-$open sets in $(U, \tau_R(X))$ contained A is called $N\hat{g}^*s-$interor of A and is denoted by $N\hat{g}^*s - int(A)$.*

Theorem 4.3. *If A is $N\hat{g}^*s-$closed set and B is nano closed in $(U, \tau_R(X))$ then $A \cup B$ in $N\hat{g}^*s-$closed in $(U, \tau_R(X))$.*

Proof. Let U/A is $N\hat{g}^*s-$open set and B is nano closed in $(U, \tau_R(X))$. Then $(U/A) \cap (U/B) = (U/A) \cup (U/B)$ is $N\hat{g}^*s-$open in $(U, \tau_R(X))$. Hence $A \cup B$ is $N\hat{g}^*s-$closed in $(U, \tau_R(X))$

Remark 4.4. *Let $(U, \tau_R(X))$ be a topological space and $A \subseteq U$*

(i) $N\hat{g}^*s - cl(A) = A$ *iff* A *is* $N\hat{g}^*s-$*closed set.*

(ii) $N\hat{g}^*s - cl(A)$ *is the smallest* $N\hat{g}^*s-$*closed set containing* A.

Proof. (i) and (ii) are obvious.

Corollary 4.5. *If* A *is* $N\hat{g}^*s-$*closed set* B *is nano open in* $(U, \tau_R(X))$ *then* A/B *in* $N\hat{g}^*s-$*closed in* $(U, \tau_R(X))$.
Proof. follow the theorem (4.3).

Remark 4.6. *Let* $(U, \tau_R(X))$ *be a nano topological space and* $A \subseteq U$ *the following statements hold*

(i) $N\hat{g}^*s - int(\varphi) = \varphi$ *and* $N\hat{g}^*s - int(U) = U$

(ii) *If* $A \subseteq B$ *then* $N\hat{g}^*s - int(A) \subseteq (N\hat{g}^*s - intB)$

(iii) $N\hat{g}^*s - int(A)$ *is the largest* $N\hat{g}^*s-$*open set contained in* A

(iv) $N\hat{g}^*s - int(A \cap B) = N\hat{g}^*s - int(A) \cap N\hat{g}^*s - int(B)$

(v) $N\hat{g}^*s - int(A \cup B) \supseteq N\hat{g}^*s - int(A) \cup N\hat{g}^*s - int(B)$

(vi) $N\hat{g}^*s - int(N\hat{g}^*s - int(A)) = N\hat{g}^*s - int(A)$

Proof. Proof follows from the definition.

Theorem 4.7. A *is* $N\hat{g}^*s-$*open if only if* $N\hat{g}^*s - int(A)$.

Remark 4.8. *For any nano subset* A *of* $(U, \tau_R(X)), N-int(A) \subseteq N\hat{g}^*s-int(A) \subseteq U$.

Theorem 4.9. *Every nano open set is* $N\hat{g}^*s-$*open.*
Proof. Let A be an nano open set in U. Then A^c is nano closed in $(U, \tau_R(X))$. Since every nano closed set is $N\hat{g}^*s-$ closed, A^c is $N\hat{g}^*s-$open in $(U, \tau_R(X))$. Then, A is $N\hat{g}^*s-$open in $(U, \tau_R(X))$.

5. $N\hat{g}^*s-$border

Definition 5.1. *Let* $(U, \tau_R(X))$ *be a nano topological space and* $A \subseteq U$. *Then* $N\hat{g}^*s-$ *border of* $A(briefly, N\hat{g}^*s-Bd(A))$ *is given by* $N\hat{g}^*s-Bd(A) = A/N\hat{g}^*s-int(A)$.

Theorem 5.2.

(i) $N\hat{g}^*s - int(A) \cap N\hat{g}^*s - Bd(A) = \varphi$

(ii) $N\hat{g}^*s - Bd(U) = N\hat{g}^*s - Bd(\varphi) = \varphi$

(iii) $N\hat{g}^*s - Bd(N\hat{g}^*s - int(A)) = \varphi$

(iv) $N\hat{g}^*s - int(N\hat{g}^*s - Bd(A)) = \varphi$

(v) $N\hat{g}^*s - Bd(N\hat{g}^*s - Bd(A)) = N\hat{g}^*s - Bd(A)$

Proof.

1. $N\hat{g}^*s{-}int(A) \cap N\hat{g}^*s{-}Bd(A) = N\hat{g}^*s{-}int(A) \cap (A/N\hat{g}^*s{-}int(A)) = N\hat{g}^*s{-}int(A) \cap A/N\hat{g}^*s{-}int(A) \cap N\hat{g}^*s{-}int(A) = N\hat{g}^*s{-}int(A)/N\hat{g}^*s{-}int(A) = \varphi$
2. $N\hat{g}^*s - Bd(U)/N\hat{g}^*s - int(U) = U/U = \varphi$ and $N\hat{g}^*s - Bd(U) = \varphi | N\hat{g}^*s - int(\varphi) = \varphi|\varphi = \varphi$.
3. $N\hat{g}^*s - Bd(N\hat{g}^*s - int(A)) = N\hat{g}^*s - int(A)/N\hat{g}^*s - int(N\hat{g}^*s - int(A)) = N\hat{g}^*s - int(A)/N\hat{g}^*s - int(A) = \varphi$.
4. $N\hat{g}^*s - int(N\hat{g}^*s - Bd(A)) = N\hat{g}^*s - int(A/N\hat{g}^*s - int(A)) = N\hat{g}^*s - int(A)/N\hat{g}^*s - int(N\hat{g}^*s - int(A)) = N\hat{g}^*s - int(A)/N\hat{g}^*s - int(A) = \varphi$.

