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PATHOS VERTEX SEMI-MIDDLE GRAPH OF A TREE

Rajendra Prasad KC, Venkanagouda M. Goudar* and Niranjan KM**

UBDT College of Engineering, Davanagere-577004
A Constituent college of Visvesvaraya Technological University, Belagavi
Department of Mathematics,
Jain Institute of Technology, Davangere - 577003,
Affiliated to Visvesvaraya Technological University,
Belagavi, Karnataka, INDIA

E-mail : rajendraprasadkp@gmail.com

*Department of Mathematics, Sri Siddhartha Institute of Technology,
Tumkur - 572105, Karnataka INDIA

E-mail : vmgouda@gmail.com

**Department of Mathematics, UBDT College of Engineering,
Davangere - 577004, Karnataka, INDIA

E-mail : niranjankm64@gmail.com

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Abstract: In this communication, the pathos vertex semi-middle graph of a tree is introduced. Its study is concentrated only on trees. We present a characterization of those graphs whose pathos vertex semi-middle graph of a tree is planar, crossing number one and crossing number two. Also we establish a characterization of graphs whose pathos vertex semi-middle graph of a tree is noneulerian, hamiltonian.

Keywords and Phrases: Crossing number; Middle graph; Planar; Semientire graph.

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1. Introduction

All graphs considered here are finite, undirected without loops or multiple edges. We refer the terminology of [1].

The concept of pathos of a graph G was introduced by Harary [2], as a collection of minimum number of edge disjoint open paths whose union is G. The path number of a graph G is the number of paths in pathos.

A graph is said to be embedded in a surface S when it is drawn on S so that no two edges intersect. A graph is planar if it can be embedded in the plane.

The crossing number $C_r(G)$ of G is the least number of intersection of pairs of edges in any embedding of G in the plane. Obviously G is planar if and only if $C_r(G) = 0$.

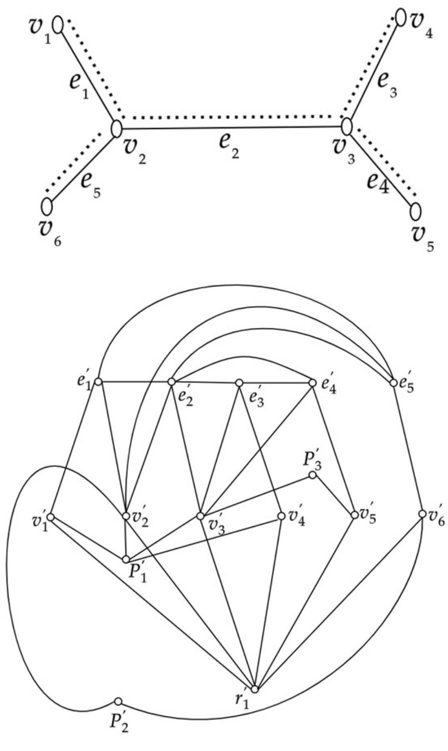

Figure 1. T and $PM_v(T)$

The *edgedegree* [5] of an edge uv of a tree T is the sum of the degrees of u and v. The pathoslength is the number of edges that lie on a particular path P_i of pathos of T. A pendent pathos is a path P_i of pathos having unit length, which corresponds to a pendent edge in T.

In the paper [4] and [6] defined the concept of on pathos litact graph of a tree and pathos cutvertex jump graph of a tree. In this context, the present work focus to define the pathos vertex semi-middle graph of a tree. The pathos vertex semi-middle graph of a tree denoted by $PM_v(T)$ is the graph whose vertex set is $V(T)UE(T)UR(T)UP_i(T)$ and two vertices of $PM_v(T)$ are adjacent if and only if they corresponds to two adjacent edges of T or one corresponds to a vertex and other to an edge incident with it or one corresponds to a vertex other to a region in which vertex lies on the region or one corresponds to a vertex and other to a path of pathos in which vertex lies on the path of pathos since the system of pathos for a tree is not unique, the corresponding pathos vertex semi-middle graph of a tree is also not unique. The tree T and its pathos vertex semi-middle graph of a tree $PM_v(T)$ is depicted in the figure 1.

2. Preliminaries.

The following results will be useful in our results.

