

ESSENTIAL ASCENT AND ESSENTIAL DESCENT OF LINEAR OPERATORS AND COMPOSITION OPERATORS

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(Received: Nov. 28, 2019 Accepted: May. 18, 2020 Published: Aug. 30, 2020)

Abstract: In this paper we prove certain results relating to essential ascent and essential descent of linear operator on an arbitrary vector space. Further, we give a complete characterization of essential ascent and essential descent of composition operators on l^p spaces.

Keywords and Phrases: Essential Ascent, Essential Descent, Composition Operator.

2010 Mathematics Subject Classification: 47B33.

1. Introduction

Let X denote an arbitrary vector space and T be a linear operator on X . Let $D(T)$, $N(T)$ and $R(T)$ denote domain, kernel and range of T respectively. Let \mathbb{N} denote the set of natural numbers.

Definition 1.1. If there is some integer $n \geq 0$ such that $N(T^n) = N(T^{n+1})$, the smallest such integer is called the ascent of T and is denoted by $a(T)$. If no such integer exists then $a(T) = \infty$; see [13].

Definition 2.2. If there is some integer $n \geq 0$ such that $R(T^{n+1}) = R(T^n)$, the smallest such integer is called the descent of T and is denoted by $d(T)$. If no such integer exists then $d(T) = \infty$; see [13].

Note that if X is a finite dimensional, then $a(T)$ and $d(T)$ are both finite. We state the following well-known result about ascent and descent of linear operators.