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A SHORT REVIEW OF ESTIMATION OF POPULATION VARIANCE THROUGH RATIO ESTIMATORS

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Dedicated to Prof. K. Srinivasa Rao on his 75^{th} ***Birth Anniversary***

Abstract: The present manuscript is a short review of the ratio type estimators of population variance of the study variable using auxiliary information on a single auxiliary variable. In this paper various ratio type estimators of population variance in the literature have been given in chronological order. The large sample properties that is biases and the mean squared errors of these ratio type estimators of population variance have been given up to the first order of approximation. The expressions of the bias and the mean squared error of every mentioned estimator have been given up to the first order of approximation.

Keywords and Phrases: Auxiliary variable, Parameter, Estimator, Bias, Mean Squared Error, Efficiency.

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1. Introduction

The variance is one of the important measures of dispersion of the main characteristic under study for the homogeneous units. The most suitable estimator for the estimation of the population parameter under consideration is the corresponding statistic and therefore the most appropriate estimator for population variance is the sample variance of the main variable under study. Although the sample variance is an unbiased estimator of population variance but it has a reasonably large amount of variation. That is its sampling distribution is not very much concentrated round the population variance. Our aim is to find the estimator which may even be biased but it should have minimum mean squared error that is its sampling distribution should be very much concentrated round the population variance. Auxiliary information fulfills this need. Auxiliary variable is highly positively or negatively correlated with the main variable under study. By the use of auxiliary information, the efficiency of the estimator is enhanced. When the main variable under study and the auxiliary variable are positively correlated, ratio type estimators are used for the estimation of parameters. Product type estimators are used when main and auxiliary variables are negatively correlated and the line of regression Y on X passes through origin and in other case regression estimators are used for the estimation of population parameters. In this short review paper, we have considered positively correlated case only.

Let the finite population under consideration consist of N distinct and identifiable units and let $(x_i, y_i),\ i = 1, 2, ..., n$ be a bivariate sample of size n taken from (X, Y) using a $SRSWOR$ scheme. Let $\overline{X}$ and $\overline{Y}$ respectively be the population means of the auxiliary and the study variables, and let $\overline{x}$ and $\overline{y}$ be the corresponding sample means.

The Natural and most suitable estimator of population variance is the sample variance given by,

$$t_0 = s_y^2, \tag{1.1}$$

It is unbiased, and its variance up to the first degree of approximation is,

$$V(t_0) = \gamma S_y^4(\lambda_{40} - 1) \tag{1.2}$$

where, $\lambda_{rs} = \dfrac{\mu_{rs}}{\mu_{20}^{r/2}\mu_{02}^{s/2}}$, $\mu_{rs} = \dfrac{1}{N-1}\sum_{i=1}^{N}(Y_i - \overline{Y})^r(X_i - \overline{X})^s$, and $\gamma = \dfrac{N-n}{nN}$

Isaki (1983) used the positively correlated auxiliary information and proposed the following usual ratio estimator of population variance as,

$$t_R = s_y^2\left(\frac{S_x^2}{s_x^2}\right), \tag{1.3}$$

where, $s_y^2 = \dfrac{1}{n-1}\sum_{i=1}^{n}(y_i - \overline{y})^2$, $s_x^2 = \dfrac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$, $S_x^2 = \dfrac{1}{N-1}\sum_{i=1}^{N}(X_i - \overline{X})^2$,

$\overline{X} = \dfrac{1}{N}\sum_{i=1}^{N}X_i$, $\overline{Y} = \dfrac{1}{N}\sum_{i=1}^{N}Y_i$, $\overline{x} = \dfrac{1}{n}\sum_{i=1}^{n}x_i$, $\overline{y} = \dfrac{1}{n}\sum_{i=1}^{n}y_i$.

