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MAX CONNECTIVITY AND INHERENT VALUE OF A SMART FUZZY GRAPH

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Abstract: The internet of things is the building blocks of the latest technology. By adding intelligence into everyday objects, it transforms them into smart objects. The internet of things acts as a podium between these smart objects and the human beings. In this paper, we have introduced the max connectivity and inherent value of quantum smart fuzzy graph. This paper also studies the comparison among quantum edge, max connectivity and inherent value of quantum smart fuzzy graph.

Keywords and Phrases: Max connectivity, inherent value.

2020 Mathematics Subject Classification: 05C78, 54C05.

1. Introduction

The uncertainty and vulnerability in the classic graph was overcome with the idea of fuzzy sets that was introduced by Zadeh in 1965. After which the subject became to gain tremendous impact and applications in engineering and technology. Using Zadehs concept of fuzzy relation, the definition of fuzzy graph was termed by Kaufmann in 1973. Later Rosenfed and other scientists notably like Yeh and Bang laid the foundation for fuzzy graph theory. This paved way for more concepts like path, tree, connectedness, bridges, etc. Fuzzy graph theory has various applications

in the cutting edge science and innovation particularly in the field of data innovation hypothesis, neutral network, medical diagnosis, networking, etc.
A developing number of physical articles are being associated with the web at an exceptional rate understanding the possibility of the web of things. In this paper, we have presented the max connectivity of quantum of smart fuzzy graph and inherent value of smart fuzzy graph along with a framework to support the idea.

2. Preliminaries

Definition 2.1. *Fuzzy Graph: Let V and E denotes the set of vertices and edges of a graph $G(V, E)$ respectively. A fuzzy graph is denoted as $G(A, R)$, where A is a fuzzy set on V and R is a fuzzy relation on V such that $R(u, v) \leq min((A(u), A(v))$, for every $u, v \in V$.*
Note that, fuzzy set A is defined by the membership function $A : V \to [0, 1]$, where $A(v)$ denotes the membership grade of a vertex $v \in V$ in fuzzy set A and fuzzy relation R is defined by membership function $R : V \times V \to [0, 1]$, where $R(u, v)$ denotes the membership grade of an edge $(u, v) \in E$ in the fuzzy relation R.

Definition 2.2. *Fuzzy Subgraph: Let $G(A, R)$ be a fuzzy set with set of vertices V. A fuzzy graph $H(\tilde{V}, B, \tilde{R})$ is called a fuzzy subgraph of $G(A, R)$ induced by $\tilde{V}$, if $\tilde{V} \subseteq V$, $B(u) = A(u)$, $\forall u \in \tilde{V}$ and $\tilde{R}(u, v) = R(u, v)$, $\forall u \in \tilde{V}$.*

Definition 2.3. *Partial Fuzzy Graph: A fuzzy graph $H(B, \tilde{R})$ is called a partial fuzzy subgraph of $G(A, R)$ if $B \subseteq A$ and $\tilde{R} \subseteq R$, i.e. $B(u) \leq A(u)$, $\forall u \in V$ and $\tilde{R}(u, v) \leq R(u, v)$, $\forall u, v \in V$.*

Definition 2.4. *Complete Fuzzy Graph: A fuzzy graph $G(A, R)$ is said to be complete if $R(u, v) = min(A(u), A(v))$, $\forall u, v \in V$, where $V^* = \{v \in V | A(v) > 0\}$.*

Definition 2.5. *Smart Fuzzy Graph: Let V be a non-empty set. Then a smart fuzzy graph is defined as a pair of functions $G = (\sigma, \mu)$ where σ is a fuzzy subset of V and μ is given as the symmetric fuzzy relation on σ. (i.e.) $\sigma : V \to [0, 1]$ and $\mu : [V \times V] \to [0, 1] \ni \mu(x, y) \leq \sigma(x) \wedge \sigma(y)$ for all u and v in V with following criterion.*

1. *If $i \neq j, \sum \mu_{ij} \leq \sum(\sigma_i \wedge \sigma_j) \leq 1$*
2. *If $i = j, \sum \mu_{ij} = \sum(\sigma_i \wedge \sigma_j) = 0$*

Definition 2.6. *Quantum of a smart fuzzy graph is defined as the average of the assigned weight of the vertices and the average of the assigned weight of the edges. Quantum analysis for edge and vertex is given separately as*
Edge Quantum: $Q_e = (\sum e_i)/N_e$ and Vertex Quantum: $Q_v = (\sum v_i)/N_v$. Where

e_i represents the edges and v_i represents the vertices in the smart fuzzy graph. And N_e is the number of edges and N_v is the number of vertices respectively, in the smart fuzzy graph.

