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TOPOLOGICAL ASPECTS OF BORON TRIANGULAR NANOTUBE AND BORON-α NANOTUBE-II

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Abstract: Graph indices have attracted great interest as they give us numerical clues for several properties of molecules. Some indices give valuable information on the molecules under consideration using mathematical calculations only. For these reasons, the calculation and properties of graph indices have been in the center of research. Naturally, the values taken by a graph index is an important problem called the inverse problem. It requires knowledge about the existence of a graph having index equal to a given number. A considerable amount of topological graph indices are the degree based ones. Probably the largest degree based class of graph indices is Zagreb indices and Randić index is one of the most famous topological graph indices. There are several variants of them. In this paper, we compute the sum connectivity index, Randić index, reciprocal Randić index, reduced second Zagreb index, reduced reciprocal Randić index, first and second Gourava indices of boron nanotubes.

Keywords and Phrases: Topological indices, Sum connectivity index, Randić index, Reciprocal Randić index, Reduced second Zagreb index, Reduced reciprocal Randić index, First and second Gourava indices, Boron triangular, Boron-α nanotubes.

2010 Mathematics Subject Classification: 05C05, 05C12, 05C75.

1. Introduction

Topological graph indices have been defined and used in many areas in the last few decades to study several properties of different objects such as atoms and molecules solely by means of some mathematical techniques. Several topological graph indices have been defined and studied by many mathematicians and chemists as most graphs are generated from molecules by replacing atoms with vertices and bonds between them with edges. These indices are defined as invariants measuring several physical, chemical, pharmaceutical, biological properties of graphs which are modelling real situations. They can be grouped mainly into three classes according to the way they are defined; by vertex degrees, by distances or by matrices. Some of the degree based topological indices are discussed below.

The sum-connectivity index was proposed in [5] and is defined as:

$$SC(G) = \sum_{uv \in E(G)} \frac{2}{\sqrt{d_u(G) + d_v(G)}}. \tag{1.1}$$

The first genuine degree-based topological index was put forward in 1975 named it as Randic$'$ or connectivity topological index and is defined as (See [4]):

$$R(G) = \sum_{uv \in E(G)} \frac{1}{\sqrt{d_u(G) d_v(G)}}. \tag{1.2}$$

In [1], Gutman et al. are presented the three vertex–degree-based graph invariants. These are the reciprocal Randić index (RR), the reduced second Zagreb index RM_2, and the reduced reciprocal Randić index RRR are respectively given by:

$$RR(G) = \sum_{uv \in E(G)} \sqrt{d_u(G) d_v(G)}, \tag{1.3}$$

$$RM_2(G) = \sum_{uv \in E(G)} (d_u(G) - 1)(d_v(G) - 1), \tag{1.4}$$

and

$$RRR(G) = \sum_{uv \in E(G)} \sqrt{(d_u(G) - 1)(d_v(G) - 1)}. \tag{1.5}$$

The first and second Gourava indices of a graph G are introduced by Kulli [3] and they are defined as follows:

$$GO_1(G) = \sum_{uv \in E(G)} (d_u(G) + d_v(G)) + (d_u(G)d_v(G)) \tag{1.6}$$

$$GO_2(G) = \sum_{uv \in E(G)} (d_u(G) + d_v(G))(d_u(G)d_v(G)). \tag{1.7}$$

In [2], the authors computed the third Zagreb index, harmonic index, forgotten index, inverse sum index, modified Zagreb index and symmetric division deg index by applying the subdivision and the semi total point graph for boron triangular and boron-α nanotubes. The extensive information about boron nanotubes, boron nanomaterials, boron triangular nanotube and boron-α nanotube, we suggest the readers to refer the paper by Hemavathi et al. [2].

2. Topological indices of a Boron triangular nanotube and subdivision (semi total point) graph of a Boron triangular nanotube

The molecular graphs of boron triangular and boron-α nanotubes are denoted by $BT[p,q]$ and $BA[p,q]$ respectively, where p and q denotes the number of rows and the number of columns in $2D$ sheet of $BT[p,q]$ and $BA[p,q]$ respectively as shown in the Figure 1. Then boron-α nanotubes can be categorized into two classes with respect to p as $BA(X)[p,q]$ and $BA(Y)[p,q]$.

