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EVEN RADIO MEAN GRACEFUL LABELING ON DEGREE SPLITTING OF SNAKE RELATED GRAPHS

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Abstract: A radio mean labeling of a connected graph G is an injection ϕ from the vertex set V(G) to N such that the condition $d(u,v) + \left\lceil \frac{\phi(u)+\phi(v)}{2} \right\rceil \geq 1 + diam(G)$ holds for any two distinct vertices u and v of G. A graph which admits radio mean labeling is called radio mean graph. The radio mean number of ϕ, rmn(ϕ), is the maximum number assigned to any vertex of G. The radio mean number of G, rmn(G), is the minimum value of rmn(ϕ) taken over all radio mean labeling ϕ of G. In this paper we introduce a new concept even radio mean graceful labeling and we investigate the even radio mean graceful labeling on degree splitting of snake related graphs.

Keywords and Phrases: Radio mean graceful labeling, even radio mean graceful labeling, degree splitting graph, triangular snake graph, quadrilateral snake graph.

2020 Mathematics Subject Classification: 05C78.

1. Introduction

A graph labeling is an assignment of integers to the vertices or edges or both subject to certain conditions. Chartrand et al developed the concept of radio labeling of graphs in [1]. Somasundaram and Ponraj introduced the notion of mean labeling of graphs in [10]. Radio mean labeling was introduced by Ponraj et al in [7]. Sampathkumar and Walikar introduced notion of the splitting graph of a graph in [9]. Ponraj and Somasundaram developed the concept of degree splitting of graphs in [6]. Somasundaram, Sandhya and Viji introduced the concept of geometric mean labeling on degree splitting graphs in [11]. Revathi found the vertex odd mean and even mean labeling of some graphs in [8]. David Raj, Sunitha and Subramanian introduced radio odd mean and even mean labeling of some graphs in [2]. Lavanya et al introduced the new concept radio mean graceful graphs in [5]. In this paper we investigate the even radio mean graceful labeling on degree splitting of snake related graphs. Throughout this paper we consider simple, undirected, finite and connected graphs. $\lceil x \rceil$ is the smallest integer greater than or equal to x, for any real x. For graph theoretic terminology, we refer to Harary [4], and for a detailed survery of graph labeling we refer to Gallian [3]. The notations V(G) is the vertex set of G, d(u, v) is the distance between the vertices u and v, diam(G) is the diameter of G, DS(G) is the degree splitting of graph G and $\left|\text{V}\right|$ is the order of a graph G.

2. Definitions

2.1. Triangular snake Graph

A triangular snake graph (T_n) is obtained from a path $s_1 s_2 ... s_n$ by joining s_i and s_{i+1} to a new vertex t_i for $1 \leq i \leq n-1$. That is, every edge of path is replaced by a triangle C_3.

2.2. Quadrilateral snake Graph

A quadrilateral snake graph (Q_n) is obtained from a path $s_1 s_2 ... s_n$ by joining s_i and s_{i+1} to two new vertices t_i and u_i, $1 \leq i \leq n-1$ respectively and join t_i and u_i. That is, every edge of path is replaced by a cycle C_4.

2.3. Degree Splitting graph of G

Let G = (V, E) be a graph with V = $S_1 \cup S_2 \cup ... S_i \cup T$, where each S_i is a set of vertices having atleast two vertices and having the same degree and $T = V - \cup S_i$. The degree splitting graph of G, denoted by DS(G), is obtained from G by adding vertices $w_1, w_2, ..., w_t$ and joining w_i to each vertex of S_i, $1 \leq i \leq t$.

2.4. Even Radio Mean Graceful graph

Even radio mean graceful labeling is a bijection $\phi :$ V(G) $\rightarrow \left\{2, 4, 6, ..., 2\left|V\right|\right\}$

satisfying the condition $d(u,v)+\left\lceil \frac{\phi(u)+\phi(v)}{2} \right\rceil \geq 1+diam(G)$, for every u, v $\in$ V(G).

