South East Asian J. of Mathematics and Mathematical Sciences
Vol. 17, Proceedings (2021), pp. 109-118

ISSN (Online): 2582-0850

ISSN (Print): 0972-7752

FUZZY CONTRA $\theta g'''$-CLOSED MAPS, $\theta g'''$-OPEN MAPS AND $\theta g'''$-HOMEOMORPHISM IN FUZZY TOPOLOGICAL SPACES

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(**Received:** *Aug. 08, 2021* **Accepted:** *Oct. 01, 2021* **Published:** *Nov. 30, 2021)*

Special Issue
Proceedings of International Virtual Conference on
"Mathematical Modelling, Analysis and Computing IC- MMAC- 2021"

Abstract: In this paper we introduce a new class of maps namely $fcta\theta g'''C$ maps, $fc\theta g'''O$ maps, $fcg'''\theta C$ maps, $fcg'''\theta O$ maps, $fc\theta g'''$-homeomorphism and $fcg'''\theta$-homeomorphism in fts's. Some of their properties have been investigated.

Keywords and Phrases: $fcta\theta g'''C$, $fctag'''\theta C$, $fcta\theta g'''O$, $fctag'''\theta O$, $fctag'''\theta$-Hom, $fcta\theta g'''$-Hom, $fctag'''_{\alpha}\theta$-Hom.

2020 Mathematics Subject Classification: 54C05.

1. Introduction and Preliminaries

As a generalisation of closed sets, Levine [14] developed generalised closed sets (g-closed sets) in general topology. Introducing and analysing g-closed maps by Malghan in 1984 [15] and g-continuous maps by Balachandran et al. [2] in 1991 enhanced various results in general topology by applying the notions of g-closed sets in general topological spaces. Gnanambal [11] proposed and explored generalised preregular closed sets and generalised preregular continuous maps for generic topological spaces in 1997.

(U, τ) or simply U refers to fuzzy topological space (abbreviated as fts) in this study. Here we recall various definitions from these papers, "fuzzy θ-closure of λ [9], fuzzy semi-θ-closure of λ [18], fuzzy θ-closed (briefly, $f\theta c$) [9], fuzzy semi-θ-closed (briefly, $fs\theta c$) [18], fuzzy regular (resp. θ, semi, semi θ & α)-open (briefly, fro [1] (resp. $f\theta o$ [9], fso [1], $fs\theta o$ [18] & $f\alpha o$ [5])), fuzzy generalized (resp. generalized semi, θ-generalized & θ generalized semi) closed (in short, fgc [3] (resp. $fgsc$ [17], $f\theta gc$ [9] & $f\theta gsc$ [12])), fuzzy semi (resp. θ-semi) generalized closed (in short, $fsgc$ [4] (resp. $f\theta sgc$ [18])), fuzzy g''' (resp. g^*s & g'''_α)-closed (briefly, $fg'''c$ [13] (resp. fg^*sc [13] & $fg'''_\alpha c$ [13])), fuzzy generalized (resp. generalized semi, θ-generalized, semi generalized, θ-semi generalized, g''', g^*s, g'''_α & θ generalized semi) open set (in short, fgo [3] (resp. $fgso$ [17], $f\theta go$ [9], $fsgo$ [4], $f\theta sgo$ [18], $fg'''o$ [13], fg^*so [13], $fg'''_\alpha o$ [13] & $f\theta gso$ [12])), fuzzy $\theta g'''$ (resp. θg^*s, $g'''\theta$, $g^*s\theta$ & $g'''_\alpha\theta$)-closed [6, 7] (briefly, $f\theta g'''c$ (resp. $f\theta g^*sc$, $fg'''\theta c$, $fg^*s\theta c$ & $fg'''_\alpha\theta c$)) set, fuzzy continuous [8] (in short $fctats$), fuzzy g (resp. θ & θgs)-continuous (in short $fgCts$ [3] (resp. $f\theta Cts$ [18] & $f\theta gsCts$ [12])) function, fuzzy $\theta g'''$ (resp. $g'''\theta$, $g'''_\alpha\theta$ & θg^*s)-continuous [6, 7] (briefly, $f\theta g'''Cts$ (resp. $fg'''\theta\ Cts$, $fg'''_\alpha\theta Cts$ & $f\theta g^*sCts$)), fuzzy $\theta g'''$ (resp. $g'''\theta$)-irresolute [6, 7] (briefly, $f\theta g'''Irr$ (resp. $fg'''\theta Irr$)), fuzzy contra continuous [10] (in short $fcCts$), fuzzy $T_{\theta g'''}$-space (briefly $fT_{\theta g'''}s$) [6, 7], fuzzy $T_{g'''\theta}$-space (briefly $fT_{g'''\theta}s$) [6, 7], fuzzy contra $\theta g'''$ (resp. $g'''\theta$, $g'''_\alpha\theta$ & θg^*s)-continuous (briefly, $fcta\theta g'''Cts$ (resp. $fctag'''\theta Cts$, $fctag'''_\alpha\theta Cts$ & $fcta\theta g^*sCts$), fuzzy contra $\theta g'''$ (resp. $g'''\theta$)-irresolute (briefly, $fcta\theta g'''Irr$ (resp. $fctag'''\theta Irr$")).