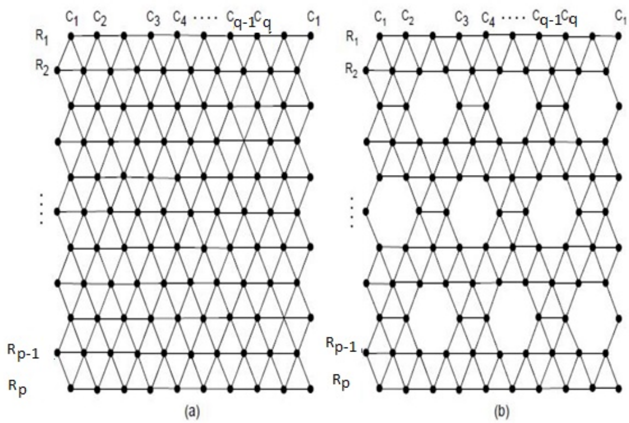

Figure 1: (a) $2D$-sheet of a boron triangular nanotube $BT[p,q]$, (b) $2D$-sheet of a boron-α nanotube $BA[p,q]$

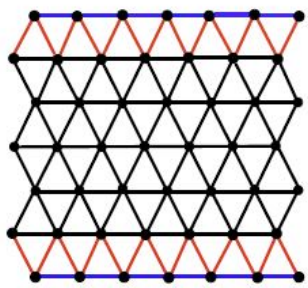

Figure 2: The edge partitions of a nanotube $BT[7,4]$ with respect to the degrees of end vertices

Molecular graph	$BT[p,q]$	$S = BT[p,q]$	$R = BT[p,q]$
Order	$\frac{3pq}{2}$	$3q(2p-1)$	$3q(2p-1)$
Size	$\frac{3q(3p-2)}{2}$	$3q(3p-2)$	$\frac{9q(3p-2)}{2}$

Table 1: The order and size of a triangular boron nanotubes

Theorem 2.1. *Consider the boron triangular nanotube $BT[p,q]$, where $p \geq 3$ and q is an even, then*

(i) $SC(BT[p,q]) = \frac{3q}{2\sqrt{2}} + \frac{6q}{\sqrt{10}} + \frac{3q(3p-8)}{4\sqrt{3}}$,

(ii) $R(BT[p,q]) = \frac{3q}{4} + \frac{3q}{\sqrt{6}} + \frac{q(3p-8)}{4}$,

(iii) $RR(BT[p,q]) = 12q + 12\sqrt{6}q + 9q(3p-8)$,

(iv) $RM_2(BT[p,q]) = 27q + 90q + \frac{75q(3p-8)}{2}$,

(v) $RRR(BT[p,q]) = 9q + 6q\sqrt{15} + \frac{15q(3p-8)}{2}$,

(vi) $GO_1(BT[p,q]) = 72q + 204q + 72q(3p-8)$,

(vii) $GO_2(BT[p,q]) = 384q + 1440q + 648(3p-8)$.

Proof. Consider the boron triangular nanotube $G = BT[p,q]$. There are three edge partitions corresponding to the degree of an end vertices which are presented as: $E_{4,4} = \{uv \in E_G \mid d_u = 4, d_v = 4\}$, $E_{4,6} = \{uv \in E_G | d_u = 4, d_v = 6\}$ and $E_{6,6} = \{uv \in E_G | d_u = 6, d_v = 6\}$.

Thus, we have $|E_{4,4}| = 3q$, $|E_{4,6}| = 6q$ and $|E_{6,6}| = \frac{3q(3p-8)}{2}$. The respective edge partitions are shown in Figure 2 in which the edges belong to $E_{4,4}$, $E_{4,6}$ and $E_{6,6}$ respectively. In view of the equations 1.1 to 1.7), we get the required results.

Theorem 2.2. *Consider the subdivision graph of a boron triangular nanotube $BT[p, q]$, then*

(i) $SC(BT[p, q]) = \frac{12q}{\sqrt{6}} + \frac{9q(p-2)}{2\sqrt{2}}$,

(ii) $R(BT[p, q]) = \frac{6q}{\sqrt{2}} + \frac{9q(p-2)}{\sqrt{12}}$,

(ii) $RR(BT[p, q]) = 24\sqrt{2}q + 18q(p-2)\sqrt{3}$,

(iv) $RM_2(BT[p, q]) = 36q + 45q(p-2)$,

(v) $RRR(BT[p, q]) = 12\sqrt{3}q + 9q\sqrt{5}(p-2)$,

(vi) $GO_1(BT[p, q]) = 168q + 180q(p-2)$,

(vii) $GO_2(BT[p, q]) = 576q + 864q(p-2)$.

