

**SIMPLE EFFICIENT BOUNDS FOR ARCSINE AND
ARCTANGENT FUNCTIONS**

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Abstract: The aim of this paper is to present new, simple and sufficiently sharp bounds for arcsine and arctangent functions. Some of the bounds are computationally efficient while others are efficient to approximate the integrals $\int_a^b \frac{\arcsin x}{x} dx$ and $\int_a^b \frac{\arctan x}{x} dx$. As a matter of interest, several other sharp and generalized inequalities for $\frac{\arcsin x}{x}$ and $\frac{\arctan x}{x}$ are also established which are efficient to give some known and other trigonometric inequalities.

Keywords and Phrases: Shafer's inequality, Shafer-Fink's inequality, arcsine function, arctangent function, approximate integral.

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1. Introduction

The inequalities

$$\frac{\arcsin x}{x} < \frac{1}{\sqrt{1-x^2}}; x \in (0, 1), \quad (1.1)$$