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QUASI-SYMMETRIC DESIGNS WITH K NUMBERS OF INTERSECTION AS UNREDUCED DESIGNS

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Abstract: In this paper we propose unreduced balanced incomplete block designs which are multiple balanced incomplete block designs of resolvable type designs with no repeated blocks and also have quasi - symmetric structure with k intersection numbers between the blocks with illustrations.

Keywords and Phrases: Balanced Incomplete Block Design, Symmetric Design, Quasi-Symmetric Design, Block Intersection, Unreduced Design, Resolvable Design, Affine Resolvable Design, μ-Resolvable Design, t-Design.

2020 Mathematics Subject Classification: 62K10.

1. Introduction

Combinatorial design theory can be applied to the area of design of experiments. Some of the basic theory of combinatorial designs originated in the Ronald Fisher's work on the design of biological experiments. Modern applications are also found in wide areas including; Finite geometry, tournament scheduling, lotteries, mathematical biology, algorithm design and analysis, networking, group testing and cryptography. In the early 1930's Prof. R. A. Fisher and F. Yates gave the concept of design of experiments. BIB design play important role in design of experiments especially in field of experiments. Many construction methods of BIB design were given by Prof. Fisher [5], Yates [27] and Bose [2]. One of the fundamental principles of experimental design is the separation of heterogeneous experimental units into subsets of more homogeneous units or blocks in order to isolate identifiable, unwanted, but unavoidable, variation in measurements made from the units.

A fundamental method of constructing 2-designs (and also the first systematic construction method) is due to Bose [2]. Several series of BIB designs have been constructed by Sprott (1954, 1956) using the method of differences. Hedyat and Kageyama [7, 8] gave some useful and important results on t-design. Teirlinck [25] has proved that non - trival t-designs without repeated blocks exist for all t. Agrawal et al. [1] gave some construction methods of α-resolvable balanced incomplete block designs. Kageyama and Mohan [11] construct some μ-resolvable balanced incomplete block designs with some restrictions. Quasi-symmetric designs and its categorization have been important in the study of design theory over the last several years. Sane and Shrikhande [23] gave many important results on quasi-symmetric designs. Ray et al. [18] proved that for a 0-design with t-intersection number, $b \leq \begin{pmatrix} v \\ t \end{pmatrix}$. Pawale [14] proved that for a fixed block size k, there exist finitely many parametrically feasible t-designs with t-numbers of intersections and $\lambda > 1$.

Here the definitions of the terms used in this paper are given below:

t - Design - A t-design is an incidence structure with v treatments, b block each have k distinct treatments, each treatment occurs in r different blocks and every set of t distinct treatments are appear in exactly λ_t blocks. It is denoted by t-(v, k, λ_t). The equation is $\lambda_i = \lambda_{i+1}$ (v-i)/(k-i) for i = 0,1,... t-1. Any t-(v,k,λ_t) design is also an s - (v,k,λ_s) design for any s with 1$\leq$ s $\leq$ t. Every t-design for t = 2 is a **Balanced Incomplete Block Design**. It is also denoted as 2- (v, k, λ). A BIB design is symmetric iff v = b and r = k. A balanced incomplete block design with parameters (v, b, r, k, λ) is known as **unreduced design** with parameters (v, ${}^{v}C_k$, ${}^{v-1}C_{k-1}$, k , ${}^{v-2}C_{k-2}$). These designs are obtained by taking all combinations of k out of v treatments.

Multiple Balanced Incomplete Block Design – By taking t copies of the BIB design we get new BIB design with parameters (v, tb, tr, k, tλ) is called the t-multiple balanced incomplete block design of the BIB (v, b, r, k , λ).

A BIB design (v, tb, tr, k , tλ) which is not obtained by taking t-copies of the BIB (v, b, r, k , λ) is known as t-quasi multiple design of the BIB (v, b, r, k , λ). A BIB design may or may not exist but its quasi multiple balanced incomplete block design may exist. If the BIB design (v, tb, tr, k, tλ) is quasi multiple design then the multiple balanced incomplete block design has no repeated blocks.