Proof. Consider the subdivision graph of a boron triangular nanotube $S = BT[p, q]$. There are two edge partitions corresponding to the degree of an end vertices which are presented as:
$E_{2,4} = \{uv \in E_S | d_u = 2, d_v = 4\}$ and $E_{2,6} = \{uv \in E_S | d_u = 2, d_v = 6\}$.

Thus, we have $|E_{2,4}| = 12q$, and $|E_{2,6}| = 9q(p-2)$. By applying the subdivision graph operator to the Figure 3, then respective edge partitions in which edges belong to $E_{2,4}$ and $E_{2,6}$ respectively. In view of the equations 1.1 to 1.7), we get the required results.

Theorem 2.3. *Consider the semi total point graph of a boron triangular nanotube $R = BT[p, q]$, then*

(i) $SC(BT[p, q]) = \frac{12q}{\sqrt{10}} + \frac{9q(p-2)}{\sqrt{14}} + \frac{3q}{4} + \frac{3q}{\sqrt{5}} + \frac{3q(3p-8)}{4\sqrt{6}}$,

(ii) $R(BT[p, q]) = 3q + \frac{9q(p-2)}{2\sqrt{6}} + \frac{3q}{8} + \frac{3q}{2\sqrt{6}} + \frac{q(3p-8)}{8}$,

(iii) $RR(BT[p, q]) = 24q + 18\sqrt{6}q(p-2) + 24q + 24\sqrt{6}q + 18q(3p-8)$,

(iv) $RM_2(BT[p, q]) = 84q + 99q(p-2) + 147q + 462q + \frac{363(3p-8)}{2}$,

(v) $RRR(BT[p,q]) = 12\sqrt{7}q + 9q\sqrt{11}(p-2) + 21q + 6\sqrt{77}q + \frac{33q(3p-8)}{2}$,

(vi) $GO_1(BT[p,q]) = 312q + 342q(p-2) + 240q + 696q + 252q(3p-8)$,

(vii) $GO_2(BT[p,q]) = 1920q + 3024q(p-2) + 3072q + 11520q + 5184q(3p-8)$.

Proof. Consider the semi total point of a boron triangular nanotube $R = BT[p,q]$. There are five edge partitions corresponding to the degrees of an end vertices which are presented as:
$E_{2,8} = \{uv \in E_R | d_u = 2, d_v = 8\}$, $E_{2,12} = \{uv \in E_R | d_u = 2, d_v = 12\}$, $E_{8,8} = \{uv \in E_R | d_u = 8, d_v = 8\}$, $E_{8,12} = \{uv \in E_R | d_u = 8, d_v = 12\}$, and $E_{12,12} = \{uv \in E_R | d_u = 12, d_v = 12\}$.

Therefore, we have $|E_{2,8}| = 12q$, $|E_{2,12}| = 9q(q-2)$, $|E_{8,8}| = 3q$, $|E_{8,12}| = 6q$, and $|E_{12,12}| = \frac{3q(3p-8)}{2}$. Applying the semi total point graph operator to the Figure 2, then respective edge partitions in which edges belongs to $E_{2,8}$, $E_{2,12}$, $E_{8,8}$, $E_{8,12}$, and $E_{12,12}$ respectively. In view of the equations 1.1 to 1.7), we get the required results.

3. Topological indices of a Boron-α nanotube and subdivision (semi total point) graph of a boron-α nanotube $BA(X)[p,q]$

In this section, we compute the basic result of a boron-α nanotube, subdivision and semi total point graph of a boron-α nanotube $BA(X)[p,q]$.