Theorem 2.1 [3]. *For any graph G, $M_v(G)$ is always non-separable.*

Theorem 2.2 [3]. *For any graph G, p vertices, q edges and l regions then $M_v(G)$ has $(p+q+r)$ vertices and $q+\sum_{i=1}^{q}\frac{1}{2}\{d(e_i)\}+\sum_{j=1}^{r}d(r_j)$ edges. Where $d(e_i)$ is the edgedegree of a edge e_i and $d(r_j)$ is the degree of a region r_j.*

Theorem 2.3 [3]. *For any graph G, $M_v(G)$ is planar if and only if G is a path.*

Theorem 2.4 [1]. *A finite graph G is Eulerian if and only if all its vertex degree are even.*

3. Pathos Vertex Semi-Middle Graph of a Tree.

We begin with the following observation.

Observation 3.1. *If v is a pendant vertex of a tree T, then the degree of a corresponding vertex v' in $PM_v(T)$ is odd.*

Theorem 3.1. *For any tree T, $PM_v(T)$ is always non-separable.*
Proof. We establish the following cases.
Case 1. Suppose T be any tree. Let $v_1', v_2', v_3'....v_n'$ be the vertices of $PM_v(T)$ corresponds to the vertices $v_1, v_2, v_3....v_n$ of T and $e_1', e_2', e_3'....e_{n-1}'$ be the vertices of $PM_v(T)$ corresponds to the edges $e_1, e_2, e_3....e_{n-1}$ of T. Let $P_1', P_2', P_3'....P_n'$ be the pathosvertices of $PM_v(T)$ corresponds to the path $P_1, P_2, P_3....P_n$ By Theorem 2.1, $M_v(G)$ is non-separable. Also in $PM_v(T)$, the pathosvertices are adjacent to the $v_1', v_2', v_3'....v_n'$ without cut vertex. Thus $PM_v(T)$ is always non-separable.
Case 2. Suppose $T = P_n : v_1, v_2, v_3, ...v_n, n > 1$. Further, $V[PM_v(T)] =$

$\{v_1^{'}, v_2^{'}, v_3^{'}....v_n^{'}, e_1^{'}, e_2^{'}, e_3^{'}....e_{n-1}^{'}, r_1^{'}, P_1^{'}\}$. By Theorem 2.1, $M_v(G)$ is non-separable. Also in $PM_v(T)$ the pathosvertex $P_1^{'}$ adjacent to the vertices $v_1^{'}, v_2^{'}, v_3^{'}....v_n^{'}$ without cut vertex. Hence $PM_v(T)$ is always non-separable.

Theorem 3.2. *For any graph G with p vertices, q edges, r regions and k path of pathos, $PM_v(T)$ has $(p + q + r + \sum_{i=1}^{k} P_i)$ vertices and $q + \sum_{i=1}^{q} \frac{1}{2}\{d(e_i)\} + \sum_{j=1}^{r} d(r_j) + \sum_{i=1}^{k} P_{v_i}$ edges. Where $d(e_i)$ is the edgedegree of a edge e_i, $d(r_j)$ is the degree of a region r_j and P_{v_i} is the number of vertices which lies on the path of pathos.*

Proof. By the definition of $PM_v(T)$, $V[PM_v(T)] = V(T)UE(T)UR(T)UP_i(T)$. Hence $V[PM_v(T)] = p + q + r + \sum_{i=1}^{k} P_i$. Further, by Theorem 2.2, $E[M_v(G)] = q + \sum_{i=1}^{q} \frac{1}{2}\{d(e_i)\} + \sum_{j=1}^{r} d(r_j)$. The degree of a pathosvertex is the sum of the number of vertices lies on the each path of pathos in T which is $\sum_{i=1}^{k} P_{v_i}$. The number of edges in $PM_v(T)$ is equal to the sum of edges in $M_v(G)$ and $\sum_{i=1}^{k} P_{v_i}$. Hence $E[PM_v(T)] = q + \sum_{i=1}^{q} \frac{1}{2}\{d(e_i)\} + \sum_{j=1}^{r} d(r_j) + \sum_{i=1}^{k} P_{v_i}$.

