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RELATIONS ON CONTINUITIES AND BOUNDEDNESS IN INTUITIONISTIC FUZZY PSEUDO NORMED LINEAR SPACES

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Abstract: In this study, different types of intuitionistic fuzzy continuities (IFCs) and intuitionistic fuzzy boundedness (IFBs) in intuitionistic fuzzy pseudo normed linear space(IFPNLS) are studied. Relations on intuitionistic fuzzy continuities and intuitionistic fuzzy boundedness are investigated.

Keywords and Phrases: Strongly intuitionistic fuzzy continuity, weakly intuitionistic fuzzy continuity, sequentially intuitionistic fuzzy continuity, strongly intuitionistic fuzzy bounded, weakly intuitionistic fuzzy bounded, uniformly intuitionistic fuzzy bounded.

2020 Mathematics Subject Classification: 47A60, 46S40.

1. Introduction

The fuzzy norm concept was originated by A. Katsaras [11, 12]. Subsequently, this notion developed by many researchers, viz. C. Falbin [9], S. C. Cheng and J. N. Moderson [3], T. Bag and S. K. Samanta [1, 2], I. Golet [10] and many others. Chasing the conviction of Cheng-Moderson [3], Bag-Samanta [1] considered another definition of fuzzy norm, it became most acceptable among researchers. Motivated by the work of Bag-Samanta [1, 2], S. Nădăban [15] introduced the idea

of fuzzy pseudo norm.
In 2006, Saadati and Park [17] introduced the notion of intuitionistic fuzzy norm. Study of intuitionistic fuzzy normed spaces attracted lots of interest in recent years [4, 5, 6, 7, 14, 18, 19]. In particular, Dinda et al. [4] studied the concept of intuitionistic fuzzy pseudo norm and showed that intuitionistic fuzzy pseudo norm is more general notion than intuitionistic fuzzy norm.
In this paper, intuitionistic fuzzy continuities and intuitionistic fuzzy boundedness of linear operator are studied in intuitionistic fuzzy pseudo normed spaces. In section 3, the concept of intuitionistic fuzzy continuities and intra relation on various types of intuitionistic fuzzy continuities are emphasized. In section 4,various types of intuitionistic fuzzy boundedness are studied. Firstly intra relations on different types of intuitionistic fuzzy boundedness is obtained. Then the interrelations on different types of continuities and boundedness are deduced.

2. Preliminaries

Definition 2.1. [16] *A pseudo norm on a linear space X over the field $\mathbb{K}$ ($=\mathbb{R}$ or $\mathbb{C}$) is a real function $\|\cdot\| : X \to \mathbb{R}$ defined on X such that for any $x, y \in X$,*
$(PN.1)$ $\|x\| \geq 0$;
$(PN.2)$ $\|x\| = 0$ *if and only if* $x = \theta$, *where θ is the zero element of X;*
$(PN.3)$ $\| cx \| \leq \|x\|, \ \forall c \in \mathbb{K}$ *with* $|c| \leq 1$;
$(PN.4)$ $\|x + y\| \leq \|x\| + \|y\|$.

Definition 2.2. [4] *Let X be linear space over the field $\mathbb{K}$ ($=\mathbb{R}$ or $\mathbb{C}$). An intuitionistic fuzzy subset (μ, ν) of $(X \times \mathbb{R}, X \times \mathbb{R})$ is said to be an intuitionistic fuzzy pseudo norm (IFPN) on X if $\forall x, y \in X$*
(IFP.1) $\forall t \in \mathbb{R}, \ \mu(x, t) + \nu(x, t) \leq 1$;
(IFP.2) $\forall t \in \mathbb{R}$ *with* $t \leq 0, \ \mu(x, t) = 0$;
(IFP.3) $\forall t \in \mathbb{R}^+, \ \mu(x, t) = 1$ *if and only if* $x = \theta$;
(IFP.4) $\forall t \in \mathbb{R}^+, \ \mu(cx, t) \geq \mu(x, t)$ *if* $|c| \leq 1, \forall c \in \mathbb{K}$;
(IFP.5) $\mu(x + y, s + t) \geq \min\{\mu(x, s), \mu(y, t)\}, \ \forall s, t \in \mathbb{R}^+$;
(IFP.6) $\lim_{t \to \infty} \mu(x, t) = 1$;
(IFP.7) *if there exists* $\alpha \in (0, 1)$ *such that* $\mu(x, t) > \alpha, \forall t \in \mathbb{R}^+$ *then* $x = \theta$;
(IFP.8) $\forall x \in X, \ \mu(x, \cdot)$ *is left continuous on* $\mathbb{R}$;
(IFP.9) $\forall t \in \mathbb{R}$ *with* $t \leq 0, \ \nu(x, t) = 1$;
(IFP.10) $\forall t \in \mathbb{R}^+, \ \nu(x, t) = 0$ *if and only if* $x = \theta$;
(IFP.11) $\forall t \in \mathbb{R}^+, \ \nu(cx, t) \leq \nu(x, t)$ *if* $|c| \leq 1, \forall c \in \mathbb{K}$;
(IFP.12) $\nu(x + y, s + t) \leq \max\{\nu(x, s), \nu(y, t)\}, \ \forall s, t \in \mathbb{R}^+$;
(IFP.13) $\lim_{t \to \infty} \nu(x, t) = 0$;

(IFP.14) *if there exists* $\alpha \in (0,1)$ *such that* $\nu(x\,,\,t) < \alpha\,, \forall\, t \in \mathbb{R}^+$ *then* $x = \theta$;
(IFP.15) $\forall\, x \in X\,,\ \nu(\,x\,,\,\cdot\,)$ *is left continuous on* $\mathbb{R}$.
Here (X, μ, ν) is called intuitionistic fuzzy pseudo normed linear space(IFPNLS).

Definition 2.3. [4] *A sequence* $\{a_n\}_{n\in\mathbb{N}}$ *in an intuitionistic fuzzy pseudo normed linear space* (X, μ, ν) *is said to converge to* $a \in X$ *if for* $0 < r < 1$, $t \in \mathbb{R}^+$ *there exists* $m_0 \in \mathbb{N}$ *such that* $\mu(a_n - a, t) > 1 - r$ *and* $\nu(a_n - a, t) < r$, $\forall\, n \geq m_0$.