A graph which admits even radio mean graceful labeling is called an even radio mean graceful graph.

Result: Any graph with diameter 1, 2 is obviously an even radio mean graceful graph.

3. Main Results

Theorem 3.1. *DS(T_n) is an even radio mean graceful graph.*
Proof. Let s_i, $1 \leq i \leq n$ be the vertices of path P_n. Join s_i and s_{i+1} to a new vertex t_i, $1 \leq i \leq n-1$. The graph thus obtained is triangular snake graph (T_n).

Case 1: n = 2,3
Introduce a new vertex u and join it with the vertices of T_n of degree two. The resultant graph is $DS(T_n)$ whose vertex set is V = $\{s_i, 1 \leq i \leq n\} \cup \{t_i, 1 \leq i \leq n-1\}$ $\cup \{u\}$. Clearly the diam$(DS(T_n))$ = $\begin{cases} 1 & if \text{ n} = 2 \\ 2 & if \text{ n} = 3 \end{cases}$

Therefore the condition $d(u,v)+\left\lceil \frac{\phi(u)+\phi(v)}{2} \right\rceil \geq 1+diam(G)$, is obviously satisfied for all the pair of vertices u, v $\in$ V(G).

Case 2: n > 3
Introduce two new vertices u, v and join them with the vertices of T_n of degree two and four respectively. The resultant graph is $DS(T_n)$ whose vertex set is V = $\{s_i, 1 \leq i \leq n\} \cup \{t_i, 1 \leq i \leq n-1\} \cup \{u, v\}$. Clearly the diam$(DS(T_n)$) = 3. Define a bijection $\phi : V(DS(T_n)) \to \{2,4,6,...2|V|\}$ by $\phi(s_i) = 2i, 1 \leq i \leq n$, $\phi(t_i) = 2n+2i+4, 1 \leq i \leq n-1$,$\phi(u) = 2n+2$, $\phi(v) = 2n+4$.
Now we find the even radio mean graceful condition for ϕ,
Subcase(i): Examine the pair (s_i, s_j), $1 \leq i \leq n-1,\ i+1 \leq j \leq n$;

$$d(s_i,s_j)+\left\lceil \frac{\phi(s_i)+\phi(s_j)}{2} \right\rceil \geq 1+\left\lceil \frac{2i+2j}{2} \right\rceil \geq 4 = 1+diam(DS(T_n)).$$

Subcase(ii): Examine the pair (s_i, t_j), $1 \leq i \leq n,\ 1 \leq j \leq n-1$;

$$d(s_i,t_j)+\left\lceil \frac{\phi(s_i)+\phi(t_j)}{2} \right\rceil \geq 1+\left\lceil \frac{2n+2i+2j+4}{2} \right\rceil \geq 4.$$

Subcase(iii): Examine the pair (s_i, u), $1 \leq i \leq n$;

$$d(s_i,u)+\left\lceil \frac{\phi(s_i)+\phi(u)}{2} \right\rceil \geq 1+\left\lceil \frac{2n+2i+2}{2} \right\rceil \geq 4.$$

Subcase(iv): Examine the pair (s_i, v), $1 \leq i \leq n$;

$d(s_i, v) + \left\lceil \frac{\phi(s_i)+\phi(v)}{2} \right\rceil \geq 1 + \left\lceil \frac{2n+2i+4}{2} \right\rceil \geq 4.$

Subcase(v): Examine the pair (t_i, t_j), $1 \leq i \leq n-2,\ i+1 \leq j \leq n-1$;

$d(t_i, t_j) + \left\lceil \frac{\phi(t_i)+\phi(t_j)}{2} \right\rceil \geq 2 + \left\lceil \frac{4n+2i+2j+8}{2} \right\rceil \geq 4.$

Subcase(vi): Examine the pair (t_i, u), $1 \leq i \leq n-1$;

$d(t_i, u) + \left\lceil \frac{\phi(t_i)+\phi(u)}{2} \right\rceil \geq 1 + \left\lceil \frac{4n+2i+6}{2} \right\rceil \geq 4.$

