South East Asian J. of Mathematics and Mathematical Sciences
Vol. 18, No. 3 (2022), pp. 33-42

DOI: 10.56827/SEAJMMS.2022.1803.4 **ISSN (Online): 2582-0850**

ISSN (Print): 0972-7752

$M-N$ ANTI HOMOMORPHISM OF AN $M-N$ FUZZY SOFT SUBGROUPS AND ITS LEVEL $M-N$ SUBGROUPS

M. Kaliraja and S. Rumenaka

PG and Research Department Mathematics,
H. H. The Rajah's College,
Pudukkottai - 622001, Tamil Nadu, INDIA

E-mail : mkr.maths009@gmail.com, rumenaka@gmail.com

(**Received:** *Oct. 26, 2021* **Accepted:** *Oct. 18, 2022* **Published:** *Dec. 30, 2022)*

Abstract: In this paper, we have discussed the concept of $M-N$ anti homomorphism of fuzzy soft subgroups, then we define the $M-N$ anti level subsets of a fuzzy soft subgroup and its some elementary properties are also discussed.

Keywords and Phrases: Fuzzy group, M-N anti fuzzy group, M-N anti fuzzy soft subgroups, $M-N$ anti level subset, $M-N$ anti homomorphism of fuzzy soft subgroups.

2020 Mathematics Subject Classification: 20N25, 06D72, 20E15, 22F05.

1. Introduction

There are various types of uncertainties in the real world, but few classical mathematical tools may not be suitable to model these uncertainties. Many intricate problems in economics, social science, engineering, medical science and many other fields involve undefined data. These problems which one comes face to face with in life cannot be solved using classical mathematical methods. In classical mathematics, a mathematical model of an object is devised and the concept of the exact solution of this model is not yet determined. Since, the classical mathematical model is too complex, the exact solution cannot be found. There are several well renowned theories available to describe uncertainty. For instance, In [15], Rosenfeld introduced the concept of fuzzy subgroup in 1971 and the theory of fuzzy sets was inspired by Zadeh [19] in addition to this, Molodtsov [11] have introduced the concept of soft sets in 1999. Furthermore, In 2009, Majiet. al., introduced the concept of fuzzy soft sets in [9] and Jacobson introduced the concept of M-group M-subgroup in [6].

VasanthaKandasamy and Smarandache [17] have introduced the Fuzzy Algebra during 2003. An introduction to the new definition of Soft sets and soft groups depending on inclusion relation and intersection of sets were exposed by Akta and Cagman [1]. MouradOqlaMassadeh, discussed the $M-N$ homomorphism and $M-N$ Anti homomorphism of an Over $M-N$ fuzzy subgroups in 2012 [12]. In 2010, R.Muthuraj et al., [10] introduced the M-Homomorphism and M-Anti Homomorphism of an M-Fuzzy Subgroup and its Level M-Subgroups. In [2] Biswas introduced the concept of anti- fuzzy subgroup of groups. Shen researched anti-fuzzy subgroups in [16] and Dong [3] studied the product of anti- fuzzy subgroups. Feng and Yao [5] studied the concept of (λ, μ) anti- fuzzy subgroups. Pandiamml et al, (2010) defined a new algebraic structure of anti L-fuzzy normal M-subgroups In our earlier work we have discussed the concept of $M-N$ anti fuzzy soft subgroups in [7, 8].

In the present manuscript, we have discussed the concept of $M-N$ anti homomorphism of fuzzy soft subgroups based on the concept of fuzzy soft groups. In section 2 , we presented the basic definition, notations on $M-N$ anti fuzzy soft subgroups and required results on fuzzy soft $M-N$ anti level subsets of a fuzzy soft subgroup. We have also discussed the concept of M-N anti homomorphism of fuzzy soft subgroups and some of its elementary properties.

2. Preliminaries

In this section, some basic definitions and results needed are given. For the sake of convenience we set out the former concepts which will be used in this paper.

Definition 2.1. [19] *Let X be a non - empty set. A fuzzy subset A of X is a function $A : X \to [0, 1]$.*

Example 2.2. Let $X = \{1, 2, 3, 4, 5\}$ be a set. Then $A = \{(1, 0.2), (2, 0.4), (3, 0.6), (4, 0.7), (5, 0.8)\}$ is a fuzzy subset of X.