Theorem 2.4. [4] *Let* (X, μ, ν) *be an intuitionistic fuzzy pseudo normed linear space. A sequence* $\{a_n\}_{n\in\mathbb{N}}$ *converges to* $a \in X$ *iff. for each* $t \in \mathbb{R}^+$, $\lim\limits_{n\to\infty} \mu(a_n - a, t) = 1$ *and* $\lim\limits_{n\to\infty} \nu(a_n - a, t) = 0$.

Theorem 2.5. [4] *Let* (X, μ, ν) *be an intuitionistic fuzzy pseudo normed linear space, and the functions* $\|\cdot\|_{\alpha}, \|\cdot\|_{\alpha}^{*} : X \to [0, \infty)$, *for* $\alpha \in (0,1)$ *defined by*

$$\begin{aligned} \|\,x\,\|_{\alpha} &= \bigwedge \{\, t > 0 \,:\, \mu\,(\,x, t\,) > \alpha \} \\ \|\,x\,\|_{\alpha}^{*} &= \bigwedge \{\, t > 0 \,:\, \nu\,(\,x, t\,) < \alpha \} \end{aligned} \tag{2.1}$$

Then the family of functions $\|\cdot\|_{\alpha} : X \to [0, \infty)$, *with* α *varying in* $(0,1)$, *is an ascending family of pseudo norms on* X. *And the family of functions* $\|\cdot\|_{\alpha}^{*} : X \to [0, \infty)$, *with* α *varying in* $(0,1)$, *is a descending family of pseudo norms on* X.

Theorem 2.6. [4] *Let* (X, μ, ν) *be an intuitionistic fuzzy pseudo normed linear space and let* $\mu'\,,\,\nu' \,:\, X \times \mathbb{R} \to [\,0\,,\,1\,]$ *be defined by*

$$\mu'(\,x\,,\,t\,) = \begin{cases} \bigvee \{\,\alpha \in (0,1) \,:\, \|\,x\,\|_{\alpha} < t\}, & if\, t > 0 \\ 0, & if\, t \leq 0 \end{cases}$$

$$\nu'(\,x\,,\,t\,) = \begin{cases} \bigwedge \{\,\alpha \in (0,1) \,:\, \|\,x\,\|_{\alpha}^{*} < t\} & if\, t > 0 \\ 1 & if\, t \leq 0 \end{cases}$$

then
(i) $(\mu'\,,\,\nu'\,)$ *is an intuitionistic fuzzy pseudo norm on* X.
(ii) $\mu' = \mu$ *and* $\nu' = \nu$, *where* $\|\cdot\|_{\alpha}$ *and* $\|\cdot\|_{\alpha}^{*}$ *are the pseudo norms defined by Equation 2.1.*

3. Intuitionistic fuzzy continuities of operators

This section deals with the study of different types of continuities of bounded linear operators and their relations in intuitionistic fuzzy pseudo normed linear spaces.

Definition 3.1. *Let (X, μ_1, ν_1), (Y, μ_2, ν_2) be intuitionistic fuzzy pseudo normed linear spaces. A mapping $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is said to be intuitionistic fuzzy continuous(IFC) at $x_0 \in X$ if for any given $\epsilon > 0$ and $\alpha \in (0, 1)$ there exists $\delta = \delta(\alpha, \epsilon) > 0$, $\beta = \beta(\alpha, \epsilon) \in (0, 1)$ such that for all $x \in X$,*

$$\mu_1(x - x_0, \delta) > 1 - \beta \Rightarrow \mu_2(T(x) - T(x_0), \epsilon) > 1 - \alpha$$

$$\nu_1(x - x_0, \delta) < \beta \Rightarrow \nu_2(T(x) - T(x_0), \epsilon) < \alpha.$$

If a linear operator T is intuitionistic fuzzy continuous at every point of a space X, then T is said to be intuitionistic fuzzy continuous on X.

Definition 3.2. *Let (X, μ_1, ν_1), (Y, μ_2, ν_2) be intuitionistic fuzzy pseudo normed linear spaces. A mapping $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is said to be sequentially IFC at $x_0 \in X$ if for any sequence $\{x_n\}_{n\in\mathbb{N}}$, $x_n \in X$ and $t > 0$,*

$$\lim_{n\to\infty} \mu_1(x_n - x_0, t) = 1 \Rightarrow \lim_{n\to\infty} \mu_2(T(x_n) - T(x_0), t) = 1,$$

$$\lim_{n\to\infty} \nu_1(x_n - x_0, t) = 0 \Rightarrow \lim_{n\to\infty} \nu_2(T(x_n) - T(x_0), t) = 0.$$

Theorem 3.3. *If a linear operator $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is sequentially IFC at a point $a_0 \in X$ then it is sequentially IFC on X, where (X, μ_1, ν_1) and (Y, μ_2, ν_2) are intuitionistic fuzzy pseudo normed linear spaces.*

Proof. Let $\{a_n\}_{n\in\mathbb{N}}$ be a sequence in X and $a_n \to a$, $(a \in X)$. Then $\forall t > 0$, $\lim_{n\to\infty} \mu_1(a_n - a, t) = 1$ and $\lim_{n\to\infty} \nu_1(a_n - a, t) = 0$, [by Theorem 2.4].
Therefore, $\lim_{n\to\infty} \mu_1((a_n - a + a_0) - a_0, t) = 1$, $\lim_{n\to\infty} \nu_1((a_n - a + a_0) - a_0, t) = 0$.
Since T is sequentially IFC at a_0, $\forall t > 0$ we have
$\lim_{n\to\infty} \mu_1(T(a_n - a + a_0) - T(a_0), t) = 1$, $\lim_{n\to\infty} \nu_1(T(a_n - a + a_0) - T(a_0), t) = 0$.
$\Rightarrow \lim_{n\to\infty} \mu_1(T(a_n) - T(a) + T(a_0) - T(a_0), t) = 1$, $\lim_{n\to\infty} \nu_1(T(a_n) - T(a) + T(a_0) - T(a_0), t) = 0$, since T is linear.
$\Rightarrow \lim_{n\to\infty} \mu_1(T(a_n) - T(a), t) = 1$, $\lim_{n\to\infty} \nu_1(T(a_n) - T(a), t) = 0$.
Since $a \in X$ is arbitrary, T is sequentially IFC on X.

