South East Asian J. of Mathematics and Mathematical Sciences
Vol. 17, No. 2 (2021), pp. 195-202

ISSN (Online): 2582-0850

ISSN (Print): 0972-7752

INTUITIONISTIC L-FUZZY SOFT SEMIRINGS

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(**Received:** *Jun. 22, 2020* **Accepted:** *May 09, 2021* **Published:** *Aug. 30, 2021)*

Abstract: The aim of this paper is to study the concept of Intuitionistic L-fuzzy soft semiring and Intuitionistic L-fuzzy soft subsemiring. The Intuitionistic L-fuzzy soft subsemiring and its level set are defined. The homomorphism of Intuitionistic L-fuzzy soft semiring defined under the Intuitionistic L-fuzzy soft function. The results based on the Intuitionistic L-fuzzy soft semiring and its homomorphism are determined.

Keywords and Phrases: Intuitionistic fuzzy set, Intuitionistic L-fuzzy set, Intuitionistic L-fuzzy soft ring, Intuitionistic L-fuzzy soft semiring.

2020 Mathematics Subject Classification: 03F55, 06D72, 08A72, 16Y60.

1. Introduction

Soft set theory is initiated by Molodtsov. D in [7]. Researchers all over the globe are working with soft sets and soft sets such as fuzzy soft sets, L-fuzzy soft sets. The theory of Intuitionistic fuzzy set plays an important role in modern mathematics. The idea of Intuitionistic L-fuzzy set (ILFS) was introduced by Atanassov. K. T (1986) [1] as a generalization of Zadeh. L. A (1965) [16] fuzzy sets. Maji P. K, Biswas. R and Roy. A. R. (2003) [5] have applied the concept of Soft set theory of groups. The concept of fuzzy soft ring was introduced by Pazar Varol. B, Ayunoglu. A and Aygun. H (2012) [13]. In this paper to introduced the concept Intuitionistic L-fuzzy soft semiring and established some results on these.

2. Preliminaries

Definition 2.1. *An intuitionistic fuzzy set (IFS) A in E is defined as an object of the following from* $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in E\}$ *where the function* $\mu_A : E \to [0,1]$ *and* $\gamma_A : E \to [0,1]$ *define the degree of membership and the degree of non-membership of the element* $x \in E$ *respectively and for every* $x \in E$*, satisfying* $0 \leq \mu_A(x) + \gamma_A(x) \leq 1$.
In addition, for all $x \in E$, $E = \{\langle x, 0, 1\rangle : x \in E\}$ *and* $\phi = \{\langle x, 0, 1\rangle : x \in E\}$ *are intuitionistic fuzzy universal and intuitionistic fuzzy empty sets, respectively.*

Definition 2.2. *Let* $(L, \leq)$ *be a complete lattice with an involutive order reversing operation* $N : L \to L$*. Let E be a non-empty set. An intuitionistic L-fuzzy set (ILFS) A in E is defined as an object of the form* $A = \{\langle x, \mu_A(x), \gamma_A(x)\rangle : x \in E\}$ *where the function* $\mu_A : E \to L$ *and* $\gamma_A : E \to L$ *define the degree of membership and the degree of non-membership of the element* $x \in X$ *respectively and for every* $x \in E$*, satisfying* $\mu_A(x) \leq (N(\gamma_A(x))$.

Definition 2.3. *Let* $(\tilde{F}, A)$ *and* $(\tilde{G}, B)$ *be two intuitionistic L-fuzzy soft sets over U. Then,* $(\tilde{F}, A)$ *is said to be an intuitionistic L-fuzzy soft subset of* $(\tilde{G}, B)$ *if*

1. $A \subset B$,
2. $\tilde{F}(\epsilon)$ *is an intuitionistic fuzzy subset of* $\tilde{G}(\epsilon)$*, for all* $\epsilon \in A$.