Quasi -Symmetric Design – Let S be a finite set of v objects (points) and γ be a finite family of distinct k subsets of S (blocks). Then the pair D = { S , γ }

is called a **Quasi- Symmetric Design** if $|B_i \cap B_j| = x$; $i \neq j$ where B_i's are the subsets (of size k) of S $0 \leq x < k$.

Affine Resolvable Design - A resolvable block design is said to be affine resolvable design if and only if b=v+r-1 and any two blocks belonging to the same resolution set have no treatment in common, say, q_1=0 whereas any two blocks belonging to different resolution sets have exactly k^2/v treatments in common say, $q_2 = k^2/v$ must be integral. Resolvable t- design is a t-(v, k, λ_t) design in which every set of t distinct treatments are appear in exactly λ_t blocks. Shrikhande and Raghavrao (1963) generalized the concept of resolvability and affine resolvability of incomplete block designs to μ-resolvability and affine α- resolvability.

α-Resolvable Design - An incomplete block design with parameters v, b, r, k, λ is said to be α-resolvable if the blocks b can be separated in to t resolution sets $(t_1),(t_2),\ldots,(t_t)$ of β blocks each, such that every treatment occurs α ($\geq$1) times in each resolution set $\{t_j\}$ where j=1,2,...,t and the parameters are v, b= βt, r = αt, k, λ. When α=1,1- Resolvable Design is known as Resolvable Design. A necessary condition for any resolvable BIB design is that b/r is an integer. In (1942) Bose gave the inequality b$\geq$ (v+r-1) for all resolvable designs.

Our main objective in this paper is to study the unreduced balanced incomplete block designs which are multiple resolvable balanced incomplete block designs with no repeated blocks and also have quasi-symmetric structure with n_μ number of blocks have μ treatments are common (k intersection numbers) between the blocks where μ = 0,1,...,k-1. Any multiple balanced incomplete block design may or may have repeated blocks.

2. Construction of Unreduced BIB Designs which are Multiple BIB Designs/Quasi- Symmetric Designs

Lemma 2.1. *An unreduced design is resolvable balanced incomplete block design if and only if v = 2k.*

Proof. An unreduced design for v = 2k have parameters v =2k, $b^* = {}^{2k}C_k$, $r^* = {}^{2k-1}C_{k-1}$, k, $\lambda^* = {}^{2(k-1)}C_{k-2}$. Here b^* is divisible by r^* and $b^* > v + r^*$ -1 and according to [Patterson and Williams] since v is multiple of k, so this unreduced design is resolvable balanced incomplete block design.

Lemma 2.2. *In an unreduced balanced incomplete block design for v = 2k, k^2/v will be an integer iff k is even.*

Proof. For v = 2k, $k^2/v = k^2/2k = k/2$. k/2 should be an integer when k is even.

Theorem 2.3. *The existence of unreduced designs D^* with parameters v = 2k, $b^* = {}^{2k}C_k$, $r^* = {}^{2k-1}C_{k-1}$, k , $\lambda^* = {}^{2k-2}C_{k-2}$ implies the existence of t-multiple balanced* *incomplete block designs with parameters (v, $b^{**} = b^* = tb$, $r^{**} = r^* = tr$, k, $\lambda^{**} = \lambda^* = t\lambda$) of resolvable type with parameters v = 2k, b = 2(2k-1), r = (2k-1), k, λ = (k-1) where t ={(v-2)!/(k-1)!k!} or ${}^{v-2}C_{k-1}/k$.*

