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N_{nc} δ-OPEN SETS

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Abstract: A new strong forms of sets called N-neutrosophic crisp δ-open sets and N-neutrosophic crisp δ-closed sets in N-neutrosophic crisp topological space are introduced in this article. Also, discuss their properties and examples are related to N-neutrosophic crisp δ open sets along with their near sets in N-neutrosophic crisp topological spaces.

Keywords and Phrases: $N_{nc}\delta os$, $N_{nc}\delta cs$, $N_{nc}\delta int(M)$, $N_{nc}\delta cl(M)$.

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1. Introduction

The concepts of neutrosophy and neutrosophic set are the recent tools in a topological space. It was first introduced by Smarandache [5, 6] in the end of 20^{th} century. In 2014, Salama, Smarandache and Kroumov [3] has provided the basic concept of neutrosophic crisp set in a topological space. After that Al-Omeri [1] also investigated some fundamental properties of neutrosophic crisp topological Spaces. Al-Hamido [2] explore the possibility of expanding the concept of neutrosophic crisp topological spaces into N-topology and investigate some of their basic properties in N-terms. In 1968, the idea of δ-interior and δ-closure operations was introduced by

Velicko [15] which are stronger than open sets. Also, it have been widely introduced some new spaces, sets and functions. Vadivel et al. [9, 10, 14] introduced δ-open sets in a neutrosophic topological spaces. Recently, Vadivel et al. introduced γ-open [7] and β-open sets [8] and their maps [11, 12, 13] in N-neutrosophic crisp topological spaces.

In this present work, we establish the concept of N-neutrosophic crisp δ-open sets and N-neutrosophic crisp δ-closed sets in $N_{nc}ts$ and also interrogate some of their basic properties along with their near open sets in N-neutrosophic crisp topological spaces.

2. Preliminaries

Some basic definitions & properties of N_{nc} topological spaces are discussed in this section.

Definition 2.1. [4] *For any non-empty fixed set X, a neutrosophic crisp set (briefly, ncs) M, is an object having the form $M = \langle M_1, M_2, M_3\rangle$ where M_1, M_2 & M_3 are subsets of X satisfying any one of the types*

(T1) $M_a \cap M_b = \phi$, $a \neq b$ & $\bigcup_{a=1}^{3} M_a \subset X$, $\forall a, b = 1, 2, 3$.

(T2) $M_a \cap M_b = \phi$, $a \neq b$ & $\bigcup_{a=1}^{3} M_a = X$, $\forall a, b = 1, 2, 3$.

(T3) $\bigcap_{a=1}^{3} M_a = \phi$ & $\bigcup_{a=1}^{3} M_a = X$, $\forall a = 1, 2, 3$.

Definition 2.2. [4] *Types of ncs's $\emptyset_N$ and X_N in X are as follows*

(i) $\emptyset_N$ *may be defined as* $\emptyset_N = \langle\emptyset, \emptyset, X\rangle$ *or* $\langle\emptyset, X, X\rangle$ *or* $\langle\emptyset, X, \emptyset\rangle$ *or* $\langle\emptyset, \emptyset, \emptyset\rangle$.

(ii) X_N *may be defined as* $X_N = \langle X, \emptyset, \emptyset\rangle$ *or* $\langle X, X, \emptyset\rangle$ *or* $\langle X, \emptyset, X\rangle$ *or* $\langle X, X, X\rangle$.

Definition 2.3. [4] *Let X be a non-empty set & the ncs's M & E in the form $M = \langle M_{11}, M_{22}, M_{33}\rangle$, $E = \langle E_{11}, E_{22}, E_{33}\rangle$, then*

(i) $M \subseteq E \Leftrightarrow M_{11} \subseteq E_{11}$, $M_{22} \subseteq E_{22}$ & $M_{33} \supseteq E_{33}$ *or* $M_{11} \subseteq E_{11}$, $M_{22} \supseteq E_{22}$ & $M_{33} \supseteq E_{33}$.