3. Level Subsets of Intuitionistic L-fuzzy Soft Subsemiring of a Semiring

Definition 3.1. *Let R be a semiring. A pair* $(\tilde{F}, A)$ *is called an intuitionitic L-fuzzy soft subsemiring over R, where* $\tilde{F}$ *is a mapping given by* $\tilde{F} : A \to ([0,1] \times [0,1])^R$, $\tilde{F}(\epsilon) : R \to [0,1] \times [0,1]$, $\tilde{F}(\epsilon) = \{(x, \mu_{\tilde{F}(\epsilon)}(x), \gamma_{\tilde{F}(\epsilon)}(x)) : x \in R\}$ *for all* $\epsilon \in A$*, if for all* $x, y \in R$ *the following conditions hold :*

1. $\mu_{\tilde{F}(\epsilon)}(x - y) \geq \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y)$ *and* $\gamma_{\tilde{F}(\epsilon)}(x - y) \leq \gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y)$,
2. $\mu_{\tilde{F}(\epsilon)}(xy) \geq \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y)$ *and* $\gamma_{\tilde{F}(\epsilon)}(xy) \leq \gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y)$.

Definition 3.2. *Let* $(\tilde{F}, A)$ *be an intuitionistic L-fuzzy soft subset in a set S, the strongest intuitionistic fuzzy relation on S, that is a intutionistic L-fuzzy soft relation on A is* ϵ *given by* $\mu_{\tilde{F}(\epsilon)}(x, y) = \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y)$ *and*

$\gamma_{\tilde{F}(\epsilon)}(x,y) = \gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y)$, *for all* $x, y \in S$.

Definition 3.3. *Let* $(\tilde{F}, A)$ *be an intuitionistic L-fuzzy soft subset of* X. *For* α, β *in* $[0,1]$ *the level soft subset of* A *is the set,* $(\tilde{F}, A)(\alpha, \beta) = \{x \in X : \mu_{\tilde{F}(\epsilon)}(x) \geq \alpha, \gamma_{\tilde{F}(\epsilon)}(x) \leq \beta\}$. *This is called an intuitionistic L- fuzzy soft level subset of* A.

Theorem 3.4. *Let* $(\tilde{F}, A)$ *be an intuitionistic L- fuzzy soft subsemiring of a semiring* R. *Then for* α *and* β *in* $[0,1]$, $A(\alpha, \beta)$ *is a soft subsemiring of* R.
Proof. Given $(\tilde{F}, A)$ is an intuitionistic L-fuzzy soft subsemiring of a semiring R. Take $x, y \in A(\alpha, \beta)$, then
$\mu_{\tilde{F}(\epsilon)}(x) \geq \alpha$, $\gamma_{\tilde{F}(\epsilon)}(x) \leq \beta$ and $\mu_{\tilde{F}(\epsilon)}(y) \geq \alpha$, $\gamma_{\tilde{F}(\epsilon)}(y) \leq \beta$. Now,
$\mu_{\tilde{F}(\epsilon)}(x-y) \geq \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y) \geq \alpha \wedge \alpha = \alpha \Rightarrow \mu_{\tilde{F}(\epsilon)}(x-y) \geq \alpha$,
$\mu_{\tilde{F}(\epsilon)}(xy) \geq \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y) \geq \alpha \wedge \alpha = \alpha \Rightarrow \mu_{\tilde{F}(\epsilon)}(xy) \geq \alpha$,
$\gamma_{\tilde{F}(\epsilon)}(x-y) \leq \gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y) \leq \beta \vee \beta = \beta \Rightarrow \gamma_{\tilde{F}(\epsilon)}(x-y) \leq \beta$
$\gamma_{\tilde{F}(\epsilon)}(xy) \leq \gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y) \leq \beta \vee \beta = \beta \Rightarrow \gamma_{\tilde{F}(\epsilon)}(xy) \leq \beta$ for all $x, y \in A(\alpha, \beta)$.
Therefore $\mu_{\tilde{F}(\epsilon)}(x-y) \geq \alpha$, $\mu_{\tilde{F}(\epsilon)}(xy) \geq \alpha$ and $\gamma_{\tilde{F}(\epsilon)}(x-y) \leq \beta$, $\gamma_{\tilde{F}(\epsilon)}(xy) \leq \beta$
$\Rightarrow x-y, xy$ are in $A(\alpha, \beta)$.
Hence $A(\alpha, \beta)$ is a soft subsemiring R.