The expressions for the Bias and Mean Square Error (MSE), up to the first order of approximation, are respectively given by

$$B(t_R) = \gamma S_y^2[(\lambda_{04} - 1) - (\lambda_{22} - 1)], \tag{1.4}$$

$$MSE(t_R) = \gamma S_y^4[(\lambda_{40}-1)+(\lambda_{04}-1)-2(\lambda_{22}-1)], \quad (1.5)$$

Table-1: Ratio type estimators of population variance, their biases and mean squared errors

Estimator	Bias	MSE
$\hat{S}_1^2 = s_y^2\left[\frac{S_x^2+C_x}{s_x^2+C_x}\right]$ Kadilar and Cingi (2006b)	$\gamma S_y^2 R_1\left[\begin{array}{l}R_1(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{l}(\lambda_{40}-1)+R_1^2(\lambda_{04}-1)\\-2R_1(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_2^2 = s_y^2\left[\frac{S_x^2+\beta_{2(x)}}{s_x^2+\beta_{2(x)}}\right]$ Upadhyaya and Singh (1999)	$\gamma S_y^2 R_2\left[\begin{array}{l}R_2(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{l}(\lambda_{40}-1)+R_2^2(\lambda_{04}-1)\\-2R_2(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_3^2 = s_y^2\left[\frac{S_x^2+\beta_{1(x)}}{s_x^2+\beta_{1(x)}}\right]$	$\gamma S_y^2 R_3\left[\begin{array}{l}R_3(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{l}(\lambda_{40}-1)+R_3^2(\lambda_{04}-1)\\-2R_3(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_4^2 = s_y^2\left[\frac{S_x^2+\rho}{s_x^2+\rho}\right]$	$\gamma S_y^2 R_4\left[\begin{array}{l}R_4(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{l}(\lambda_{40}-1)+R_4^2(\lambda_{04}-1)\\-2R_4(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_5^2 = s_y^2\left[\frac{S_x^2+S_x}{s_x^2+S_x}\right]$	$\gamma S_y^2 R_5\left[\begin{array}{l}R_5(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{l}(\lambda_{40}-1)+R_5^2(\lambda_{04}-1)\\-2R_5(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_6^2 = s_y^2\left[\frac{S_x^2+M_d}{s_x^2+M_d}\right]$ Subramani and Kumarpandiyan (2012a)	$\gamma S_y^2 R_6\left[\begin{array}{l}R_6(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{l}(\lambda_{40}-1)+R_6^2(\lambda_{04}-1)\\-2R_6(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_7^2 = s_y^2\left[\frac{S_x^2+Q_1}{s_x^2+Q_1}\right]$ Subramani and Kumarpandiyan (2012b)	$\gamma S_y^2 R_7\left[\begin{array}{l}R_7(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{l}(\lambda_{40}-1)+R_7^2(\lambda_{04}-1)\\-2R_7(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_8^2 = s_y^2\left[\frac{S_x^2+Q_3}{s_x^2+Q_3}\right]$ Subramani and Kumarpandiyan (2012b)	$\gamma S_y^2 R_8\left[\begin{array}{l}R_8(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{l}(\lambda_{40}-1)+R_8^2(\lambda_{04}-1)\\-2R_8(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_9^2 = s_y^2\left[\frac{S_x^2+Q_r}{s_x^2+Q_r}\right]$ Subramani and Kumarpandiyan (2012b)	$\gamma S_y^2 R_9\left[\begin{array}{l}R_9(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{l}(\lambda_{40}-1)+R_9^2(\lambda_{04}-1)\\-2R_9(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_{10}^2 = s_y^2\left[\frac{S_x^2+Q_d}{s_x^2+Q_d}\right]$ Subramani and Kumarpandiyan (2012b)	$\gamma S_y^2 R_{10}\left[\begin{array}{l}R_{10}(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{l}(\lambda_{40}-1)+R_{10}^2(\lambda_{04}-1)\\-2R_{10}(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_{11}^2 = s_y^2\left[\frac{S_x^2+Q_a}{s_x^2+Q_a}\right]$ Subramani and Kumarpandiyan (2012b)	$\gamma S_y^2 R_{11}\left[\begin{array}{l}R_{11}(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{l}(\lambda_{40}-1)+R_{11}^2(\lambda_{04}-1)\\-2R_{11}(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_{12}^2 = s_y^2\left[\frac{S_x^2+D_1}{s_x^2+D_1}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S_y^2 R_{12}\left[\begin{array}{l}R_{12}(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{l}(\lambda_{40}-1)+R_{12}^2(\lambda_{04}-1)\\-2R_{12}(\lambda_{22}-1)\end{array}\right]$
$\hat{S}_{13}^2 = s_y^2\left[\frac{S_x^2+D_2}{s_x^2+D_2}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S_y^2 R_{13}\left[\begin{array}{l}R_{13}(\lambda_{04}-1)\\-(\lambda_{22}-1)\end{array}\right]$	$\gamma S_y^4\left[\begin{array}{l}(\lambda_{40}-1)+R_{13}^2(\lambda_{04}-1)\\-2R_{13}(\lambda_{22}-1)\end{array}\right]$