(ii) $M \cap E = \langle M_{11} \cap E_{11}, M_{22} \cap E_{22}, M_{33} \cup E_{33}\rangle$ *or* $\langle M_{11} \cap E_{11}, M_{22} \cup E_{22}, M_{33} \cup E_{33}\rangle$

(iii) $M \cup E = \langle M_{11} \cup E_{11}, M_{22} \cup E_{22}, M_{33} \cap E_{33}\rangle$ *or* $\langle M_{11} \cup E_{11}, M_{22} \cap E_{22}, M_{33} \cap E_{33}\rangle$

Definition 2.4. [4] *Let $M = \langle M_1, M_2, M_3\rangle$ a ncs on X, then the complement of M (briefly, M^c) may be defined in three different ways:*

(C1) $M^c = \langle {M_1}^c, {M_2}^c, {M_3}^c\rangle$, *or*

(C2) $M^c = \langle M_3, M_2, M_1 \rangle$, *or*

(C3) $M^c = \langle M_3, {M_2}^c, M_1 \rangle$.

Definition 2.5. [3] *A neutrosophic crisp topology (briefly, $_{nc}t$) on a non-empty set X is a family Γ of nc subsets of X satisfying*

(i) $\emptyset_N,\ X_N \in \Gamma$.

(ii) $M_1 \cap M_2 \in \Gamma \ \forall\ M_1\ \&\ M_2 \in \Gamma$.

(iii) $\bigcup_a M_a \in \Gamma,\ \forall\ \ M_a : a \in A \subseteq \Gamma$.

Then (X, Γ) is a neutrosophic crisp topological space (briefly, ncts) in X. The neutrosophic crisp open sets (briefly, ncos) are the elements of Γ in X. A ncs C is closed (briefly, nccs) iff its complement C^c is ncos.

Definition 2.6. [2] *Let X be a non-empty set. Then $_{nc}\Gamma_1,\ _{nc}\Gamma_2,\ \cdots,\ _{nc}\Gamma_N$ are N-arbitrary crisp topologies defined on X and the collection $N_{nc}\Gamma$ is called N-neutrosophic crisp (briefly, N_{nc})-topology on X is*

$$N_{nc}\Gamma = \{A \subseteq X : A = (\bigcup_{j=1}^{N} E_j) \cup (\bigcap_{j=1}^{N} F_j),\ E_j, F_j \in {}_{nc}\Gamma_j\}$$

and it satisfies the following axioms:

(i) $\emptyset_N,\ X_N \in N_{nc}\Gamma$.

(ii) $\bigcup_{j=1}^{\infty} A_j \in N_{nc}\Gamma \ \forall\ \{A_j\}_{j=1}^{\infty} \in N_{nc}\Gamma$.

(iii) $\bigcap_{j=1}^{n} A_j \in N_{nc}\Gamma \ \forall\ \{A_j\}_{j=1}^{n} \in N_{nc}\Gamma$.

Then $(X, N_{nc}\Gamma)$ is called a N-neutrosophic crisp topological space (briefly, $N_{nc}ts$) on X. The N-neutrosophic crisp open sets (briefly, $N_{nc}os$) are the elements of $N_{nc}\Gamma$ in X and the complement of $N_{nc}os$ is called N-neutrosophic crisp closed sets (briefly, $N_{nc}cs$) in X. The elements of X are known as N-neutrosophic crisp sets ($N_{nc}s$) on X.

Definition 2.7. [2] *Let $(X, N_{nc}\Gamma)$ be $N_{nc}ts$ on X and M be an $N_{nc}s$ on X, then the N-neutrosophic crisp interior of M (briefly, $N_{nc}int(M)$) and N-neutrosophic crisp closure of M (briefly, $N_{nc}cl(M)$) are defined as*

$$N_{nc}int(M) = \cup\{A : A \subseteq M\ \&\ A \text{ is a } N_{nc}os \text{ in } X\}$$

$$N_{nc}cl(M) = \cap\{C : M \subseteq C \ \& \ C \text{ is a } N_{nc}cs \text{ in } X\}.$$

Definition 2.8. [2] *Let $(X, N_{nc}\Gamma)$ be any $N_{nc}ts$. Let M be an $N_{nc}s$ in $(X, N_{nc}\Gamma)$. Then M is said to be a N-neutrosophic crisp*

(i) regular open [7] *set (briefly, $N_{nc}ros$) if $M = N_{nc}int(N_{nc}cl(M))$.*

(ii) pre open set (briefly, $N_{nc}\mathcal{P}os$) if $M \subseteq N_{nc}int(N_{nc}cl(M))$.