Theorem 3.4. *A linear operator $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is sequentially IFC if and only if it is IFC, where (X, μ_1, ν_1) and (Y, μ_2, ν_2) are intuitionistic fuzzy pseudo normed linear spaces.*

Proof. First suppose T be IFC at $a_0 \in X$. Then for any given $\epsilon > 0$, $\alpha \in (0, 1)$ there exists $\delta = \delta(\epsilon, \alpha) > 0$ and $\beta = \beta(\epsilon, \alpha) \in (0, 1)$ such that $\forall a \in X$,

$$\mu_1(a - a_0, \delta) > 1 - \beta \Rightarrow \mu_2(T(a) - T(a_0), \epsilon) > 1 - \alpha$$

$$\nu_1(a - a_0, \delta) < \beta \Rightarrow \nu_2(T(a) - T(a_0), \epsilon) < \alpha$$

Let $\{a_n\}_{n\in\mathbb{N}}$ be a sequence in X converges to a_0. Therefore, there exists $n_0 \in \mathbb{N}$ such that $\forall\, n_0 \geq n$,
$\mu_1(a_n - a_0, \delta) > 1 - \beta,\ \nu_1(a_n - a_0, \delta) < \beta$, [by Definition 2.3].
Now, since T is IFC at $a_0 \in X$, we have
$\mu_2(T(a_n) - T(a_0), \epsilon) > 1 - \alpha,\ \ \nu_2(T(a_n) - T(a_0), \epsilon) < \alpha$
$\Rightarrow\ T(a_n) \to T(a_0)$ i.e., T is sequentially IFC at $a_0 \in X$.
Conversely, suppose T be not IFC at $a_0 \in X$. Then there exists $\epsilon > 0,\ \alpha \in (0, 1)$ such that for any $\delta > 0,\ \beta \in (0, 1)$, there exists b (depends on $\delta,\ \beta$) in X such that

$$\mu_1(b - a_0, \delta) > 1 - \beta \text{ but } \mu_2(T(b) - T(a_0), \epsilon) \leq 1 - \alpha,$$

$$\nu_1(b - a_0, \delta) < \beta \text{ but } \nu_2(T(b) - T(a_0), \epsilon) \geq 1 - \alpha.$$

Hence for $\delta = \beta = \dfrac{1}{n+1}$ there exists b_n, for $n = 1, 2, \cdots$, such that

$$\mu_1(b_n - a_0, \delta) = \mu_1(b_n - a_0, \frac{1}{n+1}) > 1 - \frac{1}{n+1} \text{ but } \mu_2(T(b_n) - T(a_0), \epsilon) \leq 1 - \alpha,$$

$$\nu_1(b_n - a_0, \delta) = \nu_1(b_n - a_0, \frac{1}{n+1}) < \frac{1}{n+1} \text{ but } \nu_2(T(b_n) - T(a_0), \epsilon) \geq \alpha.$$

Therefore, $\lim\limits_{n\to\infty} \mu_1(b_n - a_0, \delta) = 1$ but $\lim\limits_{n\to\infty} \mu_2(T(b_n) - T(a_0), \epsilon) \neq 1$,
$\lim\limits_{n\to\infty} \nu_1(b_n - a_0, \delta) = 0$ but $\lim\limits_{n\to\infty} \nu_2(T(b_n) - T(a_0), \epsilon) \neq 0$.
Hence T is not sequentially IFC at a_0. This completes the proof.

Definition 3.5. *Let (X, μ_1, ν_1), (Y, μ_2, ν_2) be intuitionistic fuzzy pseudo normed linear spaces. A mapping $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is said to be strongly IFC at $x_0 \in X$ if for any given $\epsilon > 0$ there exists $\delta(\epsilon) > 0$ such that for all $x \in X$,*

$$\mu_2(T(x) - T(x_0), \epsilon) \geq \mu_1(x - x_0, \delta),\ \nu_2(T(x) - T(x_0), \epsilon) \leq \nu_1(x - x_0, \delta).$$

Theorem 3.6. *If a linear operator $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is strongly IFC at a point $a_0 \in X$ then it is strongly IFC on X, where (X, μ_1, ν_1) and (Y, μ_2, ν_2) are intuitionistic fuzzy pseudo normed linear spaces.*

Proof. Since T is strongly IFC at a_0, for given $\epsilon > 0$ there exists $\delta(\epsilon) > 0$ such that $\forall\, a \in X$,

$$\mu_2(T(a) - T(a_0), \epsilon) \geq \mu_1(a - a_0, \delta),\ \ \nu_2(T(a) - T(a_0), \epsilon) \leq \nu_1(a - a_0, \delta).$$

Taking $b \in X$ we have $a + a_0 - b \in X$. Therefore replacing a by $a + a_0 - b$, we have

$$\begin{aligned} &\mu_2(T(a + a_0 - b) - T(a_0), \epsilon) \geq \mu_1(a + a_0 - b - a_0, \delta) \\ &\Rightarrow \mu_2(T(a) + T(a_0) - T(b) - T(a_0), \epsilon) \geq \mu_1(a - b, \delta) \\ &\Rightarrow \mu_2(T(a) - T(b), \epsilon) \geq \mu_1(a - b, \delta), \end{aligned}$$

$$\begin{aligned} &\nu_2(T(a + a_0 - b) - T(a_0), \epsilon) \leq \nu_1(a + a_0 - b - a_0, \delta) \\ &\Rightarrow \nu_2(T(a) + T(a_0) - T(b) - T(a_0), \epsilon) \leq \nu_1(a - b, \delta) \\ &\Rightarrow \nu_2(T(a) - T(b), \epsilon) \leq \nu_1(a - b, \delta). \end{aligned}$$

Hence T is strongly IFC at b.
Since $b \in X$ is arbitrary, T is strongly IFC on X.