2. Fuzzy Contra $\theta g'''$-closed and $\theta g'''$-open maps

Definition 2.1. *A function $k : U \to V$ is said to be a fuzzy contra closed (in short $fctaC$) map if $k(\lambda)$ is a $fcta$ in V, $\forall$ fo set λ in U.*

Definition 2.2. *A function $k : U \to V$ is said to be a fuzzy contra $\theta g'''$ (resp. $g'''\theta$)-closed (in short $fcta\theta g'''C$ (resp. $fctag'''\theta C$)) map if $k(\lambda)$ is a $f\theta g'''c$ (resp. $fg'''\theta c$) in V, $\forall$ fo set λ in U.*

Definition 2.3. *A function $k : U \to V$ is said to be a fuzzy contra $\theta g'''$ (resp. $g'''\theta$)-open (in short $fcta\theta g'''O$ (resp. $fctag'''\theta O$)) map if the image of every fo set in U is a $f\theta g'''c$ (resp. $fg'''\theta c$) in V.*

Example 2.1. Let $U = \{a\} = V$ and the fs's L & M are defined by $L(a) = 0.4$, $M(a) = 0.8$. Consider $\tau = \{0, L, 1\}$ and $\sigma = \{0, M, 1\}$. Then $\mathfrak{i} : (U, \tau) \to (V, \sigma)$ is both $fcta\theta g'''C$ & $fctag'''\theta C$.

Theorem 2.1. *A function $k : U \to V$ is a $fcta\theta g'''C$ (resp. $fctag'''\theta C$) iff $\forall$ fs S of V and for each $fcta$ set U containing $k^{-1}(S)$ $\exists$ a $f\theta g'''o$ (resp. $fg'''\theta o$) set V of $V \ni S \leq V$ and $k^{-1}(V) \leq U$.*

Theorem 2.2. *If $k : U \to V$ is a $fcta\theta g'''C$ map and V is a $fT_{\theta g'''}s$, then k is a*

fcC.

Theorem 2.3. *If $k : U \to V$ is a $fctag'''\theta C$ map and V is a $fT_{g'''\theta}s$, then k is a $fctaC$.*

Definition 2.4. *A function $k : U \to V$ is called fuzzy contra θgs-irresolute (briefly, $fcta\theta gsIrr$) if $k^{-1}(\eta)$ is a $f\theta gsc$ in U $\forall$ $f\theta gso$ η in V.*

Theorem 2.4. *If $k : U \to V$ is both $fcta\theta gsIrr$ and $fcta\theta g'''C$ (resp. $fctag'''\theta C$). λ is a $f\theta g'''o$ (resp. $fg'''\theta o$) of U, then $k(\lambda)$ is a $f\theta g'''o$ (resp. $fg'''\theta o$) in V.*

Example 2.2. Let $U = \{a\} = Q = R$ and the fs's A, B, D, K & E are defined by $A(a) = 0.7$; $B(a) = 0.6$; $D(a) = 0.6$; $K(a) = 0.3$; $E(a) = 0.8$. Consider $\tau = \{0, A, 1\}$, $\sigma = \{0, B, 1\}$ & $\gamma = \{0, D, E, H, 1\}$. Then $\mathfrak{i}_1 : (U, \tau) \to (V, \sigma)$ & $\mathfrak{i}_2 : (V, \sigma) \to (R, \gamma)$ as identity functions. Clearly both $\mathfrak{i}_1$ and $\mathfrak{i}_2$ are $fcta\theta g'''C$ (resp. $fctag'''\theta C$) functions but $\mathfrak{i}_2 \circ \mathfrak{i}_1 : (X, \tau) \to (Z, \gamma)$ is not a $fcta\theta g'''C$ (resp. $fctag'''\theta C$) function.