(iii) semi open set (briefly, $N_{nc}\mathcal{S}os$) if $M \subseteq N_{nc}cl(N_{nc}int(M))$.

(iv) α-open set (briefly, $N_{nc}\alpha os$) if $M \subseteq N_{nc}int(N_{nc}cl(N_{nc}int(M)))$.

(v) β-open [8] *set (briefly, $N_{nc}\beta os$) if $M \subseteq N_{nc}cl(N_{nc}int(N_{nc}cl(M)))$.*

The complement of a $N_{nc}ros$ (resp. $N_{nc}\mathcal{P}os$, $N_{nc}\mathcal{S}os$, $N_{nc}\alpha os$ & $N_{nc}\beta os$) is called a N-neutrosophic crisp regular (resp. pre, semi, α & β) closed set (briefly, $N_{nc}rcs$ (resp. $N_{nc}\mathcal{P}cs$, $N_{nc}\mathcal{S}cs$, $N_{nc}\alpha cs$ & $N_{nc}\beta cs$)) in X.

The family of all $N_{nc}\mathcal{P}os$ (resp. $N_{nc}\mathcal{P}cs$, $N_{nc}\mathcal{S}os$, $N_{nc}\mathcal{S}cs$, $N_{nc}\alpha os$, $N_{nc}\alpha cs$, $N_{nc}\beta os$ & $N_{nc}\beta cs$) of X is denoted by $N_{nc}\mathcal{P}OS(X)$ (resp. $N_{nc}\mathcal{P}CS(X)$, $N_{nc}\mathcal{S}OS(X)$, $N_{nc}\mathcal{S}CS(X)$, $N_{nc}\alpha OS(X)$, $N_{nc}\alpha CS(X)$, $N_{nc}\beta OS(X)$ & $N_{nc}\beta CS(X)$).

3. δ-open sets in $N_{nc}ts$

Throughout the section, let $(X, N_{nc}\Gamma)$ be any $N_{nc}ts$. Let M and E be an $N_{nc}s$'s in $(X, N_{nc}\Gamma)$.

Definition 3.1. *A set M is said to be a N-neutrosophic crisp*

(i) δ interior of M (briefly, $N_{nc}\delta int(M)$) is defined by $N_{nc}\delta int(M) = \cup\{A : A \subseteq M \ \& \ A$ is a $N_{nc}ros\}$.

(ii) δ closure of M (briefly, $N_{nc}\delta cl(M)$) is defined by $N_{nc}\delta cl(M) = \cap\{C : M \subseteq C \ \& \ C$ is a $N_{nc}rcs$ in $X\}$.

Definition 3.2. *A set M is said to be a N-neutrosophic crisp*

(i) δ-open set (briefly, $N_{nc}\delta os$) if $M = N_{nc}\delta int(M)$.

(ii) δ-pre open set (briefly, $N_{nc}\delta\mathcal{P}os$) if $M \subseteq N_{nc}int(N_{nc}\delta cl(M))$.

(iii) δ-semi open set (briefly, $N_{nc}\delta\mathcal{S}os$) if $M \subseteq N_{nc}cl(N_{nc}\delta int(M))$.

(iv) δ-α-open set (briefly, $N_{nc}\delta\alpha os$) if $M \subseteq N_{nc}int(N_{nc}cl(N_{nc}\delta int(M)))$.

(v) δ-β-open set (briefly, $N_{nc}\delta\beta os$) if $M \subseteq N_{nc}cl(N_{nc}int(N_{nc}\delta cl(M)))$.