Theorem 3.5. *Let* $(\tilde{F}, A)$ *be an intuitionistic L- fuzzy soft subsemiring of a semiring* R. *Then two level soft subsemiring* $(\tilde{F}, A)(\alpha_1, \beta_1), (\tilde{F}, A)(\alpha_2, \beta_2)$ *and* α_1, α_2, β_1, β_2 *in* $[0,1]$ *with* $\alpha_2 < \alpha_1$ *and* $\beta_1 < \beta_2$ *of* $(\tilde{F}, A)$ *are equal if and only if there is no* x *in* R *such that* $\alpha_1 > \mu_{\tilde{F}(\epsilon)}(x) > \alpha_2$ *and* $\beta_1 < \gamma_{\tilde{F}(\epsilon)}(x) < \beta_2$.

Proof. Assume that $(\tilde{F}, A)(\alpha_1, \beta_1) = (\tilde{F}, A)(\alpha_2, \beta_2)$. Suppose there exists $x \in R$ such that $\alpha_1 > \mu_{\tilde{F}(\epsilon)}(x) > \alpha_2$ and $\beta_1 < \gamma_{\tilde{F}(\epsilon)}(x) < \beta_2$. Then $(\tilde{F}, A)(\alpha_1, \beta_1) \subseteq (\tilde{F}, A)(\alpha_2, \beta_2)$ which implies $x \in (\tilde{F}, A)(\alpha_2, \beta_2)$ but $x \notin (\tilde{F}, A)(\alpha_1, \beta_1)$ which implies a contradiction to $(\tilde{F}, A)(\alpha_1, \beta_1) = (\tilde{F}, A)(\alpha_2, \beta_2)$ therefore there is no $x \in R$ such that $\alpha_1 > \mu_{\tilde{F}(\epsilon)}(x) > \alpha_2$ and $\beta_1 < \gamma_{\tilde{F}(\epsilon)}(x) < \beta_2$. Conversely if there is no $x \in R$ such that $\alpha_1 > \mu_{\tilde{F}(\epsilon)}(x) > \alpha_2$ and $\beta_1 < \gamma_{\tilde{F}(\epsilon)}(x) < \beta_2$. Then $A(\alpha_1, \beta_1) = A(\alpha_2, \beta_2)$.

Theorem 3.6. *Let* R *be a soft semiring and* $(\tilde{F}, A)$ *be an intuitionistic L-fuzzy soft subset of* R *such that* $(\tilde{F}, A)(\alpha, \beta)$ *be a soft subsemiring of* R. *If* α *and* β *in* $[0,1]$ *then* $(\tilde{F}, A)$ *is an intuitionistic L-fuzzy soft subsemiring of* R.
Proof. Let R be a soft subsemiring. For x and y in R.
Let $\mu_{\tilde{F}(\epsilon)}(x) = \alpha_1$, $\mu_{\tilde{F}(\epsilon)}(y) = \alpha_2$, $\gamma_{\tilde{F}(\epsilon)}(x) = \beta_1$ and $\gamma_{\tilde{F}(\epsilon)}(y) = \beta_2$.
Case (i):
If $\alpha_1 < \alpha_2$ and $\beta_1 > \beta_2$ then $x, y \in (\tilde{F}, A)(\alpha_1, \beta_1)$.
As $(\tilde{F}, A)(\alpha_1, \beta_1)$ is a soft subsemiring of R , $x-y$ and xy in $(\tilde{F}, A)(\alpha_1, \beta_1)$
$\mu_{\tilde{F}(\epsilon)}(x-y) \geq \alpha_1 = \alpha_1 \wedge \alpha_2$ which implies $\mu_{\tilde{F}(\epsilon)}(x-y) \geq \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y)$,

$\mu_{\tilde{F}(\epsilon)}(xy) \geq \alpha_1 = \alpha_1 \wedge \alpha_2$ which implies $\mu_{\tilde{F}(\epsilon)}(xy) \geq \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y)$,
$\gamma_{\tilde{F}(\epsilon)}(x-y) \leq \beta_1 = \beta_1 \vee \beta_2$ which implies $\gamma_{\tilde{F}(\epsilon)}(x-y) \leq \gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y)$,
$\gamma_{\tilde{F}(\epsilon)}(xy) \leq \beta_1 = \beta_1 \vee \beta_2$ which implies $\gamma_{\tilde{F}(\epsilon)}(xy) \leq \gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y)$,
for all $x, y \in R$.