Definition 2.3. [19] *The standard union of two fuzzy subset A and B of a set X is defined by $(A \cup B)(x) = \max(A(x), B(x))$ for all x in X.*

Example 2.4. Let $A = \{(1, 0.2), (2, 0.6), (3, 0.3)\}$ and $B = \{(1, 0.4), (2, 0.2), (3, 0.8)\}$ be two fuzzy subset of a set $X = \{1, 2, 3\}$. The standard union of the fuzzy subsets A and B is $A \cup B = \{(1, 0.4), (2, 0.6), (3, 0.8)\}$.

Definition 2.5. [19] *The standard intersection of two fuzzy subset A and B of a set X is defined by $(A \cap B)(x) = \min\{A(x), B(x)\}$ for all x in X.*

Example 2.6. Let $A=\{(1,0.1),(2,0.5),(3,0.9)\}$ and $B=\{(1,0.8),(2,0.6),(3,0.4)\}$ be two fuzzy subset of a set $X=\{1,2,3\}$. The standard intersection of the fuzzy subsets A and B is $A\cap B=\{(1,0.1),(2,\ 0.5),(3,0.4)\}$.

Definition 2.7. [7] *Let G be a group. A fuzzy subset A of G is called an anti fuzzy subgroup if for $x,y\in G$*

1. $A(xy)\leq\min\{A(x),A(y)\}$

2. $A\left(x^{-1}\right)=A(x)$

Theorem 2.8. [7] *Let G be an $M-N$ group, A and B both be $M-N$ anti fuzzy subgroup of G. Then $A\cup B$ is an $M-N$ anti fuzzy subgroup of G.*

Example 2.9. $A:G\to[0,1]$ where $G=\{1,-1,i,-i\}$ defined as $A(1)=0.2$, $A(-1)=0.4, A(i)=0.6, A(-i)=0.6$, where A is an anti fuzzy subgroup.

Definition 2.10. [7] *Let A be an anti fuzzy subgroup of a group G. For $t\in[0,1]$ the anti level subset of A is the set $A_t=\{x\in G/A(x)\leq t\}$. This is called an anti level subset of A.*

Definition 2.11. [1] *Let U be an initial universe set and E be the set of parameter. Let $P(U)$ denote the power set of U. A pair (F,E) is called a soft set over U, where F is mapping given by $F:E\to P(U)$.*

Example 2.12. [1] Let U is the set of house under consideration. E is the set of parameters. Each parameter is a word of a sentence $E=\{$ expensive, beautiful, wooden, and cheap, in the green surrounding, modern, in a good repair, in bad repair $\}$. In this case, to define a set of means to point out expensive houses, beautiful houses and so on. The soft set (F,E) describes the attractiveness of the houses which Mr (say) X is going to buy.

Definition 2.13. [1] *Let (F,A) be a soft set over G. Then (F,A) is called a soft group over G if $F(a)$ is a group G for all $a\in A$. Here (F,A) means f_a, f_a is a fuzzy soft subgroup.*

Definition 2.14. [1] *Let U be an initial universe set and E be the set of parameters. Let $A\sqsubseteq E$. A pair (F,A) is called fuzzy soft over U, where F is a mapping given by $F:A\to I^U$ where I^U denote the collection of all fuzzy subset of U. Here I means $[0,1]$.*

Example 2.15. [1] Let $U=\{C_1,C_2,C_3,C_4,C_5\}$ be the set of four cars under consideration and $E=\{$ e_1(Costly), e_2(Beautiful), e_3(Fuel Efficient), e_4(Modern Technology), e_5 (Luxurious), $\}$ be the set of parameters and $A=\{e_1,e_2,e_3\}\sqsubseteq E$, then

$$(F, A) = \{F(e) = \{C_1/0.7, C_2/0.1, C_3/0.2, C_4/0.6\}$$
$$F(e_2) = \{C_1/0.8, C_2/0.6, C3/0.1, C_4/0.5\}$$
$$F(e_3) = \{C_1/0.1, C_2/0.2, C_3/0.7, C_4/0.3\}\}$$

In this fuzzy soft set representing the attractiveness of the car which $Mr.X$ is going to buy.