Definition 2.7. *Super Connectivity: The strongest connection in the quantum network is termed as the super connectivity. It is denoted as $S = max[Q_e^H \wedge Q_e^L]$.*

3. Max Connectivity of Quantum of Smart Fuzzy Graph

Definition 3.1. *Max connectivity is defined as the average weight of the quantum edges or the quantum vertices whose values are obtained from super connectivity. It is denoted as*

$$M = \sum S/N_m$$

where S is the super connectivity and N_m is the number values in super connectivity.
Note: A smart fuzzy graph has a max connectivity in the quantum Q, when there exist a strongest connection between two vertices whose connection is greater than the super connectivity of the same quantum network.
The way towards deciding the max connectivity of the quantum of a smart fuzzy graph is to obtain the strongest connectivity in an arrangement of components. In this procedure, the components in a framework are viewed as the vertices and the connections among them are taken to be the edges. The quantum ends with the value of max connectivity in the system which is considered as the strongest connection than the super connectivity in the given system.

4. Algorithm to find the Max Connectivity through Quantum method

Step 1. Take the given network and check for the conditions of smart fuzzy graph. Allot the weight-age according to the connection in the network.

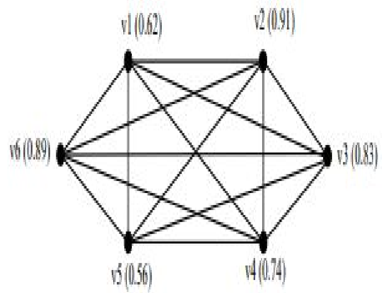

Figure 4.1 Smart fuzzy graph

Table 4.1 Table of Smart Fuzzy Graph

-	-	0.62	0.91	0.83	0.74	0.56	0.89	-
-	-	V1	V2	V3	V4	V5	V6	$\sum \mu_{ij}$
0.62	V1	0	0.05	0.12	0.07	0.16	0.04	0.44
0.91	V2	0.05	0	0.06	0.18	0.03	0.14	0.46
0.83	V3	0.12	0.06	0	0.15	0.02	0.08	0.43
0.74	V4	0.07	0.18	0.15	0	0.11	0.05	0.56
0.56	V5	0.16	0.03	0.02	0.11	0	0.17	0.49
0.89	V6	0.04	0.14	0.08	0.05	0.17	0	0.48
-	$\sum \mu_{ij}$	0.44	0.46	0.43	0.56	0.49	0.48	-

Step 2. Find the quantum values for the edges and note the values of the super connectivity. (Refer J Jon Arockiaraj and Chaarumathi P M, Quantum of a Smart Fuzzy Graph)

Table 4.2 Edge Quantum Frequency

Edge Quantum Frequency	High Frequency Quantum Q_e^H	Low Frequency Quantum Q_e^L
(0.10)	0.15	0.05
(0.05)	0.06	0.03
(0.15)	0.17	0.12
(0.03)	0.04	0.02
(0.02)	repeats	nil
(0.06)	0.07	0.05
(0.05)	repeats	nil
(0.12)	0.13	0.11
(0.17)	0.16	0.18
(0.04)	0.04	0.03
(0.11)	repeats	nil
(0.07)	0.07	0.06
(0.04)	repeats	nil
(0.03)	repeats	nil
(0.06)	repeats	nil
(0.07)	repeats	nil
(0.13)	0.14	0.12
(0.18)	0.18	0.17
(0.16)	0.16	0.15
(0.14)	repeats	nil
(0.18)	repeats	nil
(0.16)	repeats	nil
(0.12)	repeats	nil
(0.17)	repeats	nil
(0.15)	repeats	nil

Step 3. Assign the obtain values in a structure of a tree as below.

