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BINARY AND SEMI-FIBONACCI PARTITIONS

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Dedicated to Prof. A.K. Agarwal on his 70^{th} ***Birth Anniversary***

Abstract: It is proved that the partitions of n into powers of two with all parts appearing an odd number of times equals the number of Semi-Fibonacci partition of n. The parity of the number of such partitions is also exhibited.

Keyword and Phrases: Binary partitions, semi-Fibonacci partitions.

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1. Introduction

The set $\mathfrak{SF}(n)$ of semi-Fibonacci partitions is defined as follows: $\mathfrak{SF}(1) = \{1\}$, $\mathfrak{SF}(2) = \{2\}$. If $n > 2$ and even the $\mathfrak{SF}(n)$ consists of the partitions of $\mathfrak{SF}(\frac{n}{2})$ wherein each part has been multiplied by 2. If n is odd, $\mathfrak{SF}(n)$ arises from two sources: first a 1 is inserted in each partition of $n-1$ and second a 2 is added to the single odd part of $\mathfrak{S}(n-2)$ (note: it is easily seen by induction that semi-Fibonacci partitions have at most one odd part).

Thus here are the first seven $\mathfrak{SF}(n)$:

$$\begin{aligned}
\mathfrak{SF}(1) &= \{1\}\\
\mathfrak{SF}(2) &= \{2\}\\
\mathfrak{SF}(3) &= \{2+1, 3\}\\
\mathfrak{SF}(4) &= \{4\}\\
\mathfrak{SF}(5) &= \{4+1, 3+2, 5\}\\
\mathfrak{SF}(6) &= \{4+2, 6\}\\
\mathfrak{SF}(7) &= \{4+2+1, 6+1, 4+3, 5+2, 7\}
\end{aligned}$$