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ON THE SOLUTION OF A CLASS OF EXPONENTIAL DIOPHANTINE EQUATIONS

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Abstract: In this note, we show that for $n = 4N + 3, N \in \mathbb{N} \cup \{0\}$, the exponential Diophantine equation $n^x + 24^y = z^2$ has exactly two solutions if $n + 1$ or equivalently $N + 1$ is an square. When $N + 1 = m^2$, the solutions are given by $(0, 1, 5)$ and $(1, 0, 2m)$. Otherwise it has a unique solution $(0, 1, 5)$ in non-negative integers. Finally, we leave an open problem to explore.

Keywords and Phrases: Catalan's Conjecture solutions, Exponential Diophantine equations, Integer solutions.

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1. Introduction

Many authors have studied the exponential Diophantine equation for a long time [7]. In 1844, the great Mathematician, Eugene Charles Catalan formulated a conjecture that the exponential Diophantine equation $a^x - b^y = 1$ where $a, b, x, y \in \mathbb{Z}$ with $min\{a, b, x, y\} > 1$ has a unique solution $(a, b, x, y) = (3, 2, 2, 3)$ [8]. Since then, numerous mathematicians have attempted to solve it, with varying degrees of success (see for instance [2, 10, 13, 14]). Eventually, Preda Mihăilescu [15] proved