Theorem 2.5. *Let $k : (U, \tau) \to (V, \sigma)$ be a $fcta\theta g'''C$ (resp. $fctag'''\theta C$) & $g : (V, \sigma) \to (R, \gamma)$ be both $fcta\theta g'''C$ (resp. $fctag'''\theta C$) & $fcta\theta gsIrr$, then $g \circ h : (U, \tau) \to (R, \gamma)$ is a $fcta\theta g'''C$ (resp. $fctag'''\theta C$).*

Theorem 2.6. *Let $k : (U, \tau) \to (V, \sigma)$, $g : (V, \sigma) \to (R, \gamma)$ be $fcta\theta g'''C$ (resp. $fctag'''\theta C$) functions and (V, σ) be a $fT_{\theta g'''}s$ (resp. $fT_{g'''\theta}s$). Then $g \circ h : (U, \tau) \to (R, \gamma)$ is a $fcta\theta g'''C$ (resp. $fctag'''\theta C$).*

Theorem 2.7. *Let $k : (U, \tau) \to (V, \sigma)$ be a $fctaC$ and $g : (V, \sigma) \to (R, \gamma)$ be a $fcta\theta g'''C$ (resp. $fctag'''\theta C$), then $g \circ h : (U, \tau) \to (R, \gamma)$ is a $fcta\theta g'''C$ (resp. $fctag'''\theta C$).*

Remark 2.1. *If $k : (U, \tau) \to (V, \sigma)$ be a $fcta\theta g'''C$ and $g : (V, \sigma) \to (R, \gamma)$ be a $fctaC$, then $g \circ h$ need not be a $fcta\theta g'''C$.*

Example 2.3. Let $U = \{a\} = V = R$ and the fs's A, B, K and E are defined by $A(a) = 0.7$; $B(a) = 0.6$; $K(a) = 0.3$; $E(a) = 0.8$. Consider $\tau = \{0, A, 1\}$, $\sigma = \{0, B, 1\}$ and $\gamma = \{0, B, E, H, 1\}$. Then (U, τ) & (V, σ) are fts. Then $\mathfrak{i}_1 : (U, \tau) \to (V, \sigma)$ is a $fcta\theta g'''C$ (resp. $fctag'''\theta C$) and $\mathfrak{i}_2 : (V, \sigma) \to (R, \gamma)$ is a $fctaC$ map but $\mathfrak{i}_2 \circ \mathfrak{i}_1 : (U, \tau) \to (R, \gamma)$ is not a $fcta\theta g'''C$ (resp. not $fctag'''\theta C$) function.

Theorem 2.8. *The map $g \circ h : (U, \tau) \to (R, \gamma)$, where $k : (U, \tau) \to (V, \sigma)$ and $g : (V, \sigma) \to (R, \gamma)$, is a $fcta\theta g'''C$ (resp. $fctag'''\theta C$).*

(i) If k is surjective $fctaCts$, then g is a $fcta\theta g'''C$ (resp. $fctag'''\theta C$).

(ii) If g is injective $fcta\theta g'''Irr$ (resp. $fctag'''\theta Irr$), then k is a $fcta\theta g'''C$ (resp. $fctag'''\theta C$).

Proof. (i) Let λ be a $fcta$ in (V,σ). Then $k^{-1}(\lambda)$ is a $fcta$ in (U,τ), as k is a $fctaCts$. Since $g\circ h$ is both $fcta\theta g'''C$ (resp. $fctag'''\theta C$) and surjective, $(g\circ k)(k^{-1}(\lambda)) = g(\lambda)$ is a $f\theta g'''c$ (resp. $fg'''\theta c$) in R. Hence g is a $fcta\theta g'''C$ (resp. $fctag'''\theta C$).

(ii)Let λ be a $fcta$ in (U,τ). Then $(g\circ k)(\lambda)$ is a $f\theta g'''c$ (resp. $fg'''\theta c$) in R. Since g is both $fcta\theta g'''Irr$ (resp. $fctag'''\theta Irr$) and injective $g^{-1}(g\circ k)(k^{-1}(\lambda)) = k(\lambda)$ is a $f\theta g'''c$ (resp. $fg'''\theta c$) in V. Hence k is a $fcta\theta g'''C$ (resp. $fctag'''\theta C$).