Theorem 3.3. *For any tree T, $PM_v(T)$ is planar if and only if T is P_2.*

Proof. Consider a tree T is not a path P_2. Let $T = P_3$, $T = P_3 : v_1, v_2, v_3$. Further, $V[PM_v(T)] = \{v_1^{'}, v_2^{'}, v_3^{'}, e_1^{'}, e_2^{'}, r_1^{'}, P_1^{'}\}$. By Theorem 2.3, $M_v(P_3)$ is planar. Further in $PM_v(T)$, the pathos vertex $P_1^{'}$ is adjacent to the vertices $v_1^{'}, v_2^{'}, v_3^{'}$, of $M_v(P_3)$. The edge between $v_2^{'}$ and $P_1^{'}$ is crossing over the edge already drawn in $M_v(P_3)$. Which is non-planar, a contradiction. Conversely, suppose $G = P_2 : v_1, v_2$. Further, $V[PM_v(T)] = \{v_1^{'},\ v_2^{'},\ e_1^{'},\ r_1^{'},\ P_1^{'}\}$. By Theorem 2.3, $M_v(P_2)$ is planar. Further in $PM_v(T)$, $i[PM_v(P_2)] = 01$. Hence $PM_v(P_2)$ is planar.

Theorem 3.4. *For any tree T, $PM_v(T)$ has crossing number one if and only if T is P_3.*

Proof. Consider $T = P_3$, then $M_v(P_3) =$ 1-minimally non outer planar. Further, in $PM_v(P_3)$, the pathos vertex $P_1^{'}$ is adjacent to the vertices $v_1^{'}, v_2^{'}, v_3^{'}$ of $M_v(P_3)$. Which gives a crossing number one. Hence $PM_v(T)$ has crossing number one.
Conversely, suppose $T = P_4 : v_1, v_2, v_3, v_4$. Further, $V[M_v(P_4)] = \{v_1^{'},\ v_2^{'},\ v_3^{'}, v_4^{'},\ e_1^{'},\ e_2^{'},\ e_3^{'},\ r_1^{'}, P_1^{'}\}$. By Theorem 2.3, $M_v(P_3)$ is planar. Further in $PM_v(T)$, pathos vertex $P_1^{'}$ is adjacent to $v_1^{'}, v_2^{'}, v_3^{'}, v_4^{'}$ and gives crossing number two, a contradiction.

Theorem 3.5. *For any tree T, $PM_v(T)$ has crossing number two if and only if T is P_4 or $K_{1,3}$.*

Proof. Suppose that $PM_v(T)$ has crossing number two.
We have the following cases.

Case 1. Suppose $T = P_5 : v_1, v_2, v_3, v_4, v_5$. Further, $V[PM_v(T)] = \{v_1^{'}, v_2^{'}, v_3^{'}, v_4^{'}, v_5^{'}, e_1^{'}, e_2^{'}, e_3^{'}, e_4^{'}, r_1^{'}, P_1^{'}\}$. By Theorem 2.3, $M_v(P_5)$ is planar. Further in $PM_v(T)$, $P_1^{'}$ is adjacent to $v_1^{'}, v_2^{'}, v_3^{'}, v_4^{'}, v_5^{'}$ and gives crossing number three, a contradiction.
Case 2. Suppose $T = K_{1,4} : v_1, v_2, v_3, v_4, v_5$ and $deg(v_i) = 4$. Further, $V[PM_v(T)] = \{v_1^{'}, v_2^{'}, v_3^{'}, v_4^{'}, v_5^{'}, e_1^{'}, e_2^{'}, e_3^{'}, e_4^{'}, r_1^{'}, P_1^{'}, P_2^{'}\}$. In $M_v(T)$, $C_r[M_v(K_{1,4})] = 06$. Further in $PM_v(T)$, $P_1^{'}$ adjacent to $v_1^{'}, v_2^{'}, v_3^{'}$ and $P_2^{'}$ adjacent to $v_2^{'}, v_4^{'}, v_5^{'}$ of $M_v(T)$. Which gives a crossing number six, a contradiction.
Conversely,
Case 1. Suppose $T = K_{1,3} : v_1, v_2, v_3, v_4$ and $deg(v_i) = 3$. Further, $V[PM_v(T)] = \{v_1^{'}, v_2^{'}, v_3^{'}, v_4^{'}, e_1^{'}, e_2^{'}, e_3^{'}, r_1^{'}, P_1^{'}, P_2^{'}\}$. In $M_v(T)$, $C_r[M_v(K_{1,3})] = 01$. Further in $PM_v(T)$, $P_1^{'}$ adjacent to $v_1^{'}, v_2^{'}, v_3^{'}, v_4^{'}$ and $P_2^{'}$ adjacent to $v_2^{'}, v_3^{'}$ of $M_v(T)$. Which gives crossing number two. Hence $PM_v(T)$ has crossing number two.
Case 2. Suppose $T = P_4 : v_1, v_2, v_3, v_4$. Further, $V[PM_v(T)] = \{v_1^{'}, v_2^{'}, v_3^{'}, v_4^{'}, e_1^{'}, e_2^{'}, e_3^{'}, r_1^{'}, P_1^{'}\}$. By Theorem 2.3, $M_v(P_4)$ is planar. Further in $PM_v(T)$, $P_1^{'}$ is adjacent to $v_1^{'}, v_2^{'}, v_3^{'}, v_4^{'}$ and gives crossing number two. Hence $PM_v(T)$ has crossing number two.