Definition 3.7. *Let (X, μ_1, ν_1), (Y, μ_2, ν_2) be intuitionistic fuzzy pseudo normed linear spaces. A mapping $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is said to be weakly IFC at $x_0 \in X$ if for any given $\epsilon > 0$ and $\alpha \in (0, 1)$ there exists $\delta = \delta(\alpha, \epsilon) > 0$ such that for all $x \in X$,*

$$\mu_1(x - x_0, \delta) \geq \alpha \Rightarrow \mu_2(T(x) - T(x_0), \epsilon) \geq \alpha$$

$$\nu_1(x - x_0, \delta) \leq \alpha \Rightarrow \nu_2(T(x) - T(x_0), \epsilon) \leq \alpha.$$

Theorem 3.8. *If a linear operator $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is weakly IFC at a point $a_0 \in X$ then it is weakly IFC on X, where (X, μ_1, ν_1) and (Y, μ_2, ν_2) are intuitionistic fuzzy pseudo normed linear spaces.*

Proof. Since T is weakly IFC on $a_0 \in X$ then for given $\epsilon > 0$, $\alpha \in (0, 1)$ there exist $\delta(\alpha, \epsilon) > 0$ such that $\forall\, a \in X$,

$$\mu_1(a - a_0, \delta) \geq \alpha \Rightarrow \mu_2(T(a) - T(a_0), \epsilon) \geq \alpha,$$

$$\nu_1(a - a_0, \delta) \leq \alpha \Rightarrow \nu_2(T(a) - T(a_0), \epsilon) \leq \alpha.$$

Taking $b \in X$ we have $a + a_0 - b \in X$. Therefore replacing a by $a + a_0 - b$, we have

$$\begin{aligned} \mu_1(a - b, \delta) \geq \alpha &\Rightarrow \mu_2(T(a + a_0 - b) - T(a_0), \epsilon) \geq \alpha \\ &\Rightarrow \mu_2(T(a) + T(a_0) - T(b) - T(a_0), \epsilon) \geq \alpha \\ &\Rightarrow \mu_2(T(a) - T(b), \epsilon) \geq \alpha, \end{aligned}$$

$$\begin{aligned} \nu_1(a - b, \delta) \leq \alpha &\Rightarrow \nu_2(T(a + a_0 - b) - T(a_0), \epsilon) \leq \alpha \\ &\Rightarrow \nu_2(T(a) + T(a_0) - T(b) - T(a_0), \epsilon) \leq \alpha \\ &\Rightarrow \nu_2(T(a) - T(b), \epsilon) \leq \alpha. \end{aligned}$$

Since $b(\in X)$ is arbitrary, T is weakly IFC on X.

Theorem 3.9. *If a linear operator $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is strongly IFC then it is weakly IFC, where (X, μ_1, ν_1) and (Y, μ_2, ν_2) are intuitionistic fuzzy pseudo normed linear spaces.*

Proof. From the definitions of strongly IFC and weakly IFC the Theorem follows. The next example shows that in an intuitionistic fuzzy pseudo normed linear space weakly intuitionistic fuzzy continuity may not imply strongly intuitionistic fuzzy continuity.

Example 3.10. Let $(X, \|\cdot\|)$ be a pseudo normed linear space. Define $\mu, \nu : X \times \mathbb{R} \to [0, 1]$ by

$$\mu(x, t) = \begin{cases} 1 & \text{if } t > \|x\|, t > 0, \\ \dfrac{t}{t + \|x\|} & \text{if } t \leq \|x\|, t > 0, \\ 0 & \text{if } t \leq 0. \end{cases}$$

$$\nu(x, t) = \begin{cases} 0 & \text{if } t > \|x\|, t > 0, \\ \dfrac{\|x\|}{t + \|x\|} & \text{if } t \leq \|x\|, t > 0, \\ 1 & \text{if } t \leq 0. \end{cases}$$

then by Example 3.2 of [4], (X, μ, ν) is an intuitionistic fuzzy pseudo normed linear space (IFPNLS).

Let $T : (X, \mu, \nu) \to (X, \mu, \nu)$ be a linear operator defined by $T(x) = \dfrac{x^3}{1+x}$.
Let $x_0 \in X$, then for each $x \in X$, $\epsilon > 0$ and $\alpha \in (0, 1)$,

$$\mu(T(x) - T(x_0), \epsilon) \geq \alpha$$

$$\Leftarrow \frac{\epsilon}{\epsilon + \|T(x) - T(x_0)\|} \geq \alpha$$

$$\Leftarrow \frac{\epsilon}{\epsilon + \|\frac{x^3}{1+x} - \frac{{x_0}^3}{1+x_0}\|} \geq \alpha$$

$$\Leftarrow \frac{\epsilon\|(1+x)(1+x_0)\|}{\epsilon\|(1+x)(1+x_0)\| + \|x^3 + x_0x^3 - {x_0}^3 - x{x_0}^3\|} \geq \alpha$$

$$\Leftarrow \frac{\epsilon\|1 + x + x_0 + xx_0)}{\epsilon\|1 + x + x_0 + xx_0)\| + \|(x - x_0)(x^2 + xx_0 + x_0^2) + xx_0(x + x_0)(x - x_0)\|} \geq \alpha$$

$$\Leftarrow \frac{\epsilon\|1+x+x_0+xx_0)\|}{\epsilon\|1+x+x_0+xx_0)\|+\|(x-x_0)\|\|(x^2+xx_0+x_0^2+x^2x_0+xx_0^2)\|} \geq \alpha$$

$$\Leftarrow \frac{\epsilon\dfrac{\|1+x+x_0+xx_0)\|}{\|(x^2+xx_0+x_0^2+x^2x_0+xx_0^2)\|}}{\epsilon\dfrac{\|1+x+x_0+xx_0)\|}{\|(x^2+xx_0+x_0^2+x^2x_0+xx_0^2)\|}+\|(x-x_0)\|} \geq \alpha$$

$$\Leftarrow \epsilon\frac{\|1+x+x_0+xx_0)\|}{\|(x^2+xx_0+x_0^2+x^2x_0+xx_0^2)\|}$$