Theorem 2.9. *If $k:(U,\tau)\to(V,\sigma)$ is a bijection then (i) k^{-1} is a $fcta\theta g'''Cts$. (ii) k is a $fcta\theta g'''O$. (iii) k is a $fcta\theta g'''C$. are equivalent. And (iv) k^{-1} is a $fctag'''\theta Cts$, (v) k is a $fctag'''\theta O$, (vi) k is a $fctag'''\theta C$, are equivalent.*

Theorem 2.10. *A function $k:U\to V$ is a $fcta\theta g'''O$ (resp. $fctag'''\theta O$) iff for each fs S of V and for each $fcta$ set λ containing $k^{-1}(S)$ $\exists$ a $f\theta g'''c$ (resp. $fg'''\theta c$) set K of V containing $S \ni k^{-1}(K)\leq F$.*

Definition 2.5. *A function $k:U\to V$ is said to be a $fcta\theta g'''^{*}C$ (resp. $fctag'''^{*}\theta C$) if $k(\lambda)$ is a $f\theta g'''c$ (resp. $fg'''\theta c$) in V $\forall$ $f\theta g'''c$ (resp. $fg'''\theta c$) set λ in U.*

Remark 2.2.

- *(i) Since every $fcta$ set is a $fg'''\theta c$, we have every $fctag'''^{*}\theta C$ function is a $fctag'''\theta C$.*
- *(ii) Since every $fcta\theta g'''C$ map is a $fctag'''\theta C$, we have every $fcta\theta g'''^{*}C$ function is a $fctag'''^{*}\theta C$.*

Theorem 2.11. *A function $k:U\to V$ is a $fcta\theta g'''^{*}C$ (resp. $fctag'''^{*}\theta C$) iff $f\theta g'''Int(k(\lambda))\leq k(f\theta g'''Int(\lambda))$ (resp. $fg'''\theta\ Int(k(\lambda))\leq k(fg'''\theta Int(\lambda))$) $\forall$ fs λ of U.*

Theorem 2.12. *For any bijection mapping $k:(U,\tau)\to(V,\sigma)$,*

- *(i) k^{-1} is a $fcta\theta g'''Irr$ (resp. $fctag'''\theta Irr$),*
- *(ii) k is a $fcta\theta g'''^{*}O$ (resp. $fctag'''^{*}\theta O$),*
- *(iii) k is a $fcta\theta g'''^{*}C$ (resp. $fctag'''^{*}\theta C$),*

are equivalent.

Proof. $(i)\to(ii)$ Let U be a $f\theta g'''o$ set in U. Assume that k^{-1} is a $fcta\theta g'''Irr$, thus we have $(k^{-1})^{-1}(U) = k(U)$ is a $f\theta g'''o$ in V.

$(ii)\to(iii)$ and

$(iii)\to(i)$ are similar.

Theorem 2.13. *If $k : U \to V$ is both $fcta\theta gsIrr$ and $fcta\theta g'''C$, then k is a $fcta\theta g'''^*C$.*
Proof. Suppose k is both $fcta\theta gsIrr$ and $fcta\theta g'''C$. By Theorem 2.4, $k(\lambda)$ is a $f\theta g'''c$ in Y, $\forall$ $f\theta g'''c$ λ in U. Then by definition k is a $fcta\theta g''^{\prime *}C$.

Theorem 2.14. *If $k : U \to V$ is both $fcta\theta gsIrr$ and $fctag'''\theta C$, then k is a $fctag'''^*\theta C$.*

3. Fuzzy Contra $\theta g'''$-homeomorphism in Fuzzy Topological Space

Definition 3.1. *A function $k : U \to V$ is called fuzzy contra homeomorphism (in short $fcta$-Hom) if k and k^{-1} are $fctaCts$.*

Definition 3.2. *A function $k : U \to V$ is called fuzzy contra $g'''\theta$ (resp. $\theta g'''$ and $g'''_\alpha\theta$)-homeomorphism (in short $fctag'''\theta$-Hom (resp. $fcta\theta g'''$-Hom and $fctag'''_\alpha\theta$-Hom)) if k and k^{-1} are $fctag'''\theta Cts$ (resp. $fcta\theta g'''Cts$ and $fctag'''_\alpha\theta Cts$).*
$FCG'''\theta$-$k(U,\tau)$ (resp. $FC\theta G'''$-$k(U,\tau)$ and $FCG'''_\alpha\theta$-$k(U,\tau)$) denote the family of all $fctag'''\theta$-Hom (resp. $fcta\theta g'''$-Hom and $fctag'''_\alpha\theta$-Hom) of a fts (U,τ) onto itself.