Definition 2.16. [1] *Let (F, A) and (G, B) be two fuzzy soft set over U. Then (F, A) is called a fuzzy soft subset of (G, B) denoted by $(F, A) \subseteq (G, B)$ if*

1. *$A \subseteq B$*
2. *$F(a)$ is a fuzzy subset of $G(a)$, for each $a \in A$.*

Definition 2.17. [12] *Let M, N be left and right operator sets of group G respectively, if $(mx)n = m(xn)$ for all $x \in G$, $m \in M, n \in N$. Then G is said be an $M - N$ group.*

Definition 2.18. [7] *Let G be a group and (f, A) be an $M - N$ fuzzy soft set over G. Then (f, A) is said to be a $M - N$ anti fuzzy soft group over G iff for each $a \in A$ and $x, y \in G$,*

1. *$f_a(m(xy)n\} \leq \min\{f_a(x), f_a(y)\}$*
2. *$f_a\{(mx^{-1})n\} \leq f_a(x)$ hold for each $a \in A, m \in M, n \in N, f_a$ is a $M - N$ anti fuzzy soft subgroup of a group G.*

Definition 2.19. [7] *Let G be an $M - N$ group and (f, A) be a fuzzy soft subgroup of G if*

(1) $f_a(mx) \leq f_a(x)$

(2) $f_a(xn) \leq f_a(x)$ hold for any $x \in G, m \in M, n \in N$ and $a \in A$, then (f, A) is said be an $M - N$ anti fuzzy soft subgroup of G.

Definition 2.20. [10] *A mapping f from a group G into a group G' is said be an anti homomorphism if for all $x, y \in G$, $f(xy) = f(x)f(y)$.*

3. $M - N$ fuzzy soft subgroups of an M − N group G under M − N anti homomorphism

In this section, we shall define the $M - N$ anti homomorphism of fuzzy soft subgroup and define the $M - N$ anti level subsets of a fuzzy soft subgroup. We have also discussed the concept of $M-N$ anti homomorphism of fuzzy soft groups and some of its elementary properties.

Definition 3.1. *Let G and G' be any two $M-N$ groups. If (f, A) is an fuzzy soft subgroup of an $M-N$ group G, then the function $f_a : G \to G^1$ is said be an $M-N$ anti homomorphism of fuzzy soft subgroup if*

(1) $f_a(xy) = f_a(x)f_a(y)$ *for all* $x, y \in G, a \in A$

(2) $f_a(mx) = mf_a(x)$, *for all* $x \in G, a \in A$ *and* $m \in M$

(3) $f_a(yn) = nf_a(y)$, *for all* $y \in G, a \in A$ *and* $n \in N$

Note 3.2. *If λ is a constant and* $\ker f_a$ *is an $M-N$ anti fuzzy soft subgroup, then*

(1) $f_a(\lambda)f_a(mx) = \lambda(mx) = \lambda(x)$, *for all* $x \in G, a \in A$ *and* $m \in M$.

(2) $f_a(\lambda)f_a(xn) = \lambda(xn) = \lambda(x)$, *for all* $x \in Ga \in A$ *and* $n \in N$.

Theorem 3.3. *Let f_a be an $M-N$ anti homomorphism of fuzzy soft subgroup from an $M-N$ group G onto an $M-N$ group G'. If λ is an $M-N$ fuzzy subgroup of G and λ is an f_a- soft invariant, then $f_a(\lambda)$, the image of λ under f_a is an $M-N$ anti fuzzy soft subgroup of G^l.*

Proof. We know that λ is a constant and $\ker f_a$ is an $M-N$ anti fuzzy soft subgroup. Now,

$$\begin{aligned} f_a(\lambda)\left(f_a(x)f_a(y)\right) &= f_a(\lambda)\left(f_a(xy)\right), \text{ for all } x, y \in G, a \in A \\ &= \lambda(xy), \text{ since by the Note 3.2} \\ &\leq \max\{\lambda(x), \lambda(y)\}, \text{ since by the Definition 2.7} \\ &\leq \max\left\{f_a(\lambda)\left(f_a(x), f_a(\lambda)f_a(y)\right)\right\}, \text{ since by the Note 3.2 .} \end{aligned}$$