The complement of a $N_{nc}\delta os$ (resp. $N_{nc}\delta\mathcal{P}os$, $N_{nc}\delta\mathcal{S}os$, $N_{nc}\delta\alpha os$ & $N_{nc}\delta\beta os$) is called a N-neutrosophic crisp δ (resp. δ-pre, δ-semi, δ-α & δ-β) closed set (briefly, $N_{nc}\delta cs$ (resp. $N_{nc}\delta\mathcal{P}cs$, $N_{nc}\delta\mathcal{S}cs$, $N_{nc}\delta\alpha cs$ & $N_{nc}\delta\beta cs$)) in X.

The family of all $N_{nc}\delta\mathcal{P}os$ (resp. $N_{nc}\delta\mathcal{P}cs$, $N_{nc}\delta\mathcal{S}os$, $N_{nc}\delta\mathcal{S}cs$, $N_{nc}\delta\alpha os$, $N_{nc}\delta\alpha cs$, $N_{nc}\delta\beta os$ & $N_{nc}\delta\beta cs$) of X is denoted by $N_{nc}\delta\mathcal{P}OS(X)$ (resp. $N_{nc}\delta\mathcal{P}CS(X)$, $N_{nc}\delta\mathcal{S}OS(X)$, $N_{nc}\delta\mathcal{S}CS(X)$, $N_{nc}\delta\alpha OS(X)$, $N_{nc}\delta\alpha CS(X)$, $N_{nc}\delta\beta OS(X)$ & $N_{nc}\delta \beta\ CS(X)$).

Proposition 3.1. *Every $N_{nc}\delta os$ (resp. $N_{nc}\delta cs$) is a $N_{nc}os$ (resp. $N_{nc}cs$).*
Proof. Let M is a $N_{nc}\delta os$, then $M = N_{nc}\delta int(M) \subseteq N_{nc}int(M)$. $\therefore M$ is a $N_{nc}os$.

Similar for their respective closed sets.

Proposition 3.2. *Every $N_{nc}\delta os$ (resp. $N_{nc}\delta cs$) is a $N_{nc}os$ (resp. $N_{nc}cs$). Every $N_{nc}os$ (resp. $N_{nc}cs$) is a $N_{nc}\delta\alpha os$ (resp. $N_{nc}\delta\alpha cs$).*
Proof. Let M is a $N_{nc}os$ then $M = N_{nc}int(M)$ and so $M \subseteq N_{nc}int(N_{nc}cl(N_{nc}\ \delta\ int(M)))$. $\therefore M$ is a $N_{nc}\delta\alpha os$.

Similar for their respective closed sets.

Proposition 3.3. *Every $N_{nc}\delta\alpha os$ (resp. $N_{nc}\delta\alpha cs$) is a $N_{nc}\delta\mathcal{S}os$ (resp. $N_{nc}\delta\mathcal{S}cs$).*
Proof. Let M is a $N_{nc}\delta\alpha os$ then $M \subseteq N_{nc}int(N_{nc}cl(N_{nc}\delta int(M)))$. So $M \subseteq N_{nc}int(N_{nc}cl(N_{nc}\delta int(M))) \subseteq N_{nc}cl(N_{nc}\ \delta int(M))$. $\therefore M$ is a $N_{nc}\delta\mathcal{S}os$.

Similar for their respective closed sets.

Proposition 3.4. *Every $N_{nc}\delta\alpha os$ (resp. $N_{nc}\delta\alpha cs$) is a $N_{nc}\delta\mathcal{P}os$ (resp. $N_{nc}\delta\mathcal{P}cs$).*
Proof. Let M is a $N_{nc}\delta\alpha os$ then $M \subseteq N_{nc}int(N_{nc}cl(N_{nc}\delta int(M)))$. So $M \subseteq N_{nc}int(N_{nc}cl(N_{nc}\delta int(M))) \subseteq N_{nc}int(N_{nc}\ \delta cl(M))$. $\therefore M$ is a $N_{nc}\delta\mathcal{P}os$.

Similar for their respective closed sets.