Theorem 3.1. *Every $fcta$-Hom (resp. $fcta\theta g'''$-Hom and $fctag'''\theta$-Hom) is a $fctag'''\theta$-Hom (resp. $fctag'''\theta$-Hom and $fctag'''_\alpha\theta$-Hom).*
Proof. (i) Let $k : U \to V$ be a $fcta$-Hom. Then k and k^{-1} are $fctaCts$. By Theorem 3.8 [16], k and k^{-1} are $fctag'''\theta Cts$. Hence k is a $fctag'''\theta$-Hom.

(ii) Let $k : U \to V$ be a $fcta\theta g'''$-Hom. Then k and k^{-1} are $fcta\theta g'''Cts$. By Theorem 3.8 [16], k and k^{-1} are $fctag'''\theta Cts$. Hence k is a $fctag'''\theta$-Hom.

(iii) Let $k : U \to V$ be a $fctag'''\theta$-Hom. Then k and k^{-1} are $fctag'''\theta$-Cts. By Theorem 3.8 [16], k and k^{-1} are $fctag'''_\alpha\theta Cts$. Hence k is a $fctag'''_\alpha\theta$-Hom.

Example 3.1. Let $X = \{r, s\} = Y$ and the fs's U, Q, R and S are defined by $U(r) = 0.6, U(s) = 0.6$; $Q(r) = 0.5, Q(s) = 0.6$; $R(r) = 0.6, R(s) = 0.5$; $S(r) = 0.4, S(s) = 0.4$. Consider $\tau = \{0, U, Q, 1\}$ and $\sigma = \{0, R, 1\}$. Then $k : (X,\tau) \to (Y,\sigma)$ as $k(r) = s,\ k(s) = r$, is a $fctag'''\theta$-Hom but not a $fcta$-Hom as U^c is a fc in X, $(k^{-1})^{-1}(U^c) = S$ is not a fc in (Y,σ). $k^{-1} : (Y,\sigma) \to (X,\tau)$ is not a $fctaCts$.

Example 3.2. Let $X = \{r, s\} = Y$ and the fs's U, Q, R and S are defined by $U(r) = 0.6, U(s) = 0.6; Q(r) = 0.5, Q(s) = 0.6; R(r) = 0.6, R(s) = 0.5$ & $S(r) = 0.5, S(s) = 0.4$. Consider $\tau = \{0, U, Q, 1\}$ and $\sigma = \{0, R, 1\}$. Then $k : (X,\tau) \to (Y,\sigma)$ as $k(r) = s, k(s) = r$, is a $fctag'''\theta$-Hom but not a $fcta\theta g'''$-Hom as R^c is a fc in Y, $k^{-1}(R^c) = S$ is not a $f\theta g'''c$. k is not a $fcta\theta g'''Cts$.

Example 3.3. Let $X = \{p\}$ and the fs's U, V and R are defined by $U(p) = 0.5; Q(p) = 0.7; R(p) = 0.6$. Consider $\tau = \{0, P, Q, 1\}$ and $\sigma = \{0, R, 1\}$. Then

$\mathfrak{i} : (X, \tau) \to (Y, \sigma)$ is a $fctag'''_\alpha\theta$-Hom but not a $fctag'''\theta$-Hom, since for a fc set R^c in $Y, \mathfrak{i}^{-1}(R^c) = R^c$ is not a $fg'''\theta c$. Hence $k : (X, \tau) \to (Y, \sigma)$ is not a $fctag'''\theta Cts$.

From the Examples 3.1 to 3.3, we get

$$\begin{array}{ccccc} & & fcta\textbf{-Hom} & & \\ & & \Downarrow & & \\ fcta\theta g'''\textbf{-Hom} & \Longrightarrow & fctag'''\theta\textbf{-Hom} & \Longrightarrow & fctag'''_\alpha\theta\textbf{-Hom} \end{array}$$

Theorem 3.2. *If $k : U \to V$ is a $fctag'''\theta$-Hom and U and V are $fT_{g'''\theta}s$ then k is a $fcta$-Hom.*

Proof. Let $k : U \to V$ be a $fctag'''\theta$-Hom. Then k and k^{-1} are $fctag'''\theta Cts$. To prove that k and k^{-1} are $fctaCts$. Let F, in V, be a fc. Then $k^{-1}(F)$, in U, is a $fg'''\theta c$, since k is a $fctag'''\theta Cts$. Also since U is $fT_{g'''\theta}s$, $k^{-1}(F)$, in U, is a fc. Hence k is a $fctaCts$.