Therefore $f_a(\lambda)\left(f_a(x)f_a(y)\right) \leq \max\left\{f_a(\lambda)\left(f_a(x), f_a(\lambda)f_a(y)\right)\right\}$.
Clearly $f_a(\lambda)$ is an anti fuzzy soft subgroup of G^l.
To prove that $f_a(\lambda)$ is an $M-N$ anti fuzzy soft subgroup of G^1.
Let $f_a(\lambda) \in G^l$, then

(1) $$\begin{aligned} f_a(\lambda)\left(mf_a(x)\right) &= f_a(\lambda)\left(f_a(mx)\right) \\ &= \lambda(mx) \\ &\leq \lambda(x), \text{ by the definition } A(mx) \leq A(x) \\ &= f_a(\lambda)f_a(x), \text{ by the Note 3.2} \end{aligned}$$

Therefore $f_a(\lambda)\left(mf_a(x)\right) = f_a(\lambda)f_a(x)$.

$$(2)\ \ f_a(\lambda)\,(f_a(x)n) = f_a(\lambda)\,(f_a(xn)) = \lambda(xn) \leq \lambda(x), \text{ by the Note 3.2}$$

Hence $f_a(\lambda)$ is an $M-N$ anti fuzzy soft subgroup of G^1.

Theorem 3.4. *The $M-N$ anti homomorphic pre- image of an $M-N$ anti fuzzy soft subgroup of an $M-N$ group G^1 is an $M-N$ anti fuzzy soft subgroup of an $M-N$ group G.*

Proof. Let $f_a : G \to G^1$ is said be an $M-N$ anti homomorphism of fuzzy soft subgroup.

Let μ be a fuzzy set on the $M-N$ anti fuzzy subgroup of G^l. Now

$$\begin{aligned}\lambda(xy) &= \mu\,(f_a(xy)) \text{ for all } a \in A \text{ and } x, y \in G\\ &= \mu\,(f_a(x)f_a(y))\,, \text{ since } f_a \text{ is an anti homomorphism}\\ &\leq \max\{\mu f_a(x), \mu f_a(y)\}\,, \text{ since } \mu \text{ is an anti fuzzy subgroup of } G^1\\ &= \max\{\lambda(x), \lambda(y)\}\end{aligned}$$

That is $\lambda(xy) \leq \max\{\lambda(x), \lambda(y)\}$.
Let $x \in G$,

$$\begin{aligned}\lambda\left(x^{-1}\right) &= \mu\left(f_a\left(x^{-1}\right)\right)\\ &= \mu\,(f_a(x))^{-1}\,, \text{ since } f_a \text{ is an anti homomorphism of fuzzy soft subgroup}\\ &= \mu\,(f_a(x))\,, \text{ since } \mu \text{ is an } M-N \text{ anti fuzzy subgroup of } G^1\\ &= \lambda(x)\\ \lambda\left(x^{-1}\right) &\leq \lambda(x)\end{aligned}$$

Clearly, $n \in N \Rightarrow m \in M$

$$\begin{aligned}\lambda(mx) &= \mu\,(f_a(mx))\,, \text{ for all } n \in N \text{ and } x \in G\\ &= \mu\,(mf_a(x))\,, \text{ since } f_a \text{ is an } M-N \text{ anti homomorphism of an fuzzy soft group}\\ &\leq \mu f_a(x), \text{ since } \mu M-N \text{ anti fuzzy subgroup of } G\\ &= \lambda(x)\end{aligned}$$

That is $\lambda(mx) \leq \lambda(x)$. Next

$$\begin{aligned}\lambda(xn) &= \mu\,(f_a(xn))\,, \text{ for all } n \in N \text{ and } x \in G\\ &= \mu\,(nf_a(x))\,, \text{ since } f_a \text{ is an } M-N \text{ anti homomorphism of an fuzzy soft group}\\ &\leq \mu f_a(x), \text{ since } \mu M-N \text{ anti fuzzy subgroup of } G\\ &= \lambda(x)\end{aligned}$$

That is $\lambda(xn) \leq \lambda(x)$.
Hence λ is an $M-N$ anti fuzzy subgroup of G.

