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INTEGRAL REPRESENTATIONS OF G-FUNCTIONS THROUGH AN INTERESTING IDEA

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Abstract: Some fifteen G-functions are given two integral representations and a series form each through an interesting idea of constructing statistical distributions of products and ratios of statistically independently distributed real positive scalar random variables which are equivalent to computing the Mellin convolutions of products and ratios involving two functions. These integral representations and the explicit series forms may not be available in the literature. For special values of the parameters some integral representations and series forms are available in the literature for some of the G-functions considered in this paper. But the representations given in this paper are for general parameters.

Keywords and Phrases: G-functions, integral representations, statistical distributions, distributions of products and ratios, Mellin convolutions, series forms.

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1. Introduction

In this paper, real scalar mathematical as well as random variables, will be denoted by small letters $x, y, \dots, x_1, x_2, \dots$. Let $x_1 > 0, x_2 > 0$ be real scalar positive random variables, statistically independently distributed with density functions $f_1(x_1)$ and $f_2(x_2)$ respectively so that the joint density is $f_1(x_1)f_2(x_2)$. A density function $f(x)$ is a real-valued scalar function of x such that $f(x) \geq 0$ for all x and $\int_x f(x)dx = 1$. Let $u = x_1x_2$ the product. Then the density of u , denoted by $g(u)$, is available by considering the one to one transformation $u = x_1x_2, v = x_1$