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# On Bailey's Transform and Expansion of Hypergeometric Functions-I

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**Abstract:** In this paper we introduce a new technique to obtain expansions of basic hypergeometric functions with the help of Bailey's transform and certain known transformations of truncated hypergeometric series. These results do not look possible with the help of the traditional method. Certain interesting special cases, both, new and known, have also been deduced.
**Keywords and phrases:** Truncated series, terminating series, expansion of hypergeometric series/functions, Bailey's transform, bi-basic hypergeometric series.
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## 1. Introduction, Notations and Definitions

For $|q| < 1$ and $\alpha$, real or complex, let

$$[\alpha;q]_n \equiv [\alpha]_n = \begin{cases} (1-\alpha)(1-\alpha q)(1-\alpha q^2)...(1-\alpha q^{n-1}); & n > 0 \\ 1 & ; \quad n = 0 \end{cases} \tag{1.1}$$

Accordingly,

$$[\alpha;q]_\infty = \prod_{n=0}^{\infty}(1-\alpha q^n)$$

Also,

$$[a_1,a_2,a_3,...,a_r;q]_n = [a_1;q]_n[a_2;q]_n[a_3;q]_n...[a_r;q]_n. \tag{1.2}$$

Now, we define a basic hypergeometric function

$${}_r\Phi_s\left[\begin{matrix} a_1,a_2,...,a_r;q;z \\ b_1,b_2,...,b_s;q^\lambda \end{matrix}\right] = \sum_{n=0}^{\infty}\frac{[a_1,a_2,...,a_r;q]_n z^n q^{\lambda n(n-1)/2}}{[q,b_1,b_2,...,b_s;q]_n} \tag{1.3}$$

convergent for $|z| < \infty$ when $\lambda > 0$ and for $|z| < 1$ when $\lambda = 0$.
A generalized double basic hypergeometric function is defined as,

$$\Phi\left[\begin{matrix} a_1,a_2,...,a_r : \alpha_1,\alpha_2,...,\alpha_{u_1};\beta_1,\beta_2,...,\beta_{v_1};q;x,y \\ b_1,b_2,...,b_s : \delta_1,\delta_2,...,\delta_{u_2};\gamma_1,\gamma_2,...,\gamma_{v_2} \end{matrix}\right]$$