

On Hypergeometric Proof of Certain Continued Fraction Results

B.P. Mishra, Swatantra Shukla* and Jitendra Prasad**

Department of Mathematics,
 M.D. College, Parel, Mumbai, India

*Department of Mathematics,
 Hadia PG College, Hadia Allahabad, U.P.

**Department of Mathematics,
 J.P. University, Chapra, Bihar

Abstract: In this paper, we provide hypergeometric proof of certain results and deduce a number of new and known results. This result is equivalent to Entry 12 of Chapter XVI of Ramanujan's second Notebook.

Keywords and Phrases: Basic hypergeometric function, q-series, continued fraction, Ramanujan's theta functions.

2000 A.M.S. subject classification: 33A30, 33D15, 33D20.

1. Introduction, Notations and Definitions

Ramanujan's contribution to continued fractions associated with analytic functions is remarkable. His Notebooks contain a large number of beautiful results associated with hypergeometric functions (both, basic and ordinary) and continued fractions. Many of his continued fraction results can be provided with hypergeometric proof. In a recent publication Denis and Singh [2,3] provided hypergeometric proof of Entries 25 and 33 of Chapter XII of Ramanujan's [5] second Notebook, and also provided their basic analogues.

Motivated by the above results, we propose to provide hypergeometric proof of the following results.

$$\frac{[a^2q^3, b^2q^3; q^4]_\infty}{[a^2q, b^2q; q^4]_\infty} = \frac{1}{1 - a^2q} \frac{q(b^2 - a^2q^2)}{1 + q^2} \frac{q(a^2 - b^2q^2)}{(1 - a^2q)(1 + q^4)} - \frac{q^5(b^2 - a^2q^6)}{1 + q^6} \frac{q(a^2 - b^2q^6)}{(1 - a^2q)(1 + q^8)} \frac{q^9(b^2 - a^2q^{10})}{1 + q^{10} + \dots} \quad (1.1)$$

where

$$[\alpha, \beta; p]_\infty = [\alpha; p]_\infty [\beta; p]_\infty$$

and

$$(\alpha; q)_\infty = \prod_{r=0}^{\infty} (1 - \alpha q^r), \quad |q| < 1.$$