*J. of Ramanujan Society of Mathematics and Mathematical Sciences*
*Vol. 8, No. 2 (2021), pp. 29-38*

**ISSN (Online): 2582-5461**

**ISSN (Print): 2319-1023**

# SOME DEFINITE INTEGRAL FORMULAE INVOLVING BESSEL FUNCTION, LOG FUNCTION AND HYPERGEOMETRIC FUNCTION

**Salahuddin and Vinti**

Department of Mathematics,
PDM University, Bahadurgarh - 124507, Haryana, INDIA

E-mail : vsludn@gmail.com

(**Received:** *Feb. 23, 2021* **Accepted:** *May 12, 2021* **Published:** *Jun. 30, 2021)*

**Abstract:** In this paper, we aim to evaluate some definite integrals involving Bessel function and log function in terms of generalized hypergeometric functions.

**Keywords and Phrases:** Bessel Function, Hypergeometric Function, Pochhammer symbol.

**2020 Mathematics Subject Classification:** 33B30, 33C10,33C20.

## 1. Introduction

The following definite integral formulas are recalled (see, e.g., [3, p. 204, Entries 4.7.7-20 and 21]):

$$\int_0^1 x \log x \ J_0^2(ax)dx = -\frac{1}{2}\big[J_0^2(a) + J_1^2(a) - \frac{1}{a}J_0(a)J_1(a)\big]. \tag{1.1}$$

$$\int_0^1 x \log x \ J_1^2(ax)dx = \frac{1}{2a^2}\big[1 - (a^2+1)J_0^2(a) + a \ J_0(a)J_1(a) - a^2 \ J_1^2(a)\big]. \tag{1.2}$$

Bessel functions of the first kind, denoted as $J_\alpha(x)$, are solutions of Bessel's differential equation that are finite at the origin ($x = 0$) for integer or positive $\alpha$, and diverge as $x$ approaches zero for negative non-integer $\alpha$ (See[11]). It is possible to define the function by its Taylor series expansion around $x = 0$.

$$J_\alpha(x) = \sum_{m=0}^{\infty} \frac{(-1)^m}{m! \ \Gamma(m+\alpha+1)} \left(\frac{x}{2}\right)^{2m+\alpha} \tag{1.3}$$