Subcase(vii): Examine the pair (t_i, v), $1 \leq i \leq n-1$;

$d(t_i, v) + \left\lceil \frac{\phi(t_i)+\phi(v)}{2} \right\rceil \geq 2 + \left\lceil \frac{4n+2i+8}{2} \right\rceil \geq 4.$

Subcase(viii): Examine the pair (u, v),

$d(u, v) + \left\lceil \frac{\phi(u)+\phi(v)}{2} \right\rceil \geq 3 + \left\lceil \frac{4n+6}{2} \right\rceil \geq 4.$

Thus the even radio mean graceful condition is satisfied for all the pair of vertices. Hence $DS(T_n)$ is an even radio mean graceful graph.

Example 3.1 (a).

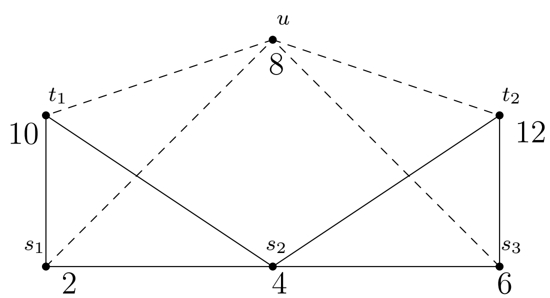

Figure 1: Even radio mean graceful labeling of $DS(T_3)$

Example 3.1 (b).

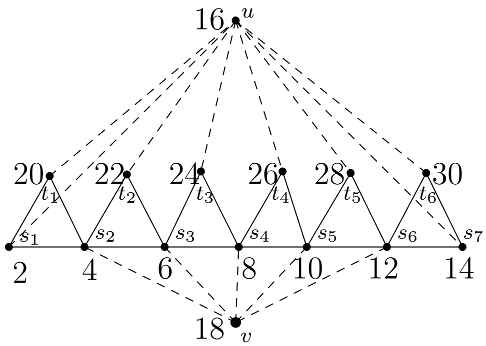

Figure 2: Even radio mean graceful labeling of $DS(T_7)$

Theorem 3.2. *DS(Q_n) is an even radio mean graceful graph.*
Proof. Let s_i, $1 \leq i \leq n$ be the vertices of path P_n. Join s_i and s_{i+1} with two new vertices t_i, and u_i respectively and then join t_i and u_i $1 \leq i \leq n-1$. The graph thus obtained is a quadrilateral snake graph (Q_n).

Case 1: n = 2, 3
Introduce a new vertex v and join it with the vertices of Q_n of degree two . The resultant graph is $DS(Q_n)$ whose vertex set is V = $\{s_i, 1 \leq i \leq n\} \cup \{t_i, u_i, 1 \leq i \leq n-1\} \cup \{v\}$. Clearly the diam($DS(Q_n)$)= 2. Therefore the condition $d(u,v) + \left\lceil \frac{\phi(u)+\phi(v)}{2} \right\rceil \geq 1 + diam(G)$, is obviously satisfied for all the pair of vertices u, v $\in$ V(G).

Case 2: n > 3
Introduce two new vertices v, w and join them with the vertices of Q_n of degree two and four respectively. The resultant graph is $DS(Q_n)$ whose vertex set is V = $\{s_i, 1 \leq i \leq n\} \cup \{t_i, u_i, 1 \leq i \leq n-1\} \cup \{v, w\}$.Clearly the diam($DS(Q_n)$)= 3. Define a bijection $\phi : V(DS(Q_n)) \to \{2, 4, 6, ...2|V|\}$ by $\phi(s_i) = 4n+2i, 1 \leq i \leq n$, $\phi(t_i) = 2i, 1 \leq i \leq n-1$, $\phi(u_i) = 2n + 2i - 2, 1 \leq i \leq n-1$, $\phi(v) = 4n - 2$, $\phi(w) = 4n$.
Now we find the even radio mean graceful condition for ϕ
Subcase(i): Examine the pair (s_i, s_j), $1 \leq i \leq n-1$, $i+1 \leq j \leq n$;

$$d(s_i, s_j) + \left\lceil \frac{\phi(s_i)+\phi(s_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{8n+2i+2j}{2} \right\rceil \geq 4 = 1 + diam(DS(Q_n)).$$