$\hat{S}^2_{14} = s^2_y\left[\frac{S^2_x + D_3}{s^2_x + D_3}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S^2_y R_{14}\begin{bmatrix} R_{14}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S^4_y\begin{bmatrix}(\lambda_{40}-1)+R^2_{14}(\lambda_{04}-1) \\ -2R_{14}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}^2_{15} = s^2_y\left[\frac{S^2_x + D_4}{s^2_x + D_4}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S^2_y R_{15}\begin{bmatrix} R_{15}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S^4_y\begin{bmatrix}(\lambda_{40}-1)+R^2_{15}(\lambda_{04}-1) \\ -2R_{15}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}^2_{16} = s^2_y\left[\frac{S^2_x + D_5}{s^2_x + D_5}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S^2_y R_{16}\begin{bmatrix} R_{16}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S^4_y\begin{bmatrix}(\lambda_{40}-1)+R^2_{16}(\lambda_{04}-1) \\ -2R_{16}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}^2_{17} = s^2_y\left[\frac{S^2_x + D_6}{s^2_x + D_6}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S^2_y R_{17}\begin{bmatrix} R_{17}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S^4_y\begin{bmatrix}(\lambda_{40}-1)+R^2_{17}(\lambda_{04}-1) \\ -2R_{17}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}^2_{18} = s^2_y\left[\frac{S^2_x + D_7}{s^2_x + D_7}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S^2_y R_{18}\begin{bmatrix} R_{18}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S^4_y\begin{bmatrix}(\lambda_{40}-1)+R^2_{18}(\lambda_{04}-1) \\ -2R_{18}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}^2_{19} = s^2_y\left[\frac{S^2_x + D_8}{s^2_x + D_8}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S^2_y R_{19}\begin{bmatrix} R_{19}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S^4_y\begin{bmatrix}(\lambda_{40}-1)+R^2_{19}(\lambda_{04}-1) \\ -2R_{19}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}^2_{20} = s^2_y\left[\frac{S^2_x + D_9}{s^2_x + D_9}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S^2_y R_{20}\begin{bmatrix} R_{20}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S^4_y\begin{bmatrix}(\lambda_{40}-1)+R^2_{20}(\lambda_{04}-1) \\ -2R_{20}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}^2_{21} = s^2_y\left[\frac{S^2_x + D_{10}}{s^2_x + D_{10}}\right]$ Subramani and Kumarpandiyan (2012c)	$\gamma S^2_y R_{21}\begin{bmatrix} R_{21}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S^4_y\begin{bmatrix}(\lambda_{40}-1)+R^2_{21}(\lambda_{04}-1) \\ -2R_{21}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}^2_{22} = s^2_y\left[\frac{S^2_x \beta_{2(x)} + C_x}{s^2_x \beta_{2(x)} + C_x}\right]$ Kadilar and Cingi (2006b)	$\gamma S^2_y R_{22}\begin{bmatrix} R_{22}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S^4_y\begin{bmatrix}(\lambda_{40}-1)+R^2_{22}(\lambda_{04}-1) \\ -2R_{22}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}^2_{23} = s^2_y\left[\frac{S^2_x C_x + \beta_{2(x)}}{s^2_x C_x + \beta_{2(x)}}\right]$ Kadilar and Cingi (2006b)	$\gamma S^2_y R_{23}\begin{bmatrix} R_{23}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S^4_y\begin{bmatrix}(\lambda_{40}-1)+R^2_{23}(\lambda_{04}-1) \\ -2R_{23}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}^2_{24} = s^2_y\left[\frac{S^2_x \beta_{1(x)} + C_x}{s^2_x \beta_{1(x)} + C_x}\right]$	$\gamma S^2_y R_{24}\begin{bmatrix} R_{24}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S^4_y\begin{bmatrix}(\lambda_{40}-1)+R^2_{24}(\lambda_{04}-1) \\ -2R_{24}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}^2_{25} = s^2_y\left[\frac{S^2_x C_x + \beta_{1(x)}}{s^2_x C_x + \beta_{1(x)}}\right]$	$\gamma S^2_y R_{25}\begin{bmatrix} R_{25}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S^4_y\begin{bmatrix}(\lambda_{40}-1)+R^2_{25}(\lambda_{04}-1) \\ -2R_{25}(\lambda_{22}-1)\end{bmatrix}$
$\hat{S}^2_{26} = s^2_y\left[\frac{S^2_x \rho + C_x}{s^2_x \rho + C_x}\right]$	$\gamma S^2_y R_{26}\begin{bmatrix} R_{26}(\lambda_{04}-1) \\ -(\lambda_{22}-1)\end{bmatrix}$	$\gamma S^4_y\begin{bmatrix}(\lambda_{40}-1)+R^2_{26}(\lambda_{04}-1) \\ -2R_{26}(\lambda_{22}-1)\end{bmatrix}$

