

**A SERIES EXPANSION FOR THE $b(s)$
BRONCKER-RAMANUJAN FUNCTION**

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Abstract: Our basic aim is to provide a power series representation for $b(s)$, $0 < s < 3$, the well-known function satisfying $b(s-1)b(s+1) = s^2$. We will do this by using integer compositions of n . In the last section, some properties involving the coefficients of s^n in the power series expansion of $b(s)$ are given, as well an expression for $\frac{4}{\pi}$.

Keywords and Phrases: Brouncker-Ramanujan function, Integer Compositions, Convergent series, Infinite Products, Functional Equations.

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1. Introduction

In 1655, the mathematician John Wallis concerning in the quadrature of the unit circle wrote a letter to William Brouncker in the attempt to solve a special problem. He want to find an arithmetical expression to

$$\int_0^1 \sqrt{1-x^2} dx,$$

wrote in contemporaneous mathematical language.

The answer of Brouncker is the $b(s)$ function satisfying $b(s-1)b(s+1) = s^2$. Writing using continued fraction, we have