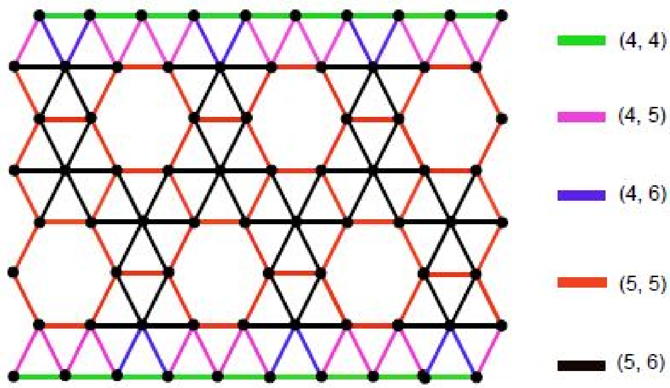

Figure 3: The edge partitions of a nanotube $BA(X)[8,6]$ with respect to degree of end vertices

Molecular graph	$BA(X)[p,q]$	$S_1 = BA(X)[p,q]$	$R_1 = BA(X)[p,q]$
Order	$\frac{q}{3}(4p+1)$	$\frac{q}{6}(29p-4)$	$\frac{q}{6}(29p-4)$
Size	$\frac{q}{2}(7p-2)$	$q(7p-2)$	$\frac{3q(7p-2)}{2}$

Table 2: The order and size of a boron-α nanotube $BA(X)[p,q]$

Theorem 3.1. *Consider the subdivision graph of a boron-α nanotube $S_1 = BA(X)$ $[p,q]$, then*

(i) $SC(BT[p,q]) = \frac{12q}{\sqrt{6}} + \frac{5q(p-2)}{\sqrt{7}} + \frac{q(p-2)}{\sqrt{2}}$,

(ii) $R(BT[p,q]) = \frac{6q}{\sqrt{2}} + \frac{5q(p-2)}{2\sqrt{10}} + \frac{q(p-2)}{\sqrt{3}}$,

(iii) $RR(BT[p,q]) = 24\sqrt{2}q + 5q(p-2)\sqrt{10} + 4q(p-2)\sqrt{3}$

(vi) $RM_2(BT[p,q]) = 36q + 20q(p-2) + 10q(p-2)$,

(v) $RRR(BT[p,q]) = 12\sqrt{3}q + 5q\sqrt{4}(p-2) + 2q(p-2)\sqrt{5}$,

(vi) $GO_1(BT[p,q]) = 168q + 85q(p-2) + 40q(p-2)$,

(vii) $GO_2(BT[p,q]) = 576q + 350q(p-2) + 192(p-2)$.

Proof. Consider the subdivision graph of a boron-α nanotube $S_1 = BA(X)[p,q]$. There are three edge partitions corresponding to the degrees of an end vertices which are presented as:
$E_{2,4} = \{uv \in E_{S_1} | d_u = 2, d_v = 4\}$, $E_{2,5} = \{uv \in E_{S_1} | d_u = 2, d_v = 5\}$ and $E_{2,6} = \{uv \in E_{S_1} | d_u = 2, d_v = 6\}$.

Therefore, we have $|E_{2,4}| = 12q$, $|E_{2,5}| = 5q(p-2)$ and $|E_{2,6}| = 2q(p-2)$. Applying the subdivision graph operator to the Figure 3, then respective edge partitions in which edges belongs to $E_{2,4}$, $E_{2,5}$ and $E_{2,6}$ respectively. In view of the equations 1.1 to 1.7), we get the required results.

Theorem 3.2. *Consider the boron-α nanotube $BA(X)[p,q]$, then*

(i) $SC(BT[p,q]) = \frac{3q}{2\sqrt{2}} + \frac{4q}{3} + \frac{2q}{\sqrt{10}} + \frac{q(3p-8)}{2\sqrt{10}} + \frac{2q(p-3)}{\sqrt{11}}$,

(ii) $R(BT[p,q]) = \frac{3q}{4} + \frac{4q}{2\sqrt{5}} + \frac{2q}{2\sqrt{6}} + \frac{q(3p-8)}{10} + \frac{2q(p-3)}{\sqrt{30}}$,

(iii) $RR(BT[p,q]) = 12q + 8q\sqrt{5} + 4q\sqrt{6} + \frac{5q(3p-8)}{2} + 2\sqrt{30}(p-3)q$,

(iv) $RM_2(BT[p,q]) = 27q + 48q + 30q + 8q(3p-8) + 40q(p-3)$,

(v) $RRR(BT[p,q]) = 9q + 8\sqrt{3}q + 2q\sqrt{15} + 2q(3p-8) + 4\sqrt{5}q(p-3)$,

(vi) $GO_1(BT[p,q]) = 72q + 116q + 68q + \frac{35q(3p-8)}{2} + 82q(p-3)$,

(vii) $GO_2(BT[p,q]) = 384q + 720q + 480q + \frac{250q(3p-8)}{2} + 660q(p-3)$.