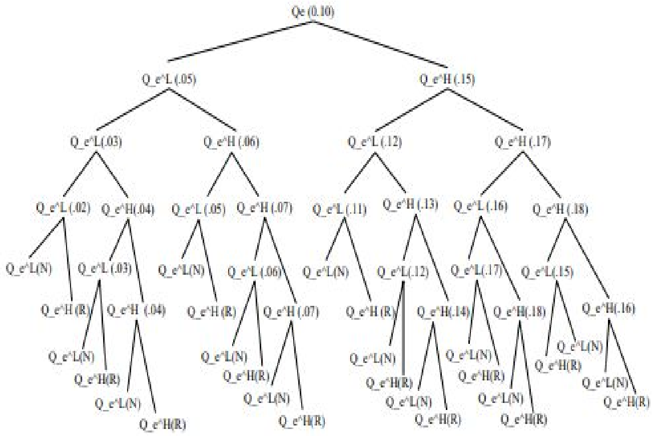

Figure 4.2 Values of edge quantum frequency

Step 4. Stage I has the strong connection in the network which is said to be the super connectivity. The super connectivity in the above tree is

Table 4.3: Super connectivity

Edge	From	To
(0.03)	V2	V5
(0.04)	V1	V6
(0.06)	V2	V3
(0.07)	V1	V4
(0.12)	V1	V3
(0.14)	V2	V6
(0.17)	V5	V6
(0.18)	V2	V6
(0.15)	V3	V4

Step 5. Now plot a graph with the values obtained from super connectivity and find the quantum for those values.

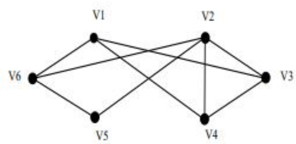

Figure 4.3 Smart Fuzzy Graph

Quantum for the edges of super connectivity.
$Q_e = \sum e_i / N_e$ $Q_e = 0.96/9 = 0.12$

Step 6. Arrange the values according to their respective quantum values into high frequency value and low frequency value.

Table 4.4 Edge quantum frequency

Edge Quantum Frequency	High Frequency Quantum Q_e^H	Low Frequency Quantum Q_e^L
(0.11)	0.05	0.15
(0.05)	0.04	0.07
(0.15)	0.13	0.17
(0.04)	0.04	0.03
(0.07)	0.07	0.06
(0.13)	0.14	0.12
(0.17)	0.18	0.16
(0.03)	repeats	nil
(0.04)	repeats	nil
(0.06)	repeats	nil
(0.07)	repeats	nil
(0.12)	repeats	nil
(0.14)	repeats	nil
(0.16)	0.16	0.15
(0.18)	0.18	0.17

Step 7. Obtained values are assigned in form of a tree

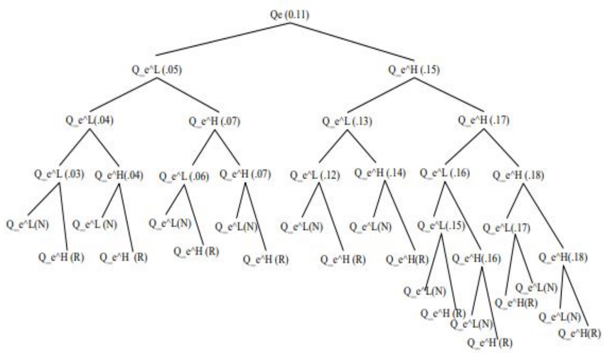

Figure 4.4 Values of edge quantum frequency

Step 8. The stage I values are assigned as the super connectivity. Applying the definition of max connectivity to these values we obtain the strongest connection in the network.

Table 4.5 Super connectivity values

Edges	From	To
(0.17)	V5	V6
(0.18)	V2	V4
(0.15)	V3	V4
(0.16)	V2	V4

Step 9. Max Connectivity: $M = \sum S/N_m$ $M = 0.66/4 = 0.17$ Therefore, the Max Connectivity in the smart fuzzy graph is between (0.17) that is form $V5$ to $V6$.

5. Application of the Max Connectivity Algorithm

Consider an airline network and its route as a smart fuzzy graph. Accept the different point in the airline as vertices and the route associating them as the edges. Here using the max connectivity in the quantum of smart fuzzy graph we can find the strongest connection between two points in the route.

