

**ON THE NUMBER OF FUZZY SUBGROUPS AND FUZZY  
NORMAL SUBGROUPS OF  $S_2$ ,  $S_3$  AND  $A_4$**

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**Abstract:** Counting fuzzy subgroups of a finite group is a fundamental problem of fuzzy group theory. Many researchers have made significant contributions to the rapid growth of this topic in recent years. The number of fuzzy subgroups of any group is infinite without the aid of equivalence relation. Some authors have used the equivalence relation of fuzzy sets to study the equivalence of fuzzy subgroups ([5], [6], [16]). The problem of counting the number of distinct fuzzy subgroups of a finite group is relative to the choice of the equivalence relation. The number of fuzzy subgroups of a particular group varies from one equivalence relation to the other. The equivalence relation applied in our computation can be seen in the existing literature. Sulaiman and Abd Ghafur [10] define an equivalence relation for counting fuzzy subgroups of group  $G$ . We have used this relation to find fuzzy subgroups and fuzzy normal subgroups of  $S_2$ ,  $S_3$  and  $A_4$ . Lattice subgroup diagrams were used in our computation.

**Keywords and Phrases:** Fuzzy Subgroups, Fuzzy Normal Subgroups, Equivalence Relation, Chain, Subgroup Lattice, Symmetric Group, Alternating Group.

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## **1. Introduction**

The idea of fuzzy set was first propounded by Lotfi A. Zadeh in 1965. Since the inception of the conception of a fuzzy set, which laid the foundations of fuzzy