$$\geq \alpha.\epsilon\frac{\|1+x+x_0+xx_0)\|}{\|(x^2+xx_0+x_0^2+x^2x_0+xx_0^2)\|}+\alpha\|(x-x_0)\|$$

$$\Leftarrow \epsilon \geq \alpha.\epsilon+\alpha\|(x-x_0)\|.\frac{\|(x^2+xx_0+x_0^2+x^2x_0+xx_0^2)\|}{\|1+x+x_0+xx_0)\|}$$

$$\geq \alpha.\epsilon+\alpha\|(x-x_0)\|, \text{ since } \frac{\|(x^2+xx_0+x_0^2+x^2x_0+xx_0^2)\|}{\|1+x+x_0+xx_0)\|} \geq 1$$

$$\Leftarrow \delta \geq \alpha.\delta+\alpha\|(x-x_0)\|, \text{ by taking } \epsilon=\delta$$

$$\Leftarrow \frac{\delta}{\delta+\|(x-x_0)\|} \geq \alpha$$

$$\Leftarrow \mu(x-x_0,\delta) \geq \alpha$$

and

$$\nu(T(x)-T(x_0),\epsilon) \leq \alpha$$

$$\Leftarrow \frac{\|T(x)-T(x_0)\|}{\epsilon+\|T(x)-T(x_0)\|} \leq \alpha$$

$$\Leftarrow \frac{\|\dfrac{x^3}{1+x}-\dfrac{{x_0}^3}{1+x_0}\|}{\epsilon+\|\dfrac{x^3}{1+x}-\dfrac{{x_0}^3}{1+x_0}\|} \leq \alpha$$

$$\Leftarrow \frac{\|x^3+x_0x^3-{x_0}^3-x{x_0}^3\|}{\epsilon\|(1+x)(1+x_0)\|+\|x^3+x_0x^3-{x_0}^3-x{x_0}^3\|} \leq \alpha$$

$$\Leftarrow \frac{\|(x-x_0)(x^2+xx_0+x_0^2)+xx_0(x+x_0)(x-x_0)\|}{\epsilon\|1+x+x_0+xx_0)\|+\|(x-x_0)(x^2+xx_0+x_0^2)+xx_0(x+x_0)(x-x_0)\|} \leq \alpha$$

$$\Leftarrow \frac{\|(x-x_0)(x^2+xx_0+x_0^2+x^2x_0+xx_0^2)\|}{\epsilon\|1+x+x_0+xx_0)\|+\|(x-x_0)(x^2+xx_0+x_0^2+x^2x_0+xx_0^2)\|} \leq \alpha$$

$$\Leftarrow \frac{\|(x-x_0)\|\|(x^2+xx_0+x_0^2+x^2x_0+xx_0^2)\|}{\epsilon\|1+x+x_0+xx_0)\|+\|(x-x_0)\|\|(x^2+xx_0+x_0^2+x^2x_0+xx_0^2)\|} \leq \alpha$$

$$\Leftarrow \frac{\|(x-x_0)\|}{\epsilon\frac{\|1+x+x_0+xx_0)\|}{\|(x^2+xx_0+x_0^2+x^2x_0+xx_0^2)\|}+\|(x-x_0)\|} \leq \alpha$$

$$\Leftarrow \alpha\,\|(x-x_0)\|+\alpha.\epsilon\frac{\|1+x+x_0+xx_0)\|}{\|(x^2+xx_0+x_0^2+x^2x_0+xx_0^2)\|} \geq \|(x-x_0)\|$$

$$\Leftarrow (1-\alpha)\|(x-x_0)\| \leq \alpha.\epsilon\frac{\|1+x+x_0+xx_0)\|}{\|(x^2+xx_0+x_0^2+x^2x_0+xx_0^2)\|}$$

$$\leq \alpha.\epsilon,\ \text{since}\ \frac{\|1+x+x_0+xx_0)\|}{\|(x^2+xx_0+x_0^2+x^2x_0+xx_0^2)\|} \leq 1$$

$$\Leftarrow \|(x-x_0)\|-\alpha\|(x-x_0)\| \leq \alpha.\delta,\ \text{by taking}\ \delta=\epsilon$$

$$\Leftarrow \|(x-x_0)\| \leq \alpha(\delta+\|(x-x_0)\|)$$

$$\Leftarrow \frac{\|(x-x_0)\|}{\delta+\|(x-x_0)\|} \leq \alpha$$

$$\Leftarrow \nu(x-x_0,\delta) \leq \alpha.$$

Thus for every $\epsilon > 0$ and $\alpha \in (0,1)$ there exists $\delta = \delta(\alpha,\epsilon) > 0$ such that for all $x \in X$ and $x_0 \in X$ $\mu(x-x_0,\delta) \geq \alpha \Rightarrow \mu(T(x)-T(x_0),\epsilon) \geq \alpha$, $\nu(x-x_0,\delta) \leq \alpha \Rightarrow \nu(T(x)-T(x_0),\epsilon) \leq \alpha$.
Hence T is weakly intuitionistic fuzzy continuous at $x_0 \in X$ and hence on X.
To show T is not strongly intuitionistic fuzzy continuous, it is enough to for any given $\epsilon > 0$ there does not exist a $\delta > 0$ such that

$$\mu(T(x)-T(x_0),\epsilon) \geq \mu(x-x_0,\delta)\ \text{or}\ \nu(T(x)-T(x_0),\epsilon) \leq \nu(x-x_0,\delta).$$

Let $\epsilon > 0$, then $\forall\, x \in X$ and $x_0 \in X$,

$$\mu(T(x)-T(x_0),\epsilon) \geq \mu(x-x_0,\delta)$$

$$\Rightarrow \mu(\frac{x^3}{1+x}-\frac{x_0^3}{1+x_0},\epsilon) \geq \frac{\delta}{\delta+\|x-x_0\|}$$

$$\Rightarrow \frac{\epsilon}{\epsilon + \frac{\|x^3 + x^3x_0 - x_0^3 - xx_0^3\|}{\|(1+x)(1+x_0)\|}} \geq \frac{\delta}{\delta + \|x - x_0\|}$$

$$\Rightarrow \frac{\epsilon\|(1 + x + x_0 + xx_0)\|}{\epsilon\|(1 + x + x_0 + xx_0)\| + \|x - x_0\|\|x^2 + xx_0 + x_0^2 + x^2x_0 + xx_0^2\|} \geq \frac{\delta}{\delta + \|x - x_0\|}$$

$$\Rightarrow \epsilon\|x-x_0\|\|(1+x+x_0+xx_0)\| \geq \delta\|x-x_0\|\|x^2+xx_0+x_0^2+x^2x_0+xx_0^2\|$$

$$\Rightarrow \delta \leq \epsilon\frac{\|(1 + x + x_0 + xx_0)\|}{\|x^2 + xx_0 + x_0^2 + x^2x_0 + xx_0^2\|}.$$