Theorem 3.5. *If $f_a : G \to G^1$ is an $M-N$ anti homomorphism of a fuzzy soft subgroup of a group G, then*

(1) $f_a(e) = e^l$, where e^l is the unit element of G^l

(2) $f_a(x^{-1}) = (f_a(x))^{-1}$ for all $x \in G$.

Proof. Given that $f_a : G \to G^1$ is an $M-N$ anti homomorphism of an fuzzy soft subgroup of a group G.

$$\begin{aligned} (1) \Rightarrow \text{ Suppose } f_a(mx)e^1 &= f_a(mx) = f_a(x), \text{ for some } m \in M,\ a \in A \text{ and } x \in G \\ &= f_a(xe), \text{ since } e \text{ is an identity element in } G \\ &= f_a(x)f_a(e), \text{ since } f_a \text{ is an anti homomorphism of} \\ &\qquad \text{an fuzzy soft subgroup} \\ f_a(x)e^1 &= f_a(x)f_a(e), \text{ by left cancellation law} \end{aligned}$$

Therefore $f_a(e) = e^1$
Similarly, we can prove that $f_a(xn)e^1 = f_a(xn) = f_a(x)$, for some $n \in N, a \in A$ and $x \in G$. That implies $f_a(e) = e^1$.

$$\begin{aligned} (2) \Rightarrow \text{ We know that } e^1 &= fa(me), \text{ since } A(mx) \leq A(x) \\ &= fa(e) \\ &= fa\left(xx^{-1}\right) \\ &= f_a(x)f_a(x^{-1}) \ f_a \text{ is an anti homomorphism of an fuzzy} \\ &\qquad \text{soft subgroup} \\ e^1(f_a(x))^{-1} &= f_a(x^{-1}) \end{aligned}$$

Similarly, we can prove that $e^1 = f_a(en)$, since $A(xn) \leq A(x)$. That implies $(f_a(x))^{-1} = f_a(x^{-1})$. Hence the proof.

Definition 3.6. *Let μ be an $M-N$ anti fuzzy subgroup of an $M-N$ group G. Then the $M-N$ subgroup μ_t for $t \in [0,1]$ and $t \leq \mu(e)$ are called anti level $M-N$ subgroup of μ.*

Theorem 3.7. *The $M-N$ anti homomorphic image of a level $M-N$ subgroup of an $M-N$ fuzzy subgroup μ of an $M-N$ group G is a level $M-N$ subgroup of an $M-N$ fuzzy soft subgroup $f_a(\mu)$ of an $M-N$ soft subgroup G^1, where μ is*

f_a- *soft invariant.*
Proof. Let G and G^1 be any two $M-N$ group.
Let $f_a : G \to G^1$ be an $M-N$ anti homomorphism of an fuzzy soft subgroup of a group G.
Let μ be an $M-N$ fuzzy subgroup of G.
Clearly $f_a(\mu)$ is an $M-N$ fuzzy soft subgroup of G^1.
Let μ_t be a level $M-N$ subgroup of an $M-N$ fuzzy subgroup μ of G.
Since f_a is an $M-N$ anti homomorphism fuzzy soft subgroup, $f_a(\mu)$ is an $M-N$ soft subgroup $f_a(\mu)$ of G^1 and $f_a(\mu_t) = (f_a(\mu))_t$.
Hence $(f_a(\mu))_t$ is a level $M-N$ soft subgroup $f_a(\mu)$ of G^1.