Theorem 3.6. *For any tree T, $PM_v(T)$ is always noneulerian.*
Proof. Let T be any tree. By observation 3.1, the degree of the corresponding vertex in $PM_v(T)$ becomes odd. By the Theorem 2.4, $PM_v(T)$ is noneulerian. Hence, $PM_v(T)$ is always non eulerian.

Theorem 3.7. *For any tree T, $PM_v(T)$ is hamiltonian if and only if T is not P_2.*
Proof. Let T be any tree. We have the following cases.
Case 1. Suppose $T = P_n : v_1, v_2, v_3, ...v_n, n \geq 3$. Further, $V[PM_v(T)]=\{v_1^{'}, v_2^{'}, v_3^{'}, ...v_n^{'}, e_1^{'}, e_2^{'}, e_3^{'}, ...e_{n-1}^{'}, r_1^{'}, P_1^{'}\}$. Then there exists a cycle $r_1^{'}, v_1^{'}, P_1^{'}, v_3^{'}, e_2^{'}, ...v_n^{'}, e_{n-1}^{'}, r_1^{'}$. Which includes all the vertices of $PM_v(T)$. Hence $PM_v(T)$ is hamiltonian.
Case 2. Suppose $T = K_{1,n} : v_1, v_2, v_3, ...v_n, n \geq 2$. Further, $V[PM_v(T)] = \{v_1^{'}, v_2^{'}, v_3^{'}, ...v_n^{'}, e_1^{'}, e_2^{'}, e_3^{'}, ...e_{n-1}^{'}, P_1^{'}, P_2^{'}, ...P_n^{'}, r_1^{'}\}$. Then there exists a cycle $r_1^{'}, v_1^{'}, P_1^{'}, v_3^{'}, e_2^{'}, ...v_n^{'}, e_{n-1}^{'}, P_n^{'}, r_1^{'}$. Which includes all the vertices of $PM_v(T)$. Hence $PM_v(T)$ is hamiltonian.
Conversely, Suppose $T = P_2 : v_1, v_2$. Further, $V[PM_v(T)]=\{v_1^{'}, v_2^{'}, e_1^{'}, r_1^{'}, P_1^{'}\}$. By the definition of $M_v(T)$, the vertices $v_1^{'}$ and $v_2^{'}$ are adjacent to the edge vertex $e_1^{'}$ and the regionvertex $r_1^{'}$. Clearly $M_v(T) = K_4$. In $PM_V(T)$, the pathos vertex $P_1^{'}$ is adjacent to $v_1^{'}$ and $v_2^{'}$. Clearly $v_1^{'}, e_1^{'}, v_2^{'}, r_1^{'}, v_1^{'}$ form a hamiltonian cycle in $PM_v(T)$. But in $PM_v(T)$, the $P_1^{'}$ is not lies on the hamiltonian cycle. Hence $PM_v(T)$ is non-hamiltonian.

4. Conclusion. In this paper we obtained the new graph valued function called pathos vertex semi-middle graph of a tree. We studied the characterization of graphs whose of pathos vertex semi-middle graph of a tree is planar, hamiltonian, crossing number one and two. Further, we obtain $PM_v(T)$ is non-eulerian.

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