Now $\inf\{\frac{\|(1 + x + x_0 + xx_0)\|}{\|x^2 + xx_0 + x_0^2 + x^2x_0 + xx_0^2\|}\} = 0, \ \forall\, x \in X.$
Therefore, $\delta = 0$, which is not possible. This shows that T is not strongly intuitionistic fuzzy continuous.

Theorem 3.11. *If a linear operator $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is strongly IFC then it is sequentially IFC, where (X, μ_1, ν_1) and (Y, μ_2, ν_2) are intuitionistic fuzzy pseudo normed linear spaces.*

Proof. Let $\{a_n\}_{n\in\mathbb{N}}$ be a sequence in X such that $a_n \to a_0$.
i.e., $\lim_{n\to\infty} \mu_1(a_n - a_0, \delta) = 1$ and $\lim_{n\to\infty} \nu_1(a_n - a_0, \delta) = 0, \ \forall\, \delta > 0,$ [by Definition 2.3] .
Now since T is strongly IFC at $a_0 \in X$. Then for $\epsilon > 0, \ \exists\, \delta(\epsilon) > 0$ such that $\forall\, a \in X$,

$$\mu_2(T(a) - T(a_0), \epsilon) \geq \mu_1(a - a_0, \delta), \ \ \nu_2(T(a) - T(a_0), \epsilon) \leq \nu_1(a - a_0, \delta).$$

Now, $\lim_{n\to\infty} \mu_2(T(a_n) - T(a_0), \epsilon) \geq \lim_{n\to\infty} \mu_1(a_n - a_0, \delta) = 1$
$\Rightarrow \lim_{n\to\infty} \mu_2(T(a_n) - T(a_0), \epsilon) = 1,$
$\lim_{n\to\infty} \nu_2(T(a_n) - T(a_0), \epsilon) \leq \lim_{n\to\infty} \nu_1(a_n - a_0, \delta) = 0$
$\Rightarrow \lim_{n\to\infty} \nu_2(T(a_n) - T(a_0), \epsilon) = 0.$
Since ϵ is arbitrary small positive number, T is sequentially IFC.
The next example shows that in an intuitionistic fuzzy pseudo normed linear space sequentially intuitionistic fuzzy continuity may not imply strongly intuitionistic fuzzy continuity.

Example 3.12. Consider the intuitionistic fuzzy pseudo normed linear space

(X, μ, ν) as Example 3.10 and the linear operator T is defined by $T(x) = \dfrac{x^3}{1+x}$.
Let $\{x_n\}_{n\in\mathbb{N}}$ be a sequence in X such that $x_n \to x_0$ in X. Now $\forall\, t > 0$,
$\lim\limits_{n\to\infty} \mu(x_n - x_0, t) = 1$, and $\lim\limits_{n\to\infty} \nu(x_n - x_0, t) = 0$.
$\Rightarrow \lim\limits_{n\to\infty} \dfrac{t}{t + \|x_n - x_0\|} = 1$, and $\lim\limits_{n\to\infty} \dfrac{\|x_n - x_0\|}{t + \|x_n - x_0\|} = 0$. Hence

$$\lim_{n\to\infty} \|x_n - x_0\| = 0 \tag{3.1}$$

Now,

$$\begin{aligned}
\mu(T(x_n)-&T(x_0), t)\\
&= \frac{t}{t + \left\| \dfrac{x_n^3}{1+x_n} - \dfrac{x_0^3}{1+x_0} \right\|}\\
&= \frac{t}{t + \left\| \dfrac{x_n^3 + x_0 x_n^3 - x_0^3 - x_n x_0^3}{(1+x_n)(1+x_0)} \right\|}\\
&= \frac{t\,\|(1+x_n)(1+x_0)\|}{t\,\|(1+x_n)(1+x_0)\| + \|x_n - x_0\|\,\|x_n^2 + x_n x_0 + x_0^2 + x_0 x_n^2 + x_n x_0^2\|} \to 1
\end{aligned}$$

as $n \to \infty$ by Equation 3.1. Also,

$$\begin{aligned}
\nu(T(x_n)-&T(x_0), t)\\
&= \frac{\left\| \dfrac{x_n^3}{1+x_n} - \dfrac{x_0^3}{1+x_0} \right\|}{t + \left\| \dfrac{x_n^3}{1+x_n} - \dfrac{x_0^3}{1+x_0} \right\|}\\
&= \frac{\left\| \dfrac{x_n^3 + x_0 x_n^3 - x_0^3 - x_n x_0^3}{(1+x_n)(1+x_0)} \right\|}{t + \left\| \dfrac{x_n^3 + x_0 x_n^3 - x_0^3 - x_n x_0^3}{(1+x_n)(1+x_0)} \right\|}\\
&= \frac{\|x_n - x_0\|\,\|x_n^2 + x_n x_0 + x_0^2 + x_0 x_n^2 + x_n x_0^2\|}{t\,\|(1+x_n)(1+x_0)\| + \|x_n - x_0\|\,\|x_n^2 + x_n x_0 + x_0^2 + x_0 x_n^2 + x_n x_0^2\|} \to 0
\end{aligned}$$

as $n \to \infty$ by Equation 3.1.
Thus T is sequentially IFC at $x_0 \in X$ and hence on X. From Example 3.1, it is

apparent that T is not strongly IFC.