Case (ii):
If $\alpha_1 < \alpha_2$ and $\beta_1 < \beta_2$ then $x, y \in (\tilde{F}, A)(\alpha_1, \beta_2)$.
As $(\tilde{F}, A)(\alpha_1, \beta_2)$ is a soft subsemiring of R , $x-y$ and xy in $(\tilde{F}, A)(\alpha_1, \beta_2)$
$\mu_{\tilde{F}(\epsilon)}(x-y) \geq \alpha_1 = \alpha_1 \wedge \alpha_2$ which implies $\mu_{\tilde{F}(\epsilon)}(x-y) \geq \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y)$,
$\mu_{\tilde{F}(\epsilon)}(xy) \geq \alpha_1 = \alpha_1 \wedge \alpha_2$ which implies $\mu_{\tilde{F}(\epsilon)}(xy) \geq \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y)$,
$\gamma_{\tilde{F}(\epsilon)}(x-y) \leq \beta_2 = \beta_1 \vee \beta_2$ which implies $\gamma_{\tilde{F}(\epsilon)}(x-y) \leq \gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y)$,
$\gamma_{\tilde{F}(\epsilon)}(xy) \leq \beta_2 = \beta_1 \vee \beta_2$ which implies $\gamma_{\tilde{F}(\epsilon)}(xy) \leq \gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y)$,
for all $x, y \in R$.

Case (iii):
If $\alpha_1 > \alpha_2$ and $\beta_1 > \beta_2$ then $x, y \in (\tilde{F}, A)(\alpha_2, \beta_1)$.
As $(\tilde{F}, A)(\alpha_2, \beta_1)$ is a soft subsemiring of R , $x-y$ and xy in $(\tilde{F}, A)(\alpha_2, \beta_1)$
$\mu_{\tilde{F}(\epsilon)}(x-y) \geq \alpha_2 = \alpha_1 \wedge \alpha_2$ which implies $\mu_{\tilde{F}(\epsilon)}(x-y) \geq \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y)$,
$\mu_{\tilde{F}(\epsilon)}(xy) \geq \alpha_2 = \alpha_1 \wedge \alpha_2$ which implies $\mu_{\tilde{F}(\epsilon)}(xy) \geq \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y)$,
$\gamma_{\tilde{F}(\epsilon)}(x-y) \leq \beta_1 = \beta_1 \vee \beta_2$ which implies $\gamma_{\tilde{F}(\epsilon)}(x-y) \leq \gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y)$,
$\gamma_{\tilde{F}(\epsilon)}(xy) \leq \beta_1 = \beta_1 \vee \beta_2$ which implies $\gamma_{\tilde{F}(\epsilon)}(xy) \leq \gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y)$,
for all $x, y \in R$.

Case (iv):
If $\alpha_1 > \alpha_2$ and $\beta_1 < \beta_2$ then $x, y \in (\tilde{F}, A)(\alpha_2, \beta_2)$.
As $(\tilde{F}, A)(\alpha_2, \beta_2)$ is a soft subsemiring of R , $x-y$ and xy in $(\tilde{F}, A)(\alpha_2, \beta_2)$
$\mu_{\tilde{F}(\epsilon)}(x-y) \geq \alpha_2 = \alpha_1 \wedge \alpha_2$ which implies $\mu_{\tilde{F}(\epsilon)}(x-y) \geq \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y)$,
$\mu_{\tilde{F}(\epsilon)}(xy) \geq \alpha_2 = \alpha_1 \wedge \alpha_2$ which implies $\mu_{\tilde{F}(\epsilon)}(xy) \geq \mu_{\tilde{F}(\epsilon)}(x) \wedge \mu_{\tilde{F}(\epsilon)}(y)$,
$\gamma_{\tilde{F}(\epsilon)}(x-y) \leq \beta_2 = \beta_1 \vee \beta_2$ which implies $\gamma_{\tilde{F}(\epsilon)}(x-y) \leq \gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y)$,
$\gamma_{\tilde{F}(\epsilon)}(xy) \leq \beta_2 = \beta_1 \vee \beta_2$ which implies $\gamma_{\tilde{F}(\epsilon)}(xy) \leq \gamma_{\tilde{F}(\epsilon)}(x) \vee \gamma_{\tilde{F}(\epsilon)}(y)$,
for all $x, y \in R$.