Proof. To get the proposed design from unreduced balanced incomplete block design D* first we have to find greatest common factor (t) of parameters (b^*, r^*, λ^*). The t of (b^*, r^*, λ^*) is {(v-2)!/(k-1)!k!} or ${}^{v-2}C_{k-1}$/k . (Integer)
Now, according to [Sane and Shrikhande]

$$b = b^*/t, r = r^*/t \text{ and } \lambda = \lambda^*/t. \tag{1}$$

Thus, we get the parameters v = 2k, b = 2(2k-1), r = (2k-1), k, λ = (k-1) of BIB design D from unreduced design D*. The obtained BIB design is self complementary design. It is also resolvable type design because b is divisible by r and b $\geq$ v+r-1, if k is even then this BIB design will be affine resolvable. From equation (1) b^* = tb, r^* = tr and λ^*= tλ . So, unreduced design can be expressed as v, tb, tr, k, tλ which is in form of t- multiple balanced incomplete block design with parameters v, b^* = tb, r^* = tr, k, λ^* = tλ. Thus, the unreduced balanced incomplete block design is t-multiple of BIB design have parameters v = 2k, b = 2(2k-1), r = (2k-1), k and λ = (k-1). Hence, the theorem is proved.

Example 2.4. Let us consider an unreduced balanced incomplete block design for v = 8 and k = 4, the designs is given by the following 70 blocks :
(0,1,2,3), (0,1,2,4), (0,1,2,5), (0,1,2,6), (0,1,2,7), (0,1,3,4), (0,1,3,5), (0,1,3,6), (0,1,3,7), (0,1,4,5), (0,1,4,6), (0,1,4,7), (0,1,5,6), (0,1,5,7), (0,1,6,7), (0,2,3,4), (0,2,3,5), (0,2,3,6), (0,2,3,7), (0,2,4,5), (0,2,4,6), (0,2,4,7), (0,2,5,6), (0,2,5,7), (0,2,6,7), (0,3,4,5), (0,3,4,6), (0,.3,4,7), (0,3,5,6), (0,3,5,7), (0,3,6,7), (0,4,5,6), (0,4,5,7), (0,4,6,7), (0,5,6,7), (1,2,3,4), (1,2,3,5), (1,2,3,6), (1,2,3,7), (1,2,4,5), (1,2,4,6), (1,2,4,7), (1,2,5,6), (1,2,5,7), (1,2,6,7), (1,3,4,5), (1,3,4,6), (1,3,4,7), (1,3,5,6), (1,3,5,7), (1,3,6,7) , (1,4,5,6), (1,4,5,7), (1,4,6,7), (1,5,6,7), (2,3,4,5), (2,3,4,6), (2,3,4,7), (2,3,5,6), (2,3,5,7), (2,3,6,7), (2,4,5,6), (2,4,5,7), (2,4,6,7), (2,5,6,7), (3,4,5,6), (3,4,5,7), (3,4,6,7), (3,5,6,7), (4,5,6,7).
The parameters of design are v = 8, b^* = 70, r^* =35, k = 4, λ^* = 15.

Example 2.5. Let us consider an unreduced balanced incomplete block design D* with parameters v = 8, $b^* = {}^8C_4$= 70, $r^* = {}^7C_3$= 35, k = 4, $\lambda^* = {}^6C_2$ =15 is 5-multiple of affine resolvable BIB design with parameters v = 8, b = 14, r = 7, k = 4, λ= 3.
Proof. According to Theorem 2.3.

Lemma 2.6. *An unreduced balanced incomplete block design D* for* ***v*** *= 2k have no repeated blocks.*

Theorem 2.7. *An unreduced design D^* with parameters $v = 2k$, $b^* = {}^{2k}C_k$, $r^* = {}^{2k-1}C_{k-1}$, k, $\lambda^* = {}^{2k-2}C_{k-2}$ is multiple quasi symmetric design with k intersection numbers.*

Proof. Let B^* be blocks of v treatments and γ be a finite family of distinct k subsets of blocks $\mathbf{B}^*$ and the pair $D^* = \{B^*, \gamma\}$ is an unreduced design with parameters (v, b^*, r^*, k, λ^*). An intersection number of D^* is x, where $0 \leq x < k$, if there exist B^*, such that $|B^*_m \cap B^*_n| = x$, where m, n = 1,2,. . . ,${}^{2k}C_k$ and $m \neq n$. Then the design D^* is quasi-symmetric design with distinct k numbers of intersection 0,1,2,...,k-1 ($0 \leq x < k$) because every two distinct blocks have common treatments either 0 or 1 or . . . or k-1.