Proof. Consider the boron-α nanotube $H = BA(X)[p,q]$. There are five edge partitions corresponding to the degrees of an end vertices which are presented as: $E_{4,4} = \{uv \in E_H | d_u = 4, d_v = 4\}$, $E_{4,5} = \{uv \in E_H | d_u = 4, d_v = 5\}$, $E_{4,6} = \{uv \in E_H | d_u = 4, d_v = 6\}$, $E_{5,5} = \{uv \in E_H | d_u = 5, d_v = 5\}$ and $E_{5,6} = \{uv \in E_H | d_u = 5, d_v = 6\}$.

Therefore, we have $|E_{4,4}| = 3q$, $|E_{4,5}| = 4q$, $|E_{4,6}| = 2q$, $|E_{5,5}| = \frac{q(3p-8)}{2}$, and $|E_{6,6}| = 2q(p-3)$. The respective edge partitions are shown in Figure 4 in which edges belong to $E_{4,4}$, $E_{4,5}$, $E_{4,6}$, $E_{5,5}$ and $E_{5,6}$ respectively. In view of the equations 1.1 to 1.7), we get the required results.

Theorem 3.3. *Consider the semi total point graph of a boron-α nanotube $R_1 = BA(X)[p,q]$, then*

(i) $SC(BT[p,q]) = \frac{12q}{2\sqrt{10}} + \frac{5q(p-2)}{12} + \frac{2q(p-2)}{\sqrt{14}} + \frac{3q}{4} + \frac{4q}{\sqrt{18}} + \frac{q}{\sqrt{5}} + \frac{q(3p-8)}{4\sqrt{5}} + \frac{2q(p-3)}{\sqrt{22}}$,

(ii) $R(BT[p,q]) = 3q + \frac{5q(p-2)}{\sqrt{20}} + \frac{q(p-2)}{\sqrt{6}} + \frac{3q}{8} + \frac{q}{\sqrt{5}} + \frac{q}{2\sqrt{24}} + \frac{q(3p-8)}{20} + \frac{q(p-3)}{\sqrt{30}}$,

(iii) $RR(BT[p,q]) = 48q + 10\sqrt{5}(p-2)q + 4q(p-2)\sqrt{6} + 24q + 16q\sqrt{5} + 8\sqrt{6}q + 5q(3p-8) + 4q(p-3)\sqrt{30}$,

(iv) $RM_2(BT[p,q]) = 84q + 45q(p-2) + 22q(p-2) + 147q + 252q + 154q + \frac{81q(3p-8)}{2} + 198q(p-3)$,

(v) $RRR(BT[p,q]) = 12q\sqrt{7} + 15q(p-2) + 2q\sqrt{11}(p-2) + 21q + 4\sqrt{63}q + 2q\sqrt{77} + \frac{9q(3p-8)}{2} + 6q\sqrt{11}(p-3)$,

(vi) $GO_1(BT[p,q]) = 312q + 160q(p-2) + 76q(p-2) + 240q + 392q + 232q + 60q(3p-8) + 284q(p-3)$,

(vii) $GO_2(BT[p,q]) = 1920q + 1200q(p-2) + 672q(p-2) + 3072q + 5760q + 384q + 1000q(3p-8) + 5280q(p-3)$.

Proof. Consider the semi total point graph of a boron-α nanotube $R_1 = BA(X)[p, q]$. There are eight edge partitions corresponding to the degrees of an end vertices which are presented as:
$E_{2,8} = \{uv \in E_{R_1} | d_u = 2, d_v = 8\}$, $E_{2,10} = \{uv \in E_{R_1} | d_u = 2, d_v = 10\}$, $E_{2,12} = \{uv \in E_{R_1} | d_u = 2, d_v = 12\}$, $E_{8,8} = \{uv \in E_{R_1} | d_u = 8, d_v = 8\}$, $E_{8,10} = \{uv \in E_{R_1} | d_u = 8, d_v = 10\}$, $E_{8,12} = \{uv \in E_{R_1} | d_u = 8, d_v = 12\}$, $E_{10,10} = \{uv \in E_{R_1} | d_u = 10, d_v = 10\}$, and $E_{10,12} = \{uv \in E_{R_1} | d_u = 10, d_v = 12\}$.