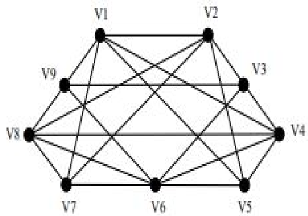

Figure 5.1 An airline network with its route and points

Table 5.1: Table for the airline network

		0.82	0.91	0.94	0.85	0.88	0.92	0.81	0.79	0.93	
		V1	**V2**	**V3**	**V4**	**V5**	**V6**	**V7**	**V8**	**V9**	$\sum \mu_{ij}$
0.82	**V1**	0	0.17	0	0.15	0.21	0	0.06	0	0.2	0.79
0.91	**V2**	0.17	0	0.14	0	0.05	0	0.07	0.09	0	0.52
0.94	**V3**	0	0.14	0	0.18	0	0.25	0	0	0.11	0.68
0.85	**V4**	0.15	0	0.18	0	0.24	0.12	0	0.05	0	0.74
0.88	**V5**	0.21	0.05	0	0.24	0	0.03	0	0	0	0.53
0.92	**V6**	0	0	0.25	0.12	0.03	0	0.16	0.13	0.1	0.79
0.81	**V7**	0.06	0.07	0	0	0	0.16	0	0.22	0.15	0.66
0.79	**V8**	0	0.09	0	0.05	0	0.13	0.22	0	0.23	0.72
0.93	**V9**	0.2	0	0.11	0	0	0.1	0.15	0.23	0	0.79
	$\sum \mu_{ij}$	0.79	0.52	0.68	0.74	0.53	0.79	0.66	0.72	0.79	

Quantum for the edges:
$Q_e = (\sum e_i)/N_e$, $Q_e = 0.14$
Assigning the values obtained in form of a tree.

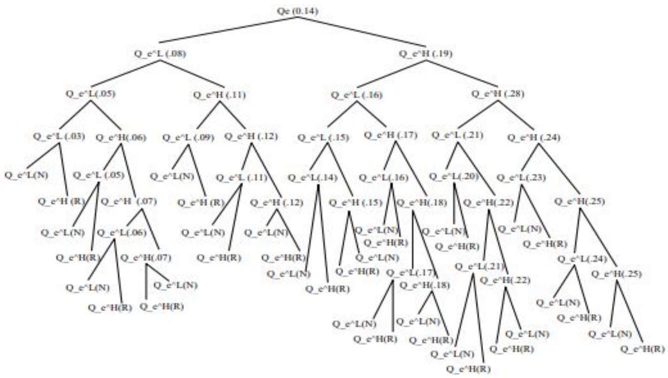

Figure 5.2 Values obtained from the above table

The super connectivity paths are (given below)

Table 5.2 Super connectivity

Edge	From	To
(0.06)	V1	V7
(0.07)	V2	V7
(0.25)	V3	V6
(0.24)	V4	V5
(0.22)	V7	V8
(0.21)	V5	V1
(0.18)	V3	V4
(0.17)	V2	V1

With the values remaining we again construct a smart fuzzy graph.

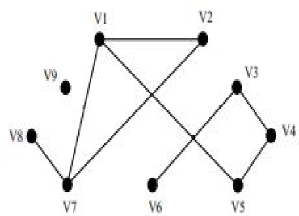

Figure 5.3 Smart Fuzzy Graph based on super connectivity

Quantum for the super connectivity edges:
$Q_e = (\sum e_i)/N_e$, $Q_e = 1.4/8 = 0.18$

Table 5.3: Edge quantum frequency

Edge Quantum Frequency	High Frequency Quantum Q_e^H	Low Frequency Quantum Q_e^L
(0.18)	0.22	0.1
(0.22)	0.24	0.2
(0.1)	0.17	0.07
(0.24)	0.25	0.22
(0.17)	Repeats	Nil
(0.22)	Repeats	Nil
(0.2)	0.21	0.18
(0.18)	Repeats	Nil
(0.21)	Repeats	Nil
(0.07)	0.07	0.06
(0.06)	Repeats	Nil
(0.07)	Repeats	Nil
(0.25)	0.25	0.24
(0.24)	Repeats	Nil
(0.25)	Repeats	Nil

Assigning the values obtained in form of a tree.
The super connectivity paths are

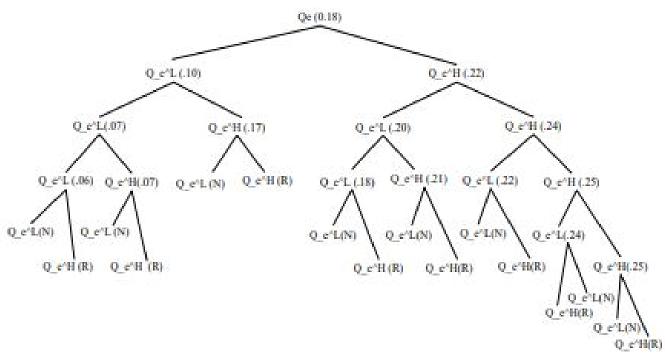

Figure 5.3 Values of edge quantum frequency

Table 5.4: Super connectivity

Edge	From	To
(0.25)	V3	V6
(0.24)	V4	V5

The Max Connectivity: $M = \sum S/N_m$
The frequently used route value is 0.25 and the frequently visited points are $V6$ and $V3$.