Corollary 3.13. *If a linear operator $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is strongly IFC then it is IFC, where (X, μ_1, ν_1) and (Y, μ_2, ν_2) are intuitionistic fuzzy pseudo normed linear spaces.*

Proof. From Theorem 3.4 and Theorem 3.11 the corollary follows.

4. Intuitionistic fuzzy boundedness of operators

Definition 4.1. *Let (X, μ_1, ν_1), (Y, μ_2, ν_2) be intuitionistic fuzzy pseudo normed linear spaces. A linear operator $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is said to be strongly intuitionistic fuzzy bounded (strongly IFB) if $\forall x \in X$ and $\forall\, t \in \mathbb{R}^+$,*

$$\mu_2(T(x), t) \geq \mu_1(x, t)\,,\ \nu_2(T(x), t) \leq \nu_1(x, t).$$

Definition 4.2. *Let (X, μ_1, ν_1), (Y, μ_2, ν_2) be intuitionistic fuzzy pseudo normed linear spaces. A mapping $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is said to be weakly IFB if for any $\alpha \in (0, 1)$, $\forall x \in X$ and $\forall\, t \in \mathbb{R}^+$,*

$$\mu_1(x, t) \geq \alpha \Rightarrow \mu_2(T(x), t) \geq \alpha\,,\ \nu_1(x, t) \leq 1 - \alpha \Rightarrow \nu_2(T(x), t) \leq 1 - \alpha.$$

Theorem 4.3. *If a linear operator $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is strongly IFB then it is weakly IFB, where (X, μ_1, ν_1) and (Y, μ_2, ν_2) are intuitionistic fuzzy pseudo normed linear spaces.*

Proof. This theorem easily perceived from the definition of strongly IFB and weakly IFB of linear operators.

Definition 4.4. *Let (X, μ_1, ν_1), (Y, μ_2, ν_2) be intuitionistic fuzzy pseudo normed linear spaces. A mapping $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is said to be uniformly IFB if there exist $c > 0$, $\alpha \in (0, 1)$ such that*

$$\|T(x)\|_\alpha^2 \leq \|\, x \,\|_\alpha^1\,,\ \|\, T(x) \,\|_\alpha^{2*} \leq \|\, x \,\|_\alpha^{1*}\,,$$

where $\|\cdot\|_\alpha^1$ and $\|\cdot\|_\alpha^{1*}$ are the pseudo norms defined by Equation 2.1. And $\|\cdot\|_\alpha^2$, $\|\cdot\|_\alpha^{2*} : Y \to (0, \infty)$ are defined by

$$\|\, T(x) \,\|_\alpha^2 = \bigwedge \{\, t > 0 \,:\, \mu_2\,(\, T(x), t\,) > \alpha\}$$

$$\|\, T(x) \,\|_\alpha^{2*} = \bigwedge \{\, t > 0 \,:\, \nu_2\,(\, T(x), t\,) < \alpha\}$$

Theorem 4.5. *A linear operator $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is strongly IFB if and only if it is uniformly IFB with respect to corresponding α-norms, $\alpha \in (0, 1)$, where*

(X, μ_1, ν_1) and (Y, μ_2, ν_2) are intuitionistic fuzzy pseudo normed linear spaces.
Proof. First we suppose T is strongly IFB. Then $\forall\, x \in X$ and $\forall\, t \in \mathbb{R}^+$,

$$\mu_2(T(x), t) \geq \mu_1(x, t)\ ,\ \ \nu_2(T(x), t) \leq \nu_1(x, t). \tag{4.1}$$

$$\begin{aligned} \text{Let } \ \| x \|^1_\alpha < s \ &\Rightarrow \bigwedge \{ t(>0) : \mu_1(x, t) > \alpha\} < s. \\ &\Rightarrow \exists\, t_0 < s \text{ such that } \mu_1(x, t_0) > \alpha. \\ &\Rightarrow \exists\, t_0 < s \text{ such that } \mu_2(T(x), t_0) > \alpha, \text{ [by Equation 4.1]} \\ &\Rightarrow \| T(x) \|^2_\alpha \leq t_0 < s. \end{aligned}$$

Hence $\| T(x) \|^2_\alpha \leq \| x \|^1_\alpha$.

$$\begin{aligned} \text{Also, let } \ \| x \|^{1*}_\alpha < s \ &\Rightarrow \bigwedge \{ t(>0) : \nu_1(x, t) < \alpha\} < s. \\ &\Rightarrow \exists\, t_0 < s \text{ such that } \nu_1(x, t_0) < \alpha. \\ &\Rightarrow \exists\, t_0 < s \text{ such that } \nu_2(T(x), t_0) < \alpha, \text{ [by Equation 4.1]} \\ &\Rightarrow \| T(x) \|^2_\alpha \leq t_0 < s. \end{aligned}$$

Hence $\| T(x) \|^{2*}_\alpha \leq \| x \|^{1*}_\alpha$.
Thus T is uniformly IFB.
Conversely, suppose T is uniformly IFB with respect to corresponding α-norms. Then for $\alpha \in (0,1)$

$$\| T(x) \|^2_\alpha \leq \| x \|^1_\alpha \quad , \quad \| T(x) \|^{2*}_\alpha \leq \| x \|^{1*}_\alpha \tag{4.2}$$

$$\begin{aligned} \text{Let } \ \mu_1(x, t) > a &\Rightarrow \bigvee \{\alpha \in (0,1) : \| x \|^1_\alpha < t\} > a \\ &\Rightarrow \exists\, \alpha_0 \in (0,1) : \alpha_0 > a \text{ and } \| x \|^1_{\alpha_0} < t \\ &\Rightarrow \exists\, \alpha_0 \in (0,1) : \alpha_0 > a \text{ and } \| T(x) \|^2_{\alpha_0} < t, \text{ [by Equation 4.2]} \\ &\Rightarrow \mu_2(T(x), t) \geq \alpha_0 > a. \end{aligned}$$

Therefore, $\mu_2(T(x), t) \geq \mu_1(x, t)$.

$$\begin{aligned} \text{Also, let } \nu_1(x, t) < b &\Rightarrow \bigwedge \{\alpha \in (0,1) : \| x \|^{1*}_\alpha < t\} < b \\ &\Rightarrow \exists\, \alpha_0 \in (0,1) : \alpha_0 < b \text{ and } \| x \|^{1*}_{\alpha_0} < t \\ &\Rightarrow \exists \alpha_0 \in (0,1) : \alpha_0 < b \text{ and } \| T(x) \|^{2*}_{\alpha_0} < t, \text{ [by Equation 4.2]} \\ &\Rightarrow \nu_2(T(x), t) \leq \alpha_0 < b. \end{aligned}$$

Therefore, $\nu_2(T(x), t) \leq \nu_1(x, t)$.
Hence T is strongly IFB.