Case (v):
If $\alpha_1 = \alpha_2$ and $\beta_1 = \beta_2$,then it is trivial.
In all the cases, $(\tilde{F}, A)$ is an intuitionistic L-fuzzy subsemiring of a semiring R.

4. Homomorphism of Intuitionistic L- fuzzy Soft Semiring

In this section we show that the homomorphism image and pre-image of a intuitionistic L-fuzzy soft semiring.

Definition 4.1. *Let $f : R \to S$ and $g : A \to B$ be two functions, where A and B are parameter sets for intuitionistic L- fuzzy soft sets R and S, respectively. Then, the pair (f, g) is called an Intuitionistic L- fuzzy soft function from R to S.*

Definition 4.2. *Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two intuitionistic L- fuzzy soft rings over R and S, respectively, Let $f : R \to S$ be a homomorphism of rings, and let $g : A \to B$ be a mapping of sets. Then, we say that $(f, g) : (\tilde{F}, A) \to (\tilde{G}, B)$ is an intuitionistic L-fuzzy soft homomorphism of intuitionistic L-fuzzy soft rings and define by $f(\tilde{F}, A) = (\tilde{G}, B)g$, if the following conditions are satisfied: $f(\mu_{\tilde{F}(\epsilon)}(x)) = (\mu_{\tilde{G}(\epsilon)}(x))g, f(\gamma_{\tilde{F}(\epsilon)}(x)) = (\gamma_{\tilde{G}(\epsilon)}(x))g$.*

Definition 4.3. *Let $(\tilde{F}, A)$ and $(\tilde{G}, B)$ be two intuitionistic L-fuzzy soft rings over R and S. Let (f, g) be an intuitionistic L-fuzzy soft function from R to S.*

1. *The image of $(\tilde{F}, A)$ under the intuitionistic L-fuzzy soft function (f, g) denoted by (f, g). $(\tilde{F}, A)$ is the intuitionistic L-fuzzy soft ring over S defined by $(f, g)(\tilde{F}, A) = (f(\tilde{F}), g(A))$, where*
$$f(\tilde{F})_k(s) = \begin{cases} \bigvee_{f(r)=s} \bigvee_{g(a)=k} \mu_{\tilde{F}(\epsilon)}(r), & if\ r \in f^{-1}(s) \\ 0, & Otherwise \end{cases}$$
$$f(\tilde{F})_k(s) = \begin{cases} \bigwedge_{f(r)=s} \bigwedge_{g(a)=k} \gamma_{\tilde{F}(\epsilon)}(r), & if\ \ r \in f^{-1}(s) \\ 1, & Otherwise \end{cases}$$
for all $k \in g(A)$ and for all $s \in S$.

2. *The preimage of $(\tilde{G}, B)$ under the intuitionistic L-fuzzy soft function (f, g) denoted by $(f, g)^{-1}(\tilde{G}, B)^{-1}$ is the intuitionistic L-fuzzy soft ring over R defined by $(f, g)^{-1}(\tilde{G}, B) = (f^{-1}(\tilde{G}), g^{-1}(B))$, where*
$f^{-1}(\tilde{G})_{(a)}(r) = (\tilde{G})_{g(a)}(f(r))$, for all $a \in g^{-1}(B)$ and for all $r \in R$.
If f and g are injective (surjective), then (f, g) is said to be injective (surjective).