Proposition 3.5. *Every $N_{nc}\delta\mathcal{S}os$ (resp. $N_{nc}\delta\mathcal{S}cs$) is a $N_{nc}\delta\beta os$ (resp. $N_{nc}\delta\beta cs$).*
Proof. Let M is a $N_{nc}\delta\mathcal{S}os$, then $M \subseteq N_{nc}cl(N_{nc}\delta int(M)) \subseteq N_{nc}\ cl(N_{nc}int(N_{nc}\ \delta cl(M)))$. $\therefore M$ is a $N_{nc}\delta\beta os$.

Similar for their respective closed sets.

Proposition 3.6. *Every $N_{nc}\delta\mathcal{P}os$ (resp. $N_{nc}\delta\mathcal{P}cs$) is a $N_{nc}\delta\beta os$ (resp. $N_{nc}\delta\beta cs$).*
Proof. Let M is a $N_{nc}\delta\mathcal{P}os$, then $M \subseteq N_{nc}int(N_{nc}\delta cl(M)) \subseteq N_{nc}\ cl(N_{nc}int(N_{nc}\ \delta\ cl(M)))$. $\therefore M$ is a $N_{nc}\delta\beta os$.

Similar for their respective closed sets.

Example 3.1. Let $X = \{l_1, m_1, n_1, o_1\}$, ${}_{nc}\tau_1 = \{\phi_N, X_N, \langle\{l_1, o_1\}, \{m_1, n_1\}, \{m_1, n_1\}\rangle\}$, ${}_{nc}\tau_2 = \{\phi_N, X_N\}$, then we have $2_{nc}\tau = \{\phi_N, X_N, \langle\{l_1, o_1\}, \{m_1, n_1\}, \{m_1, n_1\}\rangle\}$, then $\langle\{l_1, o_1\}, \{m_1, n_1\}, \{m_1, n_1\}\rangle$ is a $2_{nc}os$ but not $2_{nc}\delta os$.

Example 3.2. Let $X = \{l_1, m_1, n_1, o_1, p_1\}$, ${}_{nc}\tau_1 = \{\phi_N, X_N, A, B, C\}$, ${}_{nc}\tau_2 =$

$\{\phi_N, X_N\}$. $A = \langle\{n_1\},\{n_1\},\{l_1,m_1,o_1,p_1\}\rangle$, $B = \langle\{l_1,m_1\},\{n_1\},\{n_1,o_1,p_1\}\rangle$, $C = \langle\{l_1,m_1,n_1\},\{n_1\},\{o_1,p_1\}\rangle$, then we have $2_{nc}\tau = \{\phi_N, X_N, A, B, C\}$. The set

(i) $\langle\{l_1,m_1,n_1,o_1\},\{n_1\},\{p_1\}\rangle$ is a $2_{nc}\delta\alpha os$ but not $2_{nc}os$.

(ii) $\langle\{n_1,o_1\},\{n_1\},\{l_1,m_1,p_1\}\rangle$ is a $2_{nc}\delta\mathcal{S}os$ but not $2_{nc}\delta\alpha os$.

(iii) $\langle\{l_1,n_1\},\{n_1\},\{m_1,o_1,p_1\}\rangle$ is a $2_{nc}\delta\mathcal{P}os$ but not $2_{nc}\delta\alpha os$.

(iv) $\langle\{l_1\},\{n_1\},\{m_1,n_1,o_1,p_1\}\rangle$ is a $2_{nc}\delta\beta os$ but not $2_{nc}\delta\mathcal{S}os$.

(v) $\langle\{l_1,o_1\},\{n_1\},\{m_1,n_1,p_1\}\rangle$ is a $2_{nc}\delta\beta os$ but not $2_{nc}\delta\mathcal{P}os$.