As we know

$$\lambda_2^* = {}^{v-2}C_{k-2} = {}^{2k-2}C_{k-2}$$

$$\lambda_3^* = \frac{(k-2)}{(v-2)}\lambda_2^* = \frac{(k-2)}{(v-2)}\frac{(2k-2)!}{(k-2)!k!} = \frac{(2k-3)!}{(k-3)!k!} = {}^{v-3}C_{k-3}$$

$$\lambda_4^* = \frac{(k-3)}{(v-3)}\lambda_3^* = \frac{(k-3)}{(v-3)}\frac{(2k-3)!}{(k-3)!k!} = \frac{(2k-4)!}{(k-4)!k!} = {}^{v-4}C_{k-4}$$

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$$\lambda_i^* = \frac{\{k-(i-1)\}}{\{v-(i-1)\}}\lambda_{i-1}^* = \frac{\{k-(i-1)\}}{\{v-(i-1)\}}\frac{\{2k-(i-1)\}!}{\{k-(i-1)\}!k!} = \frac{(2k-i)!}{(k-i)!k!} = {}^{v-i}C_{k-i}$$

Thus, λ_i^*exist where i = 2,3,. . . ,k.

Now, in an unreduced design D^* the total number of blocks are ${}^{2k}C_k$ and according [Philips et al.] there are n_j the number of blocks have j treatments are common (with k types of intersection numbers,$0 \leq x < k$) between the blocks, where j = 0,1,. . . k-1.

$$n_j = \begin{pmatrix} k \\ j \end{pmatrix}\begin{pmatrix} v-k \\ k-j \end{pmatrix}$$

$$b^* - 1 = \sum_{j=0}^{k-1} n_j$$

Hence, this complete the proof of the theorem.

Example 2.8. Let us consider an unreduced design D^* with parameters 8, 70, 35, 4, 15 which is quasi-symmetric 4-design with 4 intersection numbers 0, 1, 2 and 3.

Proof. The unreduced design D* has k = 4, so it will have 4 types of intersection numbers 0,1,2 and 3. From Theorem 2.7 the given unreduced balanced incomplete design D* have
$\lambda_2^* = {}^{v-2}\mathrm{C}_{k-2}$ =15
$\lambda_3^* = {}^{v-3}\mathrm{C}_{k-3}$ = 5 and
$\lambda_4^* = {}^{v-4}\mathrm{C}_{k-4}$ = 1.
Thus, λ_2^*, λ_3^* and λ_4^* exist, so unreduced BIB design D* is quasi symmetric 4-design with 4 intersection numbers 0, 1, 2 and 3 with no repeated blocks.

Example 2.9. Consider an unreduced design D* with parameters (8, 70, 35, 4, 15) is a quasi-symmetric 4-design with 4 types of intersection numbers 0, 1, 2, 3 and from Theorem 2.7, there are n_j the number of blocks have j treatments are common between the blocks, where j = 0, 1, 2, 3. Thus,
$n_0 = 1$, $n_1 = 16$, $n_2 = 36$, $n_3 = 16$
and $b^* - 1 = \sum_{j=0}^{3} n_j = 69$.

Theorem 2.10. *Let us consider an unreduced balanced incomplete block design D* with parameters (v, ${}^{v}C_k$, ${}^{v-1}C_{k-1}$, k, ${}^{v-2}C_{k-2}$) where v =2k is t- resolvable balanced incomplete block design with parameter v, $b^{**} = tb$, $r^{**} = tr$, k, $\lambda^{**} = t\lambda$.*