Theorem 3.8. *The $M-N$ anti homomorphism pre - image of a level $M-N$ soft subgroup of an $M-N$ fuzzy subgroup μ of an $M-N$ group G^1 is a level $M-N$ subgroup of an $M-N$ fuzzy soft subgroup $f_a^{-1}(\mu)$ of an $M-N$ group G.*
Proof. Let G and G^1 be any two $M-N$ group.
Let $f_a : G \to G^1$ be an $M-N$ anti homomorphism of a fuzzy soft subgroup of a group G.
Let μ be an $M-N$ fuzzy subgroup of G^1.
Clearly $f_a^{-1}(m\mu) = f_a^{-1}(\mu)$ and $f_a^{-1}(\mu n) = f_a^{-1}(\mu)$ is an $M-N$ fuzzy soft subgroup of G.
Let μ_t be a level $M-N$ subgroup of an $M-N$ fuzzy subgroup μ of G^l.
Since f_a is an $M-N$ anti homomorphism fuzzy soft subgroup, $f_a^{-1}(\mu_t)$ is an $M-N$ soft subgroup of $f_a^{-1}(\mu)$ of G and $f_a^{-1}(\mu_t) = (f_a^{-1}(\mu))_t$, is an $M-N$ soft subgroup of an $M-N$ fuzzy soft subgroup $f_a^{-1}(\mu)$ of G.
That is $(f_a^{-1}(\mu))_t$ is a level $M-N$ subgroup of an $M-N$ fuzzy soft subgroup $f_a^{-1}(\mu)$ of G. Hence the proof.

4. Conclusion

The main results in the present manuscript are based on the concept of $M-N$ anti homomorphism of fuzzy soft subgroup [10] and [12]. We have also defined the $M-N$ level subsets of fuzzy soft subgroup and its some elementary properties are discussed.

References

[1] Aktaş, H, and Naim Çağman, Soft sets and soft groups, Information sciences, 177, 13 (2007), 2726-2735.

[2] Biswas, Ranjith, Fuzzy subgroups and anti fuzzy subgroups, Fuzzy sets and systems, 35, 1 (1990), 121-124.

[3] Das, P. Sivaramakrishna, Fuzzy groups and level subgroups, Journal of mathematical analysis and applications, 84, 1 (1981), 264-269.

[4] Dong, B., Direct product of anti fuzzy subgroups, J Shaoxing Teachers College, 5 (1992), 29-34.

[5] Feng, Yuming, and Bingxue Yao., On (λ, μ)-anti-fuzzy subgroups, Journal of Inequalities and Applications, 2012, 1 (2012), 1-5.

[6] Jacobson, Nathan, Lectures in Abstract Algebra: II. Linear Algebra, Vol. 31, Springer Science & Business Media, 2013.

[7] Kaliraja M. and Rumenaka S., M-N Anti Fuzzy Soft Groups, Journal of Computational Information Systems, (2019).

[8] Kaliraja M. and Rumenaka S., M - N Anti Fuzzy Normal Soft Groups, Int. J. Math and Appl, 6(1-E), (2018), 1035-1012.

[9] Maji, Pabitra Kumar, R. K. Biswas, and A. Roy, Fuzzy soft sets, (2001), 589-602.

[10] Muthuraj, R., M. Rajinikannan, and M. S. Muthuraman, The M - Homomorphism and M-Anti Homomorphism of an M-Fuzzy Subgroup and its Level M - Subgroups, International journal of computer applications, 2, 1 (2010), 65-70.

[11] Molodtsov, Dmitriy, Soft set theory-first results, Computers & mathematics with applications, 37, 4-5 (1999) 19-31.

[12] Massa'deh, Mourad Oqla, The MN-homomorphism and MN-anti homomorphism over MN-fuzzy subgroups, International Journal of Pure and Applied Mathematics, 78, 7 (2012), 1019-1027.

[13] Pandiammal, P., Natarajan R., and Palaniappan N., Anti L-fuzzy normal M-subgroups, International Journal of Computer Applications, 975 (2010), 8887.

[14] Patel, Hitesh R., et al., On normal fuzzy soft group, Mathematical Theory and Modeling, 5, 7 (2015), 26-32.

[15] Rosenfeld, Azriel, Fuzzy groups, Journal of mathematical analysis and applications, 35, 3 (1971), 512-517.

[16] Shen, Z., The anti-fuzzy subgroup of a group, J. Liaoning Norm. Univ. Nat. Sci, 18, 2 (1995), 99-101.

[17] Kandasamy, Vasantha. WB, Smarandache Fuzzy Algebra, (2003).

[18] Wu, Wangming, Normal fuzzy subgroups, Fuzzy Math, 1, 1 (1981), 21-30.

[19] Zadeh, Lotfi A., Fuzzy sets, Information and control, 8, 3 (1965), 338-353.