Proposition 3.7. *The union (resp. intersection) of any family of $N_{nc}\delta\mathcal{P}OS(X)$ (resp. $N_{nc}\delta\mathcal{S}OS(X)$, $N_{nc}\delta\beta OS(X)$, $N_{nc}\delta\mathcal{P}CS(X)$, $N_{nc}\delta\mathcal{S}CS(X)$, $N_{nc}\delta\beta CS(X)$) is a $N_{nc}\delta\beta OS(X)$ (resp. $N_{nc}\delta\mathcal{S}OS(X)$, $N_{nc}\delta\beta OS(X)$, $N_{nc}\delta\mathcal{P}CS(X)$, $N_{nc}\delta\ \mathcal{S}CS(X)$, $N_{nc}\delta\beta CS(X)$).*

Remark 3.1. *The intersection of two $N_{nc}\delta\mathcal{S}os$ (resp, $N_{nc}\delta\mathcal{P}os$ & $N_{nc}\delta\beta os$)'s need not be $N_{nc}\delta\mathcal{S}os$ (resp, $N_{nc}\delta\mathcal{P}os$ & $N_{nc}\delta\beta os$.*

Example 3.3. In Example 3.2, The sets

(i) $\langle\langle\{l_1,m_1,o_1\},\{n_1\},\{n_1,p_1\}\rangle\rangle$ and $\langle\{n_1,o_1,p_1\},\{n_1\},\{l_1,m_1\}\rangle$ are $2_{nc}\delta\mathcal{S}os$ but the intersection $\langle\{o_1\},\{n_1\},\{l_1,m_1,n_1,\ p_1\}\rangle$ is not $2_{nc}\delta\mathcal{S}os$.

(ii) $\langle\langle\{l_1,n_1,o_1\},\{n_1\},\{m_1,p_1\}\rangle\rangle$ and $\langle\{m_1,n_1,o_1\},\{n_1\},\{l_1,p_1\}\rangle$ are $2_{nc}\delta\mathcal{P}os$ but the intersection $\langle\{n_1,o_1\},\{n_1\},\{l_1,m_1,\ p_1\}\rangle$ is not $2_{nc}\delta\mathcal{P}os$.

(iii) $\langle\langle\{l_1,m_1,p_1\},\{n_1\},\{n_1,o_1\}\rangle\rangle$ and $\langle\{m_1,n_1,p_1\},\{n_1\},\{l_1,o_1\}\rangle$ are $2_{nc}\delta\beta os$ but the intersection $\langle\{m_1,p_1\},\{n_1\},\{l_1,n_1,\ p_1\}\rangle$ is not $2_{nc}\delta\beta os$.

Proposition 3.8. *The $N_{nc}\delta$-interior operator satisfies*

(i) $N_{nc}\delta int(M) \subseteq M$.

(ii) $M \subseteq E \Rightarrow N_{nc}\delta int(M) \subseteq N_{nc}\delta int(E)$.

(iii) $N_{nc}\delta int(M \cap E) = N_{nc}\delta int(M) \cap N_{nc}\ \delta int(E)$.

(iv) $N_{nc}\delta int(M)$ *is the largest* $N_{nc}\delta os$ *containing* M.

(v) $N_{nc}\delta int(M) = M$ *iff* M *is an* $N_{nc}\delta os$.

(vi) $N_{nc}\delta int(N_{nc}\delta int(M)) = N_{nc}\delta int(M)$.

(vii) $(X\backslash N_{nc}\delta int(M)) = N_{nc}\delta cl(X\backslash M)$.

Proof.

(i) $N_{nc}\delta int(M) = \cup\{A : A \subseteq M \; \& \; A \text{ is a } N_{nc}ros\}$. Thus, $N_{nc}\delta int(M) \subseteq M$.

(ii) $N_{nc}\delta int(E) = \cup\{A : A \subseteq E \; \& \; A \text{ is a } N_{nc}ros\} \supseteq \cup\{A : A \subseteq M \; \& \; A \text{ is a } N_{nc}ros\} \supseteq N_{nc}\ \delta int(M)$. Thus, $N_{nc}\ \delta int(M) \subseteq N_{nc}\ \delta int(E)$.

(iii) $N_{nc}\delta int(M \cap E) = \cup\{A : A \subseteq M \cap E \; \& \; A \text{ is a } N_{nc}ros\} = (\cup\{A : A \subseteq M \; \& \; A \text{ is a } N_{nc}ros\}) \cap (\cup\{A : A \subseteq E \; \& \; A \text{ is a } N_{nc}ros\}) = N_{nc}\delta int(M) \cap N_{nc}\delta int(E)$. Thus, $N_{nc}\delta int(M \cap E) = N_{nc}\delta int(M) \cap N_{nc}\delta int(E)$.

