

ON THE OUTLINES OF PLANE CURVES OF THE FORM

$$(ax)^\alpha + (by)^\alpha = r^\alpha \text{ WITH } \alpha > 0$$

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Abstract: We consider plane curves of the form $(ax)^\alpha + (by)^\alpha = r^\alpha$ defined on the first quadrant of $\mathbb{R}^2$, where $\alpha > 0$ and $a, b, r > 0$. We summarize the outlines of them by using elementary differential calculus. We will in this note understand that they are classified into three types of curves, convex, straight and concave, depending on α .

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1. Introduction

There are many famous plane curves in mathematics. We take up, in this note, plane curves represented by implicit functions of the form

$$F(x, y) = x^\alpha + y^\alpha - r^\alpha = 0$$

where $\alpha, r > 0$. They include the famous plane curves as follows:

- If $\alpha = 2$, the curve $x^2 + y^2 - r^2 = 0$ is the circle that its center is the origin and the length of its radius is r . (See Figure 1 for the outline.)
- If $\alpha = 1$, the curve $x + y - r = 0$ is the straight line whose y -intercept is r . (See Figure 2 for the outline.)