Theorem 5.3. *For a subset A of a nano topological space $(U, \tau_R(X))$, the following are equivalent*

1. *A is $N\hat{g}^*s-$open*
2. *$A = N\hat{g}^*s - int(A)$*
3. *$N\hat{g}^*s - Bd(A) = \varphi$*

6. $N\hat{g}^*s-$boundary

Definition 6.1. *Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then the $N\hat{g}^*s-$boundary of $A(briefly, N\hat{g}^*s - b(A))$ is given by $N\hat{g}^*s - b(A) = N\hat{g}^*s - cl(A) \cup N\hat{g}^*s - cl(U/A)$.*

Theorem 6.2. *Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$ the following statements hold*

(i) $N\hat{g}^*s - b(A) = N\hat{g}^*s - b(U/A)$

(ii) $N\hat{g}^*s - b(A) = N\hat{g}^*s - cl(A)/N\hat{g}^*s - int(A)$

(iii) $N\hat{g}^*s - b(A) \cap N\hat{g}^*s - int(A) = \varphi$

(iv) $N\hat{g}^*s - b(A) \cup N\hat{g}^*s - int(A) = N\hat{g}^*s - cl(A)$

Proof.

(i) $N\hat{g}^*s - b(A) = N\hat{g}^*s - cl(A) \cap N\hat{g}^*s - cl(U/A)$ and $N\hat{g}^*s - b(U/A) = N\hat{g}^*s - cl(U/A) \cap N\hat{g}^*s - cl(A)$

(ii) $N\hat{g}^*s - b(A) = N\hat{g}^*s - cl(A) \cap N\hat{g}^*s - cl(U/A) = N\hat{g}^*s - cl(A) \cap (U/N\hat{g}^*s - int(A)) = N\hat{g}^*s - cl(A) \cap (U/N\hat{g}^*s - int(A)) = N\hat{g}^*s - cl(A)/N\hat{g}^*s - int(A)$.

(iii) By using (ii) $N\hat{g}^*s - b(A) \cap N\hat{g}^*s - int(A) = (N\hat{g}^*s - cl(A)/N\hat{g}^*s - int(A)) \cap N\hat{g}^*s - int(A) = \varphi$.

(iv) By using (ii) and (iii), $N\hat{g}^*s - b(A) \cap N\hat{g}^*s - int(A) = N\hat{g}^*s - cl(A)$.

Theorem 6.3. *If A is a subset of a space $(U, \tau_R(X))$, then the following statements are hold:*

(i) *A is $N\hat{g}^*s-$open set iff $A \cap N\hat{g}^*s - b(A) = \varphi$.*

(ii) *A is $N\hat{g}^*s-$closed set iff $N\hat{g}^*s - b(A) \subset A$.*

(iii) *A is $N\hat{g}^*s-$clopen set iff $N\hat{g}^*s - b(A) = \varphi$.*

Proof. Prove the above condition using theorem (6.2) .

7. $N\hat{g}^*s-$exterior

Definition 7.1. *Let $(U, \tau_R(X))$ be a nano topological space and $A \subseteq U$. Then the $N\hat{g}^*s-$exterior of $A(briefly, N\hat{g}^*s - ext(A))$ is given by $N\hat{g}^*s - ext(A) = (U/N\hat{g}^*s - cl(A))$.*

Theorem 7.2. *Let A and B are two subsets of a nano topological space $(U, \tau_R(X))$, then the following statements are true*

(i) $N\hat{g}^*s - ext(A) - int(U/A)$

(ii) $N\hat{g}^*s - ext(A) N\hat{g}^*s$*- open*

(iii) $N\hat{g}^*s - ext(A) \cup N\hat{g}^*s - b(A) = N\hat{g}^*s - cl(U/A)$

(iv) $N\hat{g}^*s - ext(A) \cap N\hat{g}^*s - int(A) = \varnothing$

(v) $N\hat{g}^*s - ext(A) \cap N\hat{g}^*s - b(A) = \varnothing$

8. Conclusion

We have defined $N\hat{g}^*s-$closed sets and discuss its relation with other know nano-closed sets and nano generalized closed sets. The properties and characteristics of $N\hat{g}^*s-$interior, $N\hat{g}^*s-$exterior, $N\hat{g}^*s-$closure, $N\hat{g}^*s-$boundary and $N\hat{g}^*s-$border. Which act as the basic and foundation for the further of our studies of $N\hat{g}^*s-$closed sets.

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