(iv) If A is any $N_{nc}\delta os$ contained in M, then $A \subseteq N_{nc}\delta int(M)$. Hence, $N_{nc}\delta int(M)$ is the largest $N_{nc}\delta os$ containing M.

(v) Suppose M is any $N_{nc}\delta os$ of X. Then the largest $N_{nc}\delta os$ contained in M is itself. Therefore, $N_{nc}\delta int(M) = M$.

(vi) By (iv), the largest $N_{nc}\delta os$ containing $N_{nc}\delta int(M)$ is itself. Hence, $N_{nc}\delta\ int(N_{nc}\ \delta int(M)) = N_{nc}\ \delta int(M)$.

(vii) $N_{nc}\delta int(M)$ is the largest $N_{nc}\delta os$ containing M. The complement is the smallest $N_{nc}\delta cs$ contained in $X\backslash M$. Therefore, $(X\backslash N_{nc}\delta int(M)) = N_{nc}\delta\ cl(X\ \backslash M)$.

Hence the proof.

Proposition 3.9. *The* $N_{nc}\delta$*-closure operator satisfies*

(i) $M \subseteq N_{nc}\delta cl(M)$.

(ii) $M \subseteq E \Rightarrow N_{nc}\delta cl(M) \subseteq N_{nc}\delta cl(E)$.

(iii) $N_{nc}\delta cl(M \cup E) = N_{nc}\delta cl(M) \cup N_{nc}\delta cl(E)$.

(iv) $N_{nc}\delta cl(M)$ *is the smallest* $N_{nc}\delta c$ *set containing* M.

(v) $N_{nc}\delta cl(M) = M$ *iff* M *is an* $N_{nc}\delta c$ *set.*

(vi) $N_{nc}\delta cl(N_{nc}\delta cl(M)) = N_{nc}\delta cl(M)$.

(vii) $(X\backslash N_{nc}\delta cl(M)) = N_{nc}\delta int(X\backslash M)$.

(viii) $y \in N_{nc}\delta cl(M)$ *iff* $M \cap C \neq \phi$ *for every* $N_{nc}\delta os$ C *containing* y.

Proof. (viii) Suppose $y \in N_{nc}\delta cl(M)$. Let C be a $N_{nc}\delta os$ containing y. If $M \cap C = \phi$, then $X \backslash C$ is a $N_{nc}\delta cs$ containing M and so $y \notin N_{nc}\delta cl(M)$, a contradiction. Therefore, $M \cap C \neq \phi$. If $y \notin N_{nc}\delta cl(M)$, then there exists a $N_{nc}\delta cs$ D containing M such that $y \notin D$. Then $C = X \backslash D$ is a $N_{nc}\delta os$ containing y such that $M \cap C = \phi$, a contradiction. Therefore, $y \in N_{nc}\delta cl(M)$.

The other cases are follows from Proposition 3.8.

4. Conclusion

We have studied some new notions of strongly N_{nc} open (closed) sets called $N_{nc}\delta$-open and $N_{nc}\delta$-closed sets and their respective interior and closure operators in this paper. Also, $N_{nc}\delta\alpha$-open, $N_{nc}\delta\alpha$-closed, $N_{nc}\delta\mathcal{S}$-open, $N_{nc}\delta\mathcal{S}$-closed, $N_{nc}\delta\mathcal{P}$-open, $N_{nc}\delta\mathcal{P}$-closed, $N_{nc}\delta\beta$-open and $N_{nc}\delta\beta$-closed sets are introduced. Also studied some of their fundamental properties in $N_{nc}ts$. In our next work, this can be extended to $N_{nc}\delta$-continuous mappings in $N_{nc}ts$ and also their relationship with near mappings such as $N_{nc}\delta\alpha Cts$, $N_{nc}\delta\mathcal{S}Cts$, $N_{nc}\delta\mathcal{P}Cts$ and $N_{nc}\delta\beta Cts$. Also, their $N_{nc}\delta$ open and $N_{nc}\delta$ closed mappings in $N_{nc}ts$.

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