Theorem 4.6. *A linear operator $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is strongly IFC if and only if it is strongly IFB, where (X, μ_1, ν_1) and (Y, μ_2, ν_2) are intuitionistic fuzzy pseudo normed linear spaces.*
Proof. First suppose T is strongly IFB then $\forall\, x \in X$ and $\forall\, \epsilon \in \mathbb{R}^+$,
$\mu_2(T(x), \epsilon) \geq \mu_1(x, \epsilon)$, $\nu_2(T(x), \epsilon) \leq \nu_1(x, \epsilon)$.
$\Rightarrow \mu_2(T(x-\theta), \epsilon) \geq \mu_1(x-\theta, \epsilon)$, $\nu_2(T(x-\theta), \epsilon) \leq \nu_1(x-\theta, \epsilon)$
$\Rightarrow \mu_2(T(x)-T(\theta), \epsilon) \geq \mu_1(x-\theta, \delta)$, $\nu_2(T(x-\theta), \epsilon) \leq \nu_1(x-\theta, \delta)$, where $\delta = \epsilon$.
Therefore T is strongly IFC at θ and hence by Theorem 3.6, T is strongly IFC on X.
Conversely, suppose T is strongly IFC on X. Then T is strongly IFC at any point of X, say θ. Now, $\forall\, x \in X$ take $\epsilon = t = \delta$. Then
$\mu_2(T(x)-T(\theta), t) \geq \mu_1(x-\theta, t)$, $\nu_2(T(x)-T(\theta), t) \leq \nu_1(x-\theta, t)$.
$\Rightarrow \mu_2(T(x), t) \geq \mu_1(x, t)$, $\nu_2(T(x), t) \leq \nu_1(x, t)$.
If $x = \theta$, $t > 0$ then $\mu_2(T(\theta), t) = \mu_2(\theta_Y, t) = 1 = \mu_1(\theta, t)$ and $\nu_2(T(\theta), t) = \nu_2(\theta_Y, t) = 0 = \nu_1(\theta, t)$.
For any x, $t \leq 0$, $\mu_2(T(x), t) = 0 = \mu_1(x, t)$ and $\nu_2(T(x), t) = 1 = \nu_1(x, t)$.
Hence $\forall\, x \in X$, $\forall\, t \in \mathbb{R}$, T is strongly IFB.

Corollary 4.7. *A linear operator $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is strongly IFB then it is sequentially IFC, where (X, μ_1, ν_1) and (Y, μ_2, ν_2) are intuitionistic fuzzy pseudo normed linear spaces.*
Proof. From Theorem 3.11 and Theorem 4.6 the corollary follows.

Corollary 4.8. *A linear operator $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is strongly IFB then it is IFC, where (X, μ_1, ν_1) and (Y, μ_2, ν_2) are intuitionistic fuzzy pseudo normed linear spaces.*
Proof. From Corollary 3.13 and Theorem 4.6 the corollary follows.

Theorem 4.9. *A linear operator $T : (X, \mu_1, \nu_1) \to (Y, \mu_2, \nu_2)$ is weakly IFC if and only if it is weakly IFB, where (X, μ_1, ν_1) and (Y, μ_2, ν_2) are intuitionistic fuzzy pseudo normed linear spaces.*
Proof. First suppose T is weakly IFB. Then for any $\alpha \in (0, 1)$, $\forall\, x \in X$, $\forall\, t \in \mathbb{R}^+$,
$\mu_1(x, t) \geq \alpha \Rightarrow \mu_2(T(x), t) \geq \alpha$ and $\nu_1(x, t) \leq \alpha \Rightarrow \nu_2(T(x), t) \leq \alpha$.
$\mu_1(x-\theta, t) \geq \alpha \Rightarrow \mu_2(T(x-\theta), t) \geq \alpha$ and $\nu_1(x-\theta, t) \leq \alpha \Rightarrow \nu_2(T(x-\theta), t) \leq \alpha$.
$\mu_1(x-\theta, \delta) \geq \alpha \Rightarrow \mu_2(T(x)-T(\theta), \epsilon) \geq \alpha$ and $\nu_1(x-\theta, \delta) \leq \alpha \Rightarrow \nu_2(T(x)-T(\theta), \epsilon) \leq \alpha$, where $\epsilon = t = \delta$. Therefore, T is weakly IFC at θ and hence by Theorem 3.8, T is weakly IFC.

Conversely suppose T is weakly IFC on X. Then T is weakly IFC at any point of X, say θ. Now, $\forall x \in X$ take $\epsilon = t = \delta$. Then
$\mu_1(x-\theta,t) \geq \alpha \Rightarrow \mu_2(T(x)-T(\theta),t) \geq \alpha$ and $\nu_1(x-\theta,t) \leq \alpha \Rightarrow \nu_2(T(x)-T(\theta),t) \leq \alpha$
$\mu_1(x,t) \geq \alpha \Rightarrow \mu_2(T(x),t) \geq \alpha$ and $\nu_1(x,t) \leq \alpha \Rightarrow \nu_2(T(x),t) \leq \alpha$
If $x=\theta,\ t>0$ then $\mu_1(x,t) = 1 = \mu_2(T(x),t)$ and $\nu_1(x,t) = 0 = \nu_2(T(x),t)$.
For any $x,\ t \leq 0,\ \mu_1(x,t) = 0 = \mu_2(T(x),t)$ and $\nu_1(x,t) = 1 = \nu_2(T(x),t)$.
Hence for any $\alpha \in (0,1)$, $\forall x \in X$, $\forall t \in \mathbb{R}$, T is weakly IFB.

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