Now, let F, in U, be a fc. Then $(k^{-1})^{-1}(F) = k(F)$, in V, is a $fg'''\theta c$, since k^{-1} is a $fctag'''\theta Cts$. Also, since V is a $fT_{g'''\theta}s$, $k(F)$ is a fc set in V. Hence k^{-1} is a $fctaCts$, thus k is a $fcta$-Hom.

Theorem 3.3. *If $k : U \to V$ is a $fcta\theta g'''$-Hom and U and V are $fT_{\theta g'''}s$ then k is a $fcta$-Hom.*

Proof. Let $k : U \to V$ be a $fcta\theta g'''$-Hom. Then k and k^{-1} are $fcta\theta g'''Cts$. To prove that k and k^{-1} are $fctaCts$. Let F, in V, be a fc. Then $k^{-1}(F)$, in U, is a $f\theta g'''c$, since k is a $fcta\theta g'''Cts$. Also since U is a $fT_{\theta g'''}s$, $k^{-1}(F)$, in U, is a fc. Hence k is a $fctaCts$. Now, let F, in U, be a fc. Then $(k^{-1})^{-1}(F) = k(F)$ is a $f\theta g'''c$ set in V, since k^{-1} is a $fcta\theta g'''Cts$. Also, since V is a $fT_{\theta g'''}s$, $k(F)$ is a fc set in V. Hence k^{-1} is a $fctaCts$, thus k is a $fcta$-Hom.

Theorem 3.4. *Let $k : U \to V$ be a bijective function,*

(i) k is a $fcta\theta g'''$-Hom,

(ii) k is both $fcta\theta g'''Cts$ and $fcta\theta g'''O$ maps,

(iii) k is both $fcta\theta g'''Cts$ and $fcta\theta g'''C$ maps,

are equivalent.

Proof. (i) ⇒ (ii): Let k be a $fcta\theta g'''$-Hom. Then k and k^{-1} are $fcta\theta g'''Cts$. To prove that k is a $fcta\theta g'''O$ map. Let U be a fo set in U. Since $k^{-1} : Q \to U$ is a $fcta\theta g'''Cts$, $(k^{-1})^{-1}(U) = k(U)$ is a $f\theta g'''o$ in V. Hence k is a $fcta\theta g'''O$ maps.

(ii) ⇒ (i): Let k be both $fcta\theta g'''O$ and $fcta\theta g'''Cts$ map. To prove that $k^{-1} : Q \to U$ is a $fcta\theta g'''Cts$. Let V be a fo set in U. Then $k(U)$, in V, is a

$f\theta g'''o$. Since k is a $fcta\theta g'''O$. Now $(k^{-1})^{-1}(U) = k(U)$ is a $f\theta g'''o$ in V. Therefore $k^{-1} : Q \to P$ is a $fcta\theta g'''Cts$. Hence k is $fcta\theta g'''$-Hom.

(ii) $\Rightarrow$ (iii): Let k be both $fcta\theta g'''Cts$ and $fcta\theta g'''O$ map. To prove that k is a $fcta\theta g'''C$ map. Let F, in U, be a fc, then $1 - F$, in U, is a fo. Since k is a $fcta\theta g'''O$, $k(1 - F)$, in V, is $f\theta g'''o$. Now $k(1 - F) = 1 - k(F)$. Therefore $k(F)$, in V, is a $f\theta g'''c$. Hence k is a $fcta\theta g'''C$.

(iii) $\Rightarrow$ (i): Let k be both $fcta\theta g'''Cts$ and $fcta\theta g'''C$ maps. To prove that k is a $fcta\theta g'''$-Hom. Let F, in U, be a fc. Then $k(F)$, in V, is a $f\theta g'''c$, since k is a $fcta\theta g'''C$. Now $k(F) = (k^{-1})^{-1}(F)$ is a $f\theta g'''c$ set in V. Therefore $k^{-1} : Q \to U$ is a $fcta\theta g'''Cts$. Hence k is a $fcta\theta g'''$-Hom.