6. Inherent Value of a Smart Fuzzy Graph

Definition 6.1. *The Inherent Value is defined as the aggregate of the modulus estimation of the adjacency matrix of the smart fuzzy graph.*
It is denoted as $I = \sum_{i=1}^{N} |\mu_i|$.

Example 6.1.

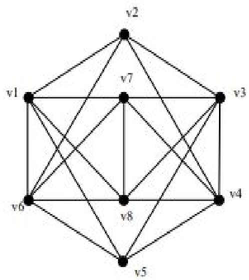

Figure 6.1 Smart Fuzzy Graph

Table 6.1: Value of the Smart fuzzy graph

		0.84	0.89	0.92	0.85	0.97	0.83	0.95	0.9	
		V1	***V2***	***V3***	***V4***	***V5***	***V6***	***V7***	***V8***	
0.84	***V1***	0	0.15	0	0	0.19	0.2	0.09	0.15	0.78
0.89	***V2***	0.15	0	0.2	0.11	0	0.3	0	0	0.76
0.92	***V3***	0	0.2	0	0.08	0.16	0	0.17	0.12	0.73
0.85	***V4***	0	0.11	0.08	0	0.21	0	0.25	0.1	0.75
0.97	***V5***	0.19	0	0.16	0.21	0	0.15	0	0.1	0.81
0.83	***V6***	0.2	0.3	0	0	0.15	0	0.1	0.02	0.77
0.95	***V7***	0.09	0	0.17	0.25	0	0.1	0	0.11	0.72
0.9	***V8***	0.15	0	0.12	0.1	0.1	0.02	0.11	0	0.6
		0.78	0.76	0.73	0.75	0.81	0.77	0.72	0.6	

The Inherent Value:

$$I = \sum_{i=1}^{N} |\mu_i|$$

$\mu_8 = 0.7726$, $\mu_7 = -0.593$, $\mu_6 = -0.2230$, $\mu_5 = 0.2230$,
$\mu_4 = -0.1484$,$u_3 = 0.0804$, $\mu_2 = -0.647$, $\mu_1 = 0.320$.

$I = (0.7726, 0.593, 0.2988, 0.2230, 0.1484, 0.0804, 0.647, 0.0320)$
$I = 2.2162 \Rightarrow 0.222 \approx 0.22$
The Inherent Value is 0.22
Note: The Inherent value is used to find the total strength of the network. It is used mostly in comparison of two or more network in communication, transportation, etc. Assigning the elements in the network as vertices and the connection between them as the edges we can easily find the inherent value for the desired network.

Theorem 6.1. *If $G(\sigma, \mu)$ is a smart fuzzy graph, then M is the strongest connection in the quantum network.*

Proof. Suppose $G(\sigma, \mu)$ is a smart fuzzy graph, then there exists a strong connection with strength greater than M. Assume $S > M$, where S is the super connectivity. If S is the strongest connection in the network, then process ends, the conditions are not satisfied. This is a contradiction to our fact. If G is a smart fuzzy graph then S is not the end of the quantum process. Hence M is the strongest connection in the quantum network.

Corollary 6.1.1. *A quantum of a smart fuzzy graph has at least one set of super connectivity values through which the max connectivity value is obtained.*

Proposition 6.1.1. *The inherent value of a smart fuzzy graph is approximately equal to the value of quantum of edges or the value of max connectivity of the same quantum network.*
i.e. $I \cong M$
i.e. $I \cong Q_e$

7. Conclusion

In this paper we have introduced the max connectivity of quantum of smart fuzzy graph that is used to find the strongest connection in the network. Also we have said that max connectivity gives the final accurate and frequent connection in a network. We have presented the inherent value of the smart fuzzy that is used to find the total value of a desired network. In the forthcoming paper, we will broaden this idea into more concepts.

Acknowledgement

The authors declare no conflict of interest associated with the work.

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