Subcase(ii): Examine the pair (s_i, t_j), $1 \leq i \leq n$, $1 \leq j \leq n-1$;

$d(s_i, t_j) + \left\lceil \frac{\phi(s_i)+\phi(t_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{4n+2i+2j}{2} \right\rceil \geq 4.$

Subcase(iii): Examine the pair (s_i, u_j), $1 \leq i \leq n, 1 \leq j \leq n-1$

$d(s_i, u_j) + \left\lceil \frac{\phi(s_i)+\phi(u_j)}{2} \right\rceil \geq 2 + \left\lceil \frac{6n+2i+2j-2}{2} \right\rceil \geq 4.$

Subcase(iv): Examine the pair (s_i, v), $1 \leq i \leq n$;

$d(s_i, v) + \left\lceil \frac{\phi(s_i)+\phi(v)}{2} \right\rceil \geq 1 + \left\lceil \frac{8n+2i-2}{2} \right\rceil \geq 4.$

Subcase(v): Examine the pair (s_i, w), $1 \leq i \leq n$;

$d(s_i, w) + \left\lceil \frac{\phi(s_i)+\phi(w)}{2} \right\rceil \geq 1 + \left\lceil \frac{8n+2i}{2} \right\rceil \geq 4.$

Subcase(vi): Examine the pair (t_i, t_j), $1 \leq i \leq n-2,\ i+1 \leq j \leq n-1$;

$d(t_i, t_j) + \left\lceil \frac{\phi(t_i)+\phi(t_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{2i+2j}{2} \right\rceil \geq 4.$

Subcase(vii): Examine the pair (t_i, u_j), $1 \leq i, j \leq n-1$;

$d(t_i, u_j) + \left\lceil \frac{\phi(t_i)+\phi(u_j)}{2} \right\rceil \geq 1 + \left\lceil \frac{2n+2i+2j-2}{2} \right\rceil \geq 4.$

Subcase(viii): Examine the pair (t_i, v), $1 \leq i \leq n-1$;

$d(t_i, v) + \left\lceil \frac{\phi(t_i)+\phi(v)}{2} \right\rceil \geq 1 + \left\lceil \frac{4n+2i-2}{2} \right\rceil \geq 4.$

Subcase(ix): Examine the pair (t_i, w), $1 \leq i \leq n-1$;

$d(t_i, w) + \left\lceil \frac{\phi(t_i)+\phi(w)}{2} \right\rceil \geq 2 + \left\lceil \frac{4n+2i}{2} \right\rceil \geq 4.$

Subcase(x): Examine the pair (v, w),

$d(v, w) + \left\lceil \frac{\phi(v)+\phi(w)}{2} \right\rceil \geq 3 + \left\lceil \frac{8n-2}{2} \right\rceil \geq 4.$

Subcase(xi): Examine the pair (u_i, u_j), $1 \leq i \leq n-2, i+1 \leq j \leq n-1$;

$d(u_i, u_j) + \left\lceil \frac{\phi(u_i)+\phi(u_j)}{2} \right\rceil \geq 2 + \left\lceil \frac{4n+2i+2j-4}{2} \right\rceil \geq 4.$

Subcase(xii): Examine the pair (u_i, v), $1 \leq i \leq n-1$;

$d(u_i, v) + \left\lceil \frac{\phi(u_i)+\phi(v)}{2} \right\rceil \geq 1 + \left\lceil \frac{6n+2i-4}{2} \right\rceil \geq 4.$

Subcase(xiii): Examine the pair (u_i, w), $1 \leq i \leq n-1$;

$d(u_i, w) + \left\lceil \frac{\phi(u_i)+\phi(w)}{2} \right\rceil \geq 2 + \left\lceil \frac{6n+2i-2}{2} \right\rceil \geq 4.$

Thus the even radio mean graceful condition is satisfied for all the pair of vertices. Hence $DS(Q_n)$ is an even radio mean graceful graph.