Therefore, we have $|E_{2,8}| = 12q$, $|E_{2,10}| = 5q(p-2)$, $|E_{2,12}| = 2q(p-2)$, $|E_{8,8}| = 3q$, $|E_{8,10}| = 4q$, $|E_{8,12}| = 2q$, $|E_{10,10}| = \frac{q(3p-8)}{2}$, and $|E_{10,12}| = 2q(p-3)$. Applying the semi total point graph operator to the Figure 3 and then applying the topological indices definitions, we get the required results.

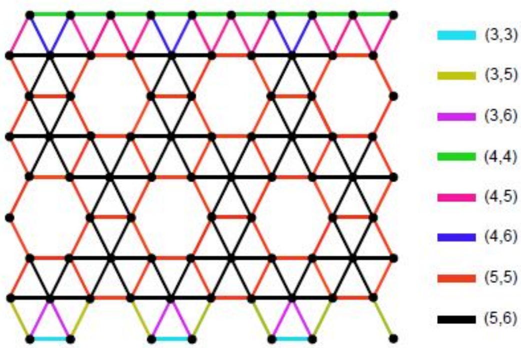

Figure 4: The edge partitions of a nanotube $BT(Y)[9, 6]$ with respect to the degrees of an end vertices

4. Topological indices of Boron-α nanotube and subdivision (semi total point) graph of boron-α nanotube $BA(Y)[p, q]$

In this section, we demonstrated the results on subdivision and semi total point graph of a boron-α nanotube $BA(Y)[p, q]$.

Molecular graph	$BA(Y)[p, q]$	$S_2 = BA(Y)[p, q]$	$R_2 = BA(Y)[p, q]$
Order	$\frac{4}{3}pq$	$\frac{q}{6}(29p - 12)$	$\frac{q}{6}(29p - 12)$
Size	$\frac{q}{2}(7p - 4)$	$q(7p - 4)$	$\frac{3q(7p-4)}{2}$

Table 3: The order and size of a boron-α nanotube $BA(Y)[p, q]$

Theorem 4.1. *Consider the boron-α nanotube $BA(Y)[p, q]$, then* *(i)* $SC(BT[p,q]) = \frac{q}{2\sqrt{6}} + \frac{q}{\sqrt{8}} + \frac{q}{3} + \frac{3q}{4\sqrt{2}} + \frac{2q}{3} + \frac{q}{\sqrt{10}} + \frac{q(3p-8)}{2\sqrt{10}} + \frac{q(2p-5)}{\sqrt{11}}$,

(ii) $R(BT[p,q]) = \frac{q}{6} + \frac{q}{\sqrt{15}} + \frac{q}{3\sqrt{2}} + \frac{3q}{8} + \frac{q}{\sqrt{5}} + \frac{q}{2\sqrt{6}} + \frac{q(3p-8)}{10} + \frac{q(2p-5)}{\sqrt{30}}$,

(iii) $RR(BT[p,q]) = \frac{3q}{2}+q\sqrt{15}+3q\sqrt{2}+6q+4q\sqrt{5}+2\sqrt{6}q+\frac{5q(3p-8)}{2}+q(2p-5)\sqrt{30}$,

(iv) $RM_2(BT[p,q]) = 2q + 8q + 10q + 6q + 24q + 15q + 8q(3p-8) + 20q(2p-5)$,

(v) $RRR(BT[p,q]) = q + 2q\sqrt{2} + q\sqrt{10} + 3q\sqrt{2} + 4q\sqrt{3} + q\sqrt{15} + 2q(3p-8) + 2q\sqrt{5}(2p-5)$,

(vi) $GO_1(BT[p,q]) = \frac{15q}{2} + 23q + 27q + 36q + 58q + 34q + \frac{35q(3p-8)}{2} + 41q(2p-5)$,

(vii) $GO_2(BT[p,q]) = 27q + 120q + 162q + 192q + 360q + 240q + 125q(3p-8) + 330q(2p-5)$.