Theorem 3.5. *Let $k : U \to V$ be a bijective function.*

(i) k is a $fctag'''\theta$-Hom,

(ii) k is both $fctag'''\theta Cts$ and $fctag'''\theta O$ maps,

(iii) k is both $fctag'''\theta Cts$ and $fctag'''\theta C$ maps,

are equivalent.

Proof. (i) $\Rightarrow$ (ii): Let k be a $fctag'''\theta$-Hom. Then k and k^{-1} are $fctag'''\theta Cts$. To prove that k is a $fctag'''\theta O$ map, let U be a fo set in U. Since $k^{-1} : Q \to U$ is a $fctag'''\theta Cts$, $(k^{-1})^{-1}(U) = k(U)$ is a $fg'''\theta o$ in Q. Hence k is a $fctag'''\theta O$ maps.

(ii) $\Rightarrow$ (i): Let k be both $fctag'''\theta O$ and $fctag'''\theta Cts$ map. To prove that $k^{-1} : Q \to U$ is a $fctag'''\theta Cts$, let V be a fo set in U. Then $k(U)$, in V, is a $fg'''\theta o$. Since k is a $fctag'''\theta O$. Now $(k^{-1})^{-1}(U) = k(U)$ is a $fg'''\theta o$ in V. Therefore $k^{-1} : Q \to U$ is a $fctag'''\theta Cts$. Hence k is a $fctag'''\theta$-Hom.

(ii) $\Rightarrow$ (iii): Let k be both $fctag'''\theta Cts$ and $fctag'''\theta O$ map. To prove that k is a $fctag'''\theta C$ map, let F, in U, be a fc, then $1 - F$, in U, is a fo. Since k is a $fctag'''\theta O$, $k(1 - F)$, in V, is a $fg'''\theta o$. Now $k(1 - F) = 1 - k(F)$. Therefore $k(F)$, in V, is a $fg'''\theta c$. Hence k is a $fctag'''\theta C$.

(iii) $\Rightarrow$ (i): Let k be both $fctag'''\theta Cts$ and $fctag'''\theta C$ maps. To prove that k is a $fctag'''\theta$-Hom, let F, in U, be a fc. Then $k(F)$, in V, is a $fg'''\theta c$ and $k^{-1} : Q \to U$ is a $fctag'''\theta Cts$. Hence k is a $fctag'''\theta$-Hom.

Theorem 3.6. *The map $g \circ h : U \to R$ is a $fcta\theta g'''$-Hom if both $k : U \to V$ and $g : Q \to R$ are $fcta\theta g'''$-Hom with V is a $fT_{\theta g'''}s$.*

Theorem 3.7. *If both $k : U \to V$ and $g : Q \to R$ are $fctag'''\theta$-Hom with V is a $fT_{g'''\theta}s$, then $g \circ h : U \to R$ is a $fctag'''\theta$-Hom.*

Theorem 3.8. *The map $g \circ h : U \to R$ is a $fcta\theta g'''Cts$ if $k : U \to V$ is a*

$fcta\theta g'''$-Hom and $g : Q \to R$ is a fcta-Hom.

Theorem 3.9. *The map $g \circ h : U \to R$ is a $fctag'''\theta Cts$ if $k : U \to V$ is a $fctag'''\theta$-Hom and $g : Q \to R$ is a fc-Hom.*

Theorem 3.10. *The map $(g \circ k)^{-1} : R \to U$ is a $fcta\theta g'''Cts$ if $k : U \to V$ is a $fcta$-Hom and $g : Q \to R$ is a $fcta\theta g'''$-Hom.*
Proof. To show that $(g \circ k)^{-1}$ is a $fcta\theta g'''Cts$, let U be a fo set in U, since $k^{-1} : Q \to U$ is a $fctaCts$, $(k^{-1})^{-1}(U)$ is a fo in V. Also since $g^{-1} : R \to V$ is a $fcta\theta g'''Cts$, $(g^{-1})^{-1}(k(U)) = g(k(U)) = ((g \circ k)^{-1})^{-1}(U)$, in R, is a $f\theta g'''o$.