Proof. Since the unreduced design D* is resolvable balanced incomplete block design (Lemma 2.1) with the block $b^* = {}^{v}\mathrm{C}_k = 2({}^{v-2}\mathrm{C}_{k-1}/k)(2k-1)$. Now, we select a block out of b* blocks of D* and then take its complement block from b*, we get a resolution set. Then, select another block from b* and take its complement block, get another resolution set. We will do this process continue up to last resolution set. Thus, we get ${}^{v-1}\mathrm{C}_{k-1} = ({}^{v-2}\mathrm{C}_{k-1}/k)(2k-1)$ total resolution sets of given unreduced design. Here we will get two types of resolvable design from ${}^{v-1}\mathrm{C}_{k-1}$ resolution sets :

1. If we divide the ${}^{v-1}\mathrm{C}_{k-1}$ resolution sets into (2k-1) groups , then each group will get $2({}^{v-2}\mathrm{C}_{k-1}/k)$ blocks ($({}^{v-2}\mathrm{C}_{k-1}/k)$ resolution sets) and each treatment will replicate $({}^{v-2}\mathrm{C}_{k-1}/k)$ times in each group. Thus this type of design is called $({}^{v-2}\mathrm{C}_{k-1}/k)$- resolvable design with parameters v, $b^* = 2({}^{v-2}\mathrm{C}_{k-1}/k)(2k-1)$, $r^* = ({}^{v-2}\mathrm{C}_{k-1}/k)(2k-1)$, k, λ^*.

2. If we divide the ${}^{v-1}\mathrm{C}_{k-1}$ resolution sets into $({}^{v-2}\mathrm{C}_{k-1}/k)$ groups , then each group will get 2(2k-1) blocks ((2k-1) resolution sets) and each treatment will replicate (2k-1) times in each group. Thus this type of design is called (2k-1)-resolvable design with parameters v, $b^* = 2(2k-1)({}^{v-2}\mathrm{C}_{k-1}/k)$, $r^* = (2k-1)({}^{v-2}\mathrm{C}_{k-1}/k)$, k , λ^*.

Example 2.11. The unreduced balanced incomplete block design D* with parameters v = 8, b* = ${}^{8}C_{4}$= 70, r* =${}^{7}C_{3}$= 35, k = 4, λ^* = ${}^{6}C_{2}$ =15 is 5-multiple balanced incomplete block design (v = 8, b**= 5b, r**= 5r, k = 4, λ^{**}= 5λ) of affine resolvable BIB design with parameters v =8, b= 14, r = 7, k = 4, λ= 3 is 5-resolvable balanced incomplete block design with parameters (8,70, 35,4,15, 5).
Proof. According to Lemma 2.1 the unreduced design D* have b* > v + r*-1 and b*/r* = 2 is an integer, so it is resolvable design. Now, from Theorem 2.10-(1) we rearrange the blocks b* of design D* in such a way that the 70 blocks are divided into 7 resolution set means 10 blocks in each set such that each treatment replicate 5 times in each resolution set.
{(0,1,2,4),(3,5,6,7),(1,2,4,7),(0,3,5,6),(2,3,4,7),(0,1,5,6),(0,1,4,5),(2,3,6,7), (0,1,3,5),(2,4,6,7)}
{(1,2,3,5),(4,6,0,7),(2,3,5,7),(1,4,6,0),(3,4,5,7),(1,2,6,0),(1,2,5,6),(3,4,0,7),(1,2,4,6) ,(3,5,0,7)}
{(2,3,4,6),(5,0,1,7),(3,4,6,7),(2,5,0,1),(4,5,6,7),(2,3,0,1),(2,3,6,0),(4,5,1,7),(2,3,5,0), (4,6,1,7)}
{(3,4,5,0),(6,1,2,7),(4,5,0,7),(3,6,1,2),(5,6,0,7),(3,4,1,2 ,(3,4,0,1),(5,6,2,7),(3,4,6,1), (5,0,2,7)}
{(4,5,6,1),(0,2,3,7),(5,6,1,7),(4,0,2,3),(6,0,1,7),(4,5,2,3),(4,5,1,2),(6,0,3,7),(4,5,0,2), (6,1,3,7)}
{(5,6,0,2),(1,3,4,7),(6,0,2,7),(5,1,3,4),(0,1,2,7),(5,6,3,4),(5,6,2,3),(0,1,4,7),(5,6,1,3), (0,2,4,7)}
{(6,0,1,3),(2,4,5,7),(0,1,3,7),(6,2,4,5),(1,2,3,7),(6,0,4,5),(6,0,3,4),(1,2,5,7),(6,0,2,4), (1,3,5,7)}
Hence, it is 5 - resolvable design with parameters v = 8, b*= 10_*7, r* = 5_*7 , k = 4 , λ^* = 15.