Theorem 4.4. *Let $(\tilde{F}, A)$ be an intuitionistic L-fuzzy soft subsemiring over R and (f, g) an intuitionistic L-fuzzy soft homomorphism from R to S. Then, $(f, g)(\tilde{F}, A)$ is an intuitionistic L- fuzzy soft subsemiring over S.*

Proof. Let $k \in g(A)$ and $y_1, y_2 \in S$. If $f^{-1}(y_1) = \phi$ or $f^{-1}(y_2) = \phi$ the proof is straightforward.
Let assume that there exist $x_1, x_2 \in R$ such that $f(x_1) = y_1, f(x_2) = y_2$.
$$\begin{aligned} f(\tilde{F})_k(y_1 - y_2) &= \bigvee_{f(t)=y_1-y_2} \bigvee_{g(a)=k} \mu_{\tilde{F}(\epsilon)}(t) \\ &\geq \bigvee_{g(a)=k} \mu_{\tilde{F}(\epsilon)}(x_1 - x_2) \\ &\geq \bigvee_{g(a)=k} \Big(\mu_{\tilde{F}(\epsilon)}(x_1) \wedge \mu_{\tilde{F}(\epsilon)}(x_2)\Big) \end{aligned}$$

$$= \bigvee_{g(a)=k} \mu_{\tilde{F}(\epsilon)}(x_1) \wedge \bigvee_{g(a)=k} \mu_{\tilde{F}(\epsilon)}(x_2)$$

This inequality is satisfied for each $x_1, x_2 \in R$, which satisfy that $f(x_1) = y_1, f(x_2) = y_2$. Then we have

$$f(\tilde{F})_k(y_1 - y_2) \geq \left(\bigvee_{f(t_1)=y_1} \bigvee_{g(a)=k} \mu_{\tilde{F}(\epsilon)}(t_1) \right) \wedge \left(\bigvee_{f(t_2)=y_2} \bigvee_{g(a)=k} \mu_{\tilde{F}(\epsilon)}(t_2) \right)$$

$$f(\tilde{F})_k(y_1 - y_2) = f(\tilde{F})_k(y_1) \wedge f(\tilde{F})_k(y_2).$$

And, we have

$$\begin{aligned} f(\tilde{F})_k(y_1 \cdot y_2) &= \bigvee_{f(t)=y_1 \cdot y_2} \bigvee_{g(a)=k} \mu_{\tilde{F}(\epsilon)}(t) \\ &\geq \bigvee_{g(a)=k} \mu_{\tilde{F}(\epsilon)}(x_1 \cdot x_2) \\ &\geq \bigvee_{g(a)=k} \left(\mu_{\tilde{F}(\epsilon)}(x_1) \wedge \mu_{\tilde{F}(\epsilon)}(x_2) \right) \\ &= \bigvee_{g(a)=k} \mu_{\tilde{F}(\epsilon)}(x_1) \wedge \bigvee_{g(a)=k} \mu_{\tilde{F}(\epsilon)}(x_2) \end{aligned}$$

This inequality is satisfied for each $x_1, x_2 \in R$, which satisfy that $f(x_1) = y_1, f(x_2) = y_2$. Then we have

$$f(\tilde{F})_k(y_1 \cdot y_2) \geq \left(\bigvee_{f(t_1)=y_1} \bigvee_{g(a)=k} \mu_{\tilde{F}(\epsilon)}(t_1) \right) \wedge \left(\bigvee_{f(t_2)=y_2} \bigvee_{g(a)=k} \mu_{\tilde{F}(\epsilon)}(t_2) \right)$$

$$f(\tilde{F})_k(y_1 \cdot y_2) = f(\tilde{F})_k(y_1) \wedge f(\tilde{F})_k(y_2).$$

Also

$$\begin{aligned} f(\tilde{F})_k(y_1 - y_2) &= \bigwedge_{f(t)=y_1 - y_2} \bigwedge_{g(a)=k} \gamma_{\tilde{F}(\epsilon)}(t) \\ &\leq \bigwedge_{g(a)=k} \gamma_{\tilde{F}(\epsilon)}(x_1 - x_2) \\ &\leq \bigwedge_{g(a)=k} \left(\gamma_{\tilde{F}(\epsilon)}(x_1) \vee \gamma_{\tilde{F}(\epsilon)}(x_2) \right) \\ &= \bigwedge_{g(a)=k} \gamma_{\tilde{F}(\epsilon)}(x_1) \vee \bigwedge_{g(a)=k} \gamma_{\tilde{F}(\epsilon)}(x_2) \end{aligned}$$