Theorem 3.11. *The map $(g \circ k)^{-1} : R \to U$ is a $fctag'''\theta Cts$ if $k : U \to V$ is a $fcta$-Hom and $g : Q \to R$ is a $fctag'''\theta$-Hom.*
Proof. To show that $(g \circ k)^{-1}$ is a $fctag'''\theta Cts$. Let U be a fo set in U. Since $k^{-1} : Q \to U$ is a $fctaCts$, $(k^{-1})^{-1}(U)$ is a fo in V. Also since $g^{-1} : R \to V$ is a $fctag'''\theta Cts$, $(g^{-1})^{-1}(k(U)) = g(k(U)) = ((g \circ k)^{-1})^{-1}(U)$ is a $fg'''\theta o$ in R. Therefore $(g \circ k)^{-1}$ is a $fctag'''\theta Cts$.

Theorem 3.12. *If a bijective function $k : U \to V$ is $fcta\theta g'''$-Hom (resp. $fctag'''\theta$-Hom) then $k(f\theta g'''Int(\lambda)) \leq Cl(k(\lambda))$ (resp. $k(fg'''\theta Int(\lambda)) \leq Cl(k(\lambda))$) $\forall$ fs λ in U.*

Theorem 3.13. *If a bijective function $k : U \to V$ is $fcta\theta g'''$-Hom (resp. $fctag'''\theta$-Hom) then $f\theta g'''Int(k^{-1}(\eta)) \leq k^{-1}(Cl(\eta))$ (resp. $fg'''\theta Int(k^{-1}(\eta)) \leq k^{-1}(Cl(\eta))$) $\forall$ fs η in U.*

Theorem 3.14. *If a bijective function $k : U \to V$ is $fcta\theta g'''$-Hom (resp. $fctag'''\theta$-Hom) then $k(f\theta g'''Cl(\lambda)) \geq Int(k(\lambda))$ (resp. $k(fg'''\theta Cl(\lambda)) \geq Int(k(\lambda))$) $\forall$ fs λ in U.*

Theorem 3.15. *If a bijective function $k : U \to V$ is $fcta\theta g'''$-Hom then $f\theta g'''Cl$ $(k^{-1}(U)) \geq k^{-1}(Int(U))$ for every fs U in U.*
Proof. Let k be a $fcta\theta g'''$-Hom.

Then k and k^{-1} are $fcta\theta g'''Cts$. Let U be any fs in Q, now $Int(U)$, in V, is a fo. As k is $fcta\theta g'''Cts$ $k^{-1}(Int(U))$, in U, is $f\theta g'''o$. From Theorem 3.14,

$$\begin{aligned} k^{-1}(Int(U)) &\leq f\theta g'''Cl(k^{-1}(Int(U))) \\ &\leq f\theta g'''Cl(k^{-1}(U)). \end{aligned}$$

Hence $f\theta g'''Cl(k^{-1}(U)) \geq k^{-1}(Int(U))$.

Theorem 3.16. *If a bijective function $k : U \to V$ is $fctag'''\theta$-Hom then $fg'''\theta Cl$ $(k^{-1}(U)) \geq k^{-1}(Int(U))$ for every fs U in U.*
Proof. Let k be a $fctag'''\theta$-Hom. Then both k and k^{-1} are $fctag'''\theta Cts$. Let U

be any fs in Q, now $Int(U)$, in V, is a fo. As k is a $fctag'''\theta Cts$, $k^{-1}(Int(U))$, in U, is a $fg'''\theta o$. From Theorem 3.14,

$$\begin{aligned} k^{-1}(Int(U)) &\leq fg'''\theta Cl(k^{-1}(int(U))) \\ &\leq fg'''\theta Cl(k^{-1}(U)). \end{aligned}$$

Hence $fg'''\theta Cl(k^{-1}(U)) \geq k^{-1}(Int(U))$.

Theorem 3.17. *The set $fcta\theta g'''$-Hom (U,τ) (resp. $fctag'''\theta$-Hom (U,τ)) is a group under the composition of functions.*

Theorem 3.18. *Let $k : (U,\tau) \to (Y,\sigma)$ be a $fcta\theta g'''$-Hom (resp. $fctag'''\theta$-Hom). Then k induces an isomorphism from the group $FC\theta G'''$-$k(U,\tau)$ (resp. $FCG'''\theta$-$k(U,\tau)$) on to the group $FC\theta G'''$-$k(U,\tau)$ (resp. $FCG'''\theta$-$k(U,\tau)$).*

4. Conclusion

In this paper, we have discussed about $fc\theta g'''$-closed maps, $fc\theta g'''$-open maps, $fcg'''\theta$-closed maps, $fcg'''\theta$-open maps and $fcg'''\theta$-homeomorphism in fts's. Also, some of their properties have been investigated.

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