Example 3.2 (a).

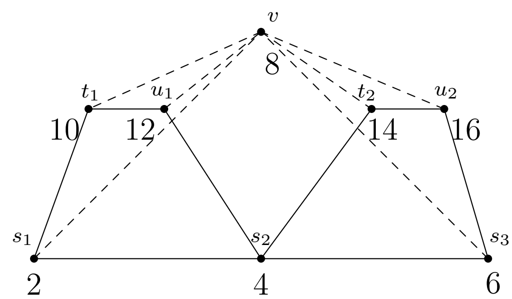

Figure 3: Even radio mean graceful labeling of $DS(Q_3)$

Example 3.2 (b).

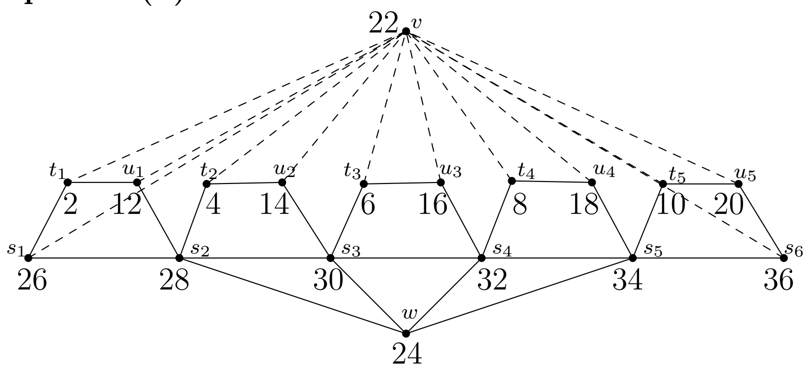

Figure 4: Even radio mean graceful labeling of $DS(Q_6)$

4. Conclusion

In this paper, we introduce a new labeling namely even radio mean graceful labeling and show that the snake graphs admit even radio mean graceful labeling.

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References

[1] Chartrand, Gray and Erwin, David and Zhang, Ping and Harary, Frank, Radio labeling of graphs, Bull. inst. Combin. Appl., 33 (2001), 77-85.

[2] David Raj C., Sunitha K. and Subramanian A., Radio odd mean and even mean labeling of some graphs, International journal of mathematical archive, 8 (11) (2017), 109-115.

[3] Gallian J. A., A dynamic survey of graph labeling, The electronic journal of combinatorics, 17 (2010) and DS6.

[4] Harary F., Graph Theory, Narosa Publishing house reading, New Delhi, 1988.

[5] Lavanya Y., Dhanyashree and Meera K. N., Radio Mean Graceful Graphs, International Conference on Applied Physics, Power and Material Science, IOP Conf. Series: Journal of Physics: Conf. Series. DOI: 10.1088/1742-6596/1172/1/012071.

[6] Ponraj R. and Somasundaram S., On the degree splitting of a graph, National Academy Science Letters, 27 (7,8) (2004), 275-278.

[7] Ponraj R. and Sathish Narayanan S. and Kala R., Radio Mean labeling of a graph, AKCE, International journal of graphs and combinatorics, 12 (2015), 224-228.

[8] Revathi N., Vertex odd mean and even mean labeling of some graphs, ISOR Journal of mathematics, 11 (2) (2015), 70-74.

[9] Sampathkumar E. and Walikar H. B., On the splitting graph of a graph, J. Karnatak Uni Sci., 25:13, (1980).

[10] Somasundaram S. and Ponraj R., Mean lableing of graphs, National Academy Science Letters, 26 (2003), 210-213.

[11] Somasudaram S., Sandhya S. S. and Viji S. P., Geometric mean labeling on Degree splitting graphs, Journal of Discrete Mathematical sciences and cryptography., 19 (2) (2015), 305-320.