Proof. Consider the boron-α nanotube $K = BA(Y)[p,q]$. There are eight edge partitions corresponding to the degrees of an end vertices which are presented as: $E_{3,3} = \{uv \in E_K | d_u = 3, d_v = 3\}$, $E_{3,5} = \{uv \in E_K | d_u = 3, d_v = 5\}$, $E_{3,6} = \{uv \in E_K | d_u = 3, d_v = 6\}$, $E_{4,4} = \{uv \in E_K | d_u = 4, d_v = 4\}$, $E_{4,5} = \{uv \in E_K | d_u = 4, d_v = 5\}$, $E_{4,6} = \{uv \in E_K | d_u = 4, d_v = 6\}$, $E_{5,5} = \{uv \in E_K | d_u = 5, d_v = 5\}$, and $E_{5,6} = \{uv \in E_K | d_u = 5, d_v = 6\}$.

Therefore, we have $|E_{3,3}| = \frac{q}{2}$, $|E_{3,5}| = q$, $|E_{3,6}| = q$, $|E_{4,4}| = \frac{3q}{2}$, $|E_{4,5}| = 2q$, $|E_{4,6}| = q$, $|E_{5,5}| = \frac{q(3p-8)}{2}$, and $|E_{5,6}| = q(2p-5)$. In view of the equations 1.1 to 1.7), we get the required results.

Theorem 4.2. *Consider the subdivision graph of a boron-α nanotube $S_2 = BA(Y)[p,q]$, then*

(i) $SC(BT[p,q]) = \frac{3q}{\sqrt{5}} + \frac{6q}{\sqrt{6}} + \frac{5q(p-2)}{\sqrt{7}} + \frac{q(2p-3)}{\sqrt{8}}$,

(ii) $R(BT[p,q]) = \frac{3q}{\sqrt{6}} + q\sqrt{3} + \frac{5q(p-2)}{\sqrt{10}} + \frac{q(2p-3)}{\sqrt{12}}$,

(iii) $RR(BT[p,q]) = 3q\sqrt{6} + 12q\sqrt{2} + 5q(p-2)\sqrt{10} + 2q(2p-3)\sqrt{3}$,

(iv) $RM_2(BT[p,q]) = 6q + 18q + 20q(p-2) + 5q(2p-3)$,

(v) $RRR(BT[p,q]) = 12\sqrt{3}q + 5q\sqrt{4}(p-2) + 2q(p-2)\sqrt{5}$,

(vi) $GO_1(BT[p,q]) = 33q + 84q + 85q(p-2) + 20q(2p-3)$,

(vii) $GO_2(BT[p,q]) = 90q + 288q + 350q(p-2) + 96q(2p-3)$.

Proof. Consider the subdivision graph of a boron-α nanotube $S_2 = \{BA(Y)[p,q]\}$. There are four edge partitions corresponding to the degrees of end vertices which are presented as:
$E_{2,3} = \{uv \in E_{S_2} | d_u = 2, d_v = 3\}$, $E_{2,4} = \{uv \in E_{S_2} | d_u = 2, d_v = 4\}$, $E_{2,5} = \{uv \in E_{S_2} | d_u = 2, d_v = 5\}$ and $E_{2,6} = \{uv \in E_{S_2} | d_u = 2, d_v = 6\}$.

Therefore, we have $|E_{2,3}| = 3q$, $|E_{2,4}| = 6q$, $|E_{2,5}| = 5q(p-2)$ and $|E_{2,6}| = q(2p-3)$. Applying the subdivision graph operator to the Figure 4, then respective edge partitions which edges belongs to $E_{2,3}$, $E_{2,4}$, $E_{2,5}$ and $E_{2,6}$ respectively. In view of the equations 1.1 to 1.7), we get the required results.

Theorem 4.3. *Consider the semi total point graph of boron-α nanotube $R_2 = BA(Y)[p,q]$, then*

(i) $SC(BT[p,q]) = \frac{3q}{\sqrt{8}} + \frac{6q}{\sqrt{10}} + \frac{5q(p-2)}{\sqrt{12}} + \frac{q(2p-3)}{\sqrt{14}} + \frac{q}{2\sqrt{12}} + \frac{q}{4} + \frac{q}{3\sqrt{2}} + \frac{3q}{8} + \frac{2q}{3\sqrt{2}} + \frac{q}{2\sqrt{5}} + \frac{q(3p-8)}{\sqrt{20}} + \frac{q(2p-5)}{\sqrt{22}}$,