Example 2.12. The unreduced balanced incomplete block design D* with parameters v = 8, b*= ${}^{8}C_{4}$= 70, r* =${}^{7}C_{3}$= 35, k = 4, λ^* = ${}^{6}C_{2}$ =15 is 5-multiple balanced incomplete block design (v = 8, b*= 5b, r*= 5r , k = 4 , λ^*= 5λ) of affine resolvable BIB design with parameters v = 8, b = 14, r = 7, k = 4, λ= 3 is also 7-resolvable balanced incomplete block design with parameter 8,70, 35,4,15, 5.
Proof. According to theorem 2.10-(2) we rearrange the blocks b* of design D* in such a way that the 70 blocks are divided into 5 resolution set means 14 blocks in each set such that each treatment replicate 7 times in each resolution set.
{(0,1,2,4),(3,5,6,7),(1,2,3,5),(4,6,0,7),(2,3,4,6),(5,0,1,7),(3,4,5,0),(6,1,2,7),(4,5,6,1), (0,2,3,7),(5,6,0,2) ,(1,3,4,7) ,(6,0,1,3),(2,4,5,7)}
{(1,2,4,7),(0,3,5,6),(2,3,5,7),(1,4,6,0),(3,4,6,7),(2,5,0,1),(4,5,0,7),(3,6,1,2),(5,6,1,7), (4,0,2,3), (6,0,2,7),(5,1,3,4) ,(0,1,3,7) ,(6,2,4,5)}
{(2,3,4,7),(0,1,5,6),(3,4,5,7),(1,2,6,0),(4,5,6,7),(2,3,0,1),(5,6,0,7),(3,4,1,2),(6,0,1,7), (4,5,2,3), (0,1,2,7),(5,6,3,4),(1,2,3,7),(6,0,4,5)}
{(0,1,4,5),(2,3,6,7),(1,2,5,6),(3,4,0,7),(2,3,6,0),(4,5,1,7),(3,4,0,1),(5,6,2,7),(4,5,1,2), (6,0,3,7), (5,6,2,3),(0,1,4,7),(6,0,3,4), (1,2,5,7)}
{(0,1,3,5),(2,4,6,7),(1,2,4,6),(3,5,0,7),(2,3,5,0),(4,6,1,7),(3,4,6,1),(5,0,2,7),(4,5,0,2), (6,1,3,7), (5,6,1,3),(0,2,4,7),(6,0,2,4),(1,3,5,7)}
Hence, it is 7- resolvable design with parameters v = 8, b*= 14_*5, r* = 7_*5, k = 4,

$\lambda^* = 15$.

3. Results

The following table provide a list of parameters of unreduced designs, t-multiple solution of resolvable designs and μ-Resolvable Design by using Theorem 2.3 and Theorem 2.10.

Table 1

S.N.	**(v, b, r, k, λ)**	**(v, b^*, r^*, k, λ^*)**	**t-multiple**	**Intersection Numbers**	**μ-Resolvable Design**
1	(8,14,7,4,3)	(8,70,35,4,15)	5	0,1,2,3	5 or 7
2	(10,18,9,5,4)	(10,252,126,5,56)	14	0,1,2,3,4	14 or 9
3	(12,22,11,6,5)	(12,924,462,6,210)	42	0,1,2,3,4,5	42 or 11

*Since, the number of blocks of unreduced designs are very high if k $\geq$ 6. Therefore, we have ignored those designs.

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