$$f(\tilde{F})_k(y_1 - y_2) \leq \left(\bigwedge_{f(t_1)=y_1} \bigwedge_{g(a)=k} \gamma_{\tilde{F}(\epsilon)}(t_1) \right) \vee \left(\bigwedge_{f(t_2)=y_2} \bigwedge_{g(a)=k} \gamma_{\tilde{F}(\epsilon)}(t_2) \right)$$

$$f(\tilde{F})_k(y_1 - y_2) = f(\tilde{F})_k(y_1) \vee f(\tilde{F})_k(y_2).$$

Again

$$\begin{aligned} f(\tilde{F})_k(y_1 \cdot y_2) &= \bigwedge_{f(t)=y_1 \cdot y_2} \bigwedge_{g(a)=k} \gamma_{\tilde{F}(\epsilon)}(t) \\ &\leq \bigwedge_{g(a)=k} \gamma_{\tilde{F}(\epsilon)}(x_1 \cdot x_2) \end{aligned}$$

$$\leq \bigwedge_{g(a)=k} \left(\gamma_{\tilde{F}(\epsilon)}(x_1) \vee \gamma_{\tilde{F}(\epsilon)}(x_2)\right)$$
$$= \bigwedge_{g(a)=k} \gamma_{\tilde{F}(\epsilon)}(x_1) \vee \bigwedge_{g(a)=k} \gamma_{\tilde{F}(\epsilon)}(x_2)$$
$$f(\tilde{F})_k(y_1 \cdot y_2) \leq \left(\bigwedge_{f(t_1)=y_1} \bigwedge_{g(a)=k} \gamma_{\tilde{F}(\epsilon)}(t_1)\right) \vee \left(\bigwedge_{f(t_2)=y_2} \bigwedge_{g(a)=k} \gamma_{\tilde{F}(\epsilon)}(t_2)\right)$$
$$f(\tilde{F})_k(y_1 \cdot y_2) = f(\tilde{F})_k(y_1) \vee f(\tilde{F})_k(y_2).$$

Thus we conclude that $(f,g)(\tilde{F},A)$ is an intuitionistic L-fuzzy soft subsemiring over S.

Theorem 4.5. *Let $(\tilde{G},B)$ be an intuitionistic L-fuzzy soft subsemiring over R and (f,g) an intuitionistic L-fuzzy soft homomorphism from R to S. Then, $(f,g)^{-1}(\tilde{G},B)$ is an intuitionistic L- fuzzy soft subsemiring over S*

Proof. Let $a \in g^{-1}(B)$ and $x_1, x_2 \in R$.

$$f^{-1}(\tilde{G})_a(x_1 \cdot x_2) = (\tilde{G})_{g(a)}(f(x_1 \cdot x_2))$$
$$= (\tilde{G})_{g(a)}(f(x_1) \cdot f(x_2))$$
$$\geq (\tilde{G})_{g(a)}(f(x_1)) \wedge (\tilde{G})_{g(a)}(f(x_2))$$
$$f^{-1}(\tilde{G})_a(x_1 \cdot x_2) = f^{-1}(\tilde{G})_{(a)}(x_1) \wedge f^{-1}(\tilde{G})_{(a)}(x_2) \text{ and}$$
$$f^{-1}(\tilde{G})_a(x_1 - x_2) = (\tilde{G})_{g(a)}(f(x_1 - x_2))$$
$$= (\tilde{G})_{g(a)}(f(x_1) - f(x_2))$$
$$\geq (\tilde{G})_{g(a)}(f(x_1)) \wedge (\tilde{G})_{g(a)}(f(x_2))$$
$$f^{-1}(\tilde{G})_a(x_1 - x_2) = f^{-1}(\tilde{G})_{(a)}(x_1) \wedge f^{-1}(\tilde{G})_{(a)}(x_2)$$

So $(f,g)^{-1}(\tilde{G},B)$ is an intuitionistic L- fuzzy soft subsemiring over S.

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