(ii) $R(BT[p,q]) = \frac{3q}{\sqrt{12}} + \frac{3q}{2} + \frac{5q(p-2)}{\sqrt{20}} + \frac{q(2p-3)}{\sqrt{24}} + \frac{q}{12} + \frac{q}{\sqrt{60}} + \frac{q}{\sqrt{72}} + \frac{3q}{16} + \frac{q}{2\sqrt{5}} + \frac{q}{\sqrt{96}} + \frac{q(3p-8)}{10} + \frac{q(2p-5)}{\sqrt{120}}$,

(iii) $RR(BT[p,q]) = 3q\sqrt{12} + 42q + 5q(p-2)\sqrt{20} + q(2p-3)\sqrt{24} + 3q + q\sqrt{60} + q\sqrt{72} + 12q + 2q\sqrt{80} + q\sqrt{96} + 10q(3p-8) + q(2p-5)\sqrt{120}$,

(iv) $RM_2(BT[p,q]) = 15q + 42q + 45q(p-2) + 11q(2p-3) + \frac{25q}{2} + 45q + 55q + \frac{147q}{2} + 126q + 77q + \frac{81q(3p-8)}{2} + 99q(2p-5)$,

(v) $RRR(BT[p,q]) = 3q\sqrt{5} + 6q\sqrt{7} + 15(p-2) + q(2p-3)\sqrt{11} + \frac{5q}{2} + 3q\sqrt{5} + q\sqrt{55} + \frac{21}{2} + 6q\sqrt{7} + q\sqrt{77} + \frac{9q(3p-8)}{2} + q(2p-5)\sqrt{99}$,

(vi) $GO_1(BT[p,q]) = 60q + 156q + 160q(p-2) + 38q(2p-8) + 24q + 76Q + 90Q + 120Q + 196q + 116q + 60q(3p-8) + 240q + 392q + 232q + 60q(3p-8) + 284q(p-3) + 142q(2p-5)$,

(vii) $GO_2(BT[p,q]) = 288q + 960q + 1680q(p-2) + 336q(2p-3) + 216q + 960q + 1296q + 1536q + 2880q + 1920q + 1000q(3p-8) + 2640q(2p-5)$.

Proof. Consider the semi total point graph of a boron-α nanotube $R_2 = BA(Y)[p,q]$. There are twelve edge partitions corresponding to the degrees of an end vertices which are presented as:
$E_{2,6} = \{uv \in E_{R_2} | d_u = 2, d_v = 6\}$, $E_{2,8} = \{uv \in E_{R_2} | d_u = 2, d_v = 8\}$, $E_{2,10} = \{uv \in E_{R_2} | d_u = 2, d_v = 10\}$, $E_{2,12} = \{uv \in E_{R_2} | d_u = 2, d_v = 12\}$, $E_{6,6} = \{uv \in$

$E_{R_2}|d_u = 6, d_v = 6\}$, $E_{6,10} = \{uv \in E_{R_2}|d_u = 6, d_v = 10\}$, $E_{6,12} = \{uv \in E_{R_2}|d_u = 6, d_v = 12\}$, $E_{8,8} = \{uv \in E_{R_2}|d_u = 8, d_v = 8\}$, $E_{8,10} = \{uv \in E_{R_2}|d_u = 8, d_v = 10\}$, $E_{8,12} = \{uv \in E_{R_2}|d_u = 8, d_v = 12\}$, $E_{10,10} = \{uv \in E_{R_2}|d_u = 10, d_v = 10\}$, and $E_{10,12} = \{uv \in E_{R_2}|d_u = 10, d_v = 12\}$.

Therefore, we have $|E_{2,6}| = 3q$, $|E_{2,8}| = 6q$, $|E_{2,10}| = 5q(p-2)$, $|E_{2,12}| = q(2p-3)$, $|E_{6,6}| = \frac{q}{2}$, $|E_{6,10}| = q$, $|E_{6,12}| = q$, $|E_{8,8}| = \frac{3q}{2}$, $|E_{8,10}| = 2q$, $|E_{8,12}| = q$, $|E_{10,10}| = \frac{q(3p-8)}{2}$, and $|E_{10,12}| = q(2p-5)$. Applying the semi total point graph operator to the Figure 4 and then applying the definitions of the topological indices (1.1 to 1.7), we get the required results.

5. Conclusion

In this paper, we have studied the important classes of boron nanotubes and formulated $SC(G)$, $R(G)$, $RR(G)$ RM_2, RRR, GO_1, GO_2 indices of their molecular graphs by using edge partition technique. These results can be used in detecting some physical and chemical properties of these boron nanotubes.

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