$\hat{S}^2_{27} = s^2_y\left[\frac{S^2_x C_x + \rho}{s^2_x C_x + \rho}\right]$	$\gamma S^2_y R_{27}\left[R_{27}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S^4_y\left[(\lambda_{40}-1) + R^2_{27}(\lambda_{04}-1) - 2R_{27}(\lambda_{22}-1)\right]$
$\hat{S}^2_{28} = s^2_y\left[\frac{S^2_x S_x + C_x}{s^2_x S_x + C_x}\right]$	$\gamma S^2_y R_{28}\left[R_{28}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S^4_y\left[(\lambda_{40}-1) + R^2_{28}(\lambda_{04}-1) - 2R_{28}(\lambda_{22}-1)\right]$
$\hat{S}^2_{29} = s^2_y\left[\frac{S^2_x C_x + S_x}{s^2_x C_x + S_x}\right]$	$\gamma S^2_y R_{29}\left[R_{29}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S^4_y\left[(\lambda_{40}-1) + R^2_{29}(\lambda_{04}-1) - 2R_{29}(\lambda_{22}-1)\right]$
$\hat{S}^2_{30} = s^2_y\left[\frac{S^2_x M_d + C_x}{s^2_x M_d + C_x}\right]$	$\gamma S^2_y R_{30}\left[R_{30}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S^4_y\left[(\lambda_{40}-1) + R^2_{30}(\lambda_{04}-1) - 2R_{30}(\lambda_{22}-1)\right]$
$\hat{S}^2_{31} = s^2_y\left[\frac{S^2_x C_x + M_d}{s^2_x C_x + M_d}\right]$	$\gamma S^2_y R_{31}\left[R_{31}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S^4_y\left[(\lambda_{40}-1) + R^2_{31}(\lambda_{04}-1) - 2R_{31}(\lambda_{22}-1)\right]$
Subramani and Kumarpandiyan (2013)		
$\hat{S}^2_{32} = s^2_y\left[\frac{S^2_x \beta_{1(x)} + \beta_{2(x)}}{s^2_x \beta_{1(x)} + \beta_{2(x)}}\right]$	$\gamma S^2_y R_{32}\left[R_{32}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S^4_y\left[(\lambda_{40}-1) + R^2_{32}(\lambda_{04}-1) - 2R_{32}(\lambda_{22}-1)\right]$
$\hat{S}^2_{33} = s^2_y\left[\frac{S^2_x \beta_{2(x)} + \beta_{1(x)}}{s^2_x \beta_{2(x)} + \beta_{1(x)}}\right]$	$\gamma S^2_y R_{33}\left[R_{33}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S^4_y\left[(\lambda_{40}-1) + R^2_{33}(\lambda_{04}-1) - 2R_{33}(\lambda_{22}-1)\right]$
$\hat{S}^2_{34} = s^2_y\left[\frac{S^2_x \rho + \beta_{2(x)}}{s^2_x \rho + \beta_{2(x)}}\right]$	$\gamma S^2_y R_{34}\left[R_{34}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S^4_y\left[(\lambda_{40}-1) + R^2_{34}(\lambda_{04}-1) - 2R_{34}(\lambda_{22}-1)\right]$
$\hat{S}^2_{35} = s^2_y\left[\frac{S^2_x \beta_{2(x)} + \rho}{s^2_x \beta_{2(x)} + \rho}\right]$	$\gamma S^2_y R_{35}\left[R_{35}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S^4_y\left[(\lambda_{40}-1) + R^2_{35}(\lambda_{04}-1) - 2R_{35}(\lambda_{22}-1)\right]$
$\hat{S}^2_{36} = s^2_y\left[\frac{S^2_x S_x + \beta_{2(x)}}{s^2_x S_x + \beta_{2(x)}}\right]$	$\gamma S^2_y R_{36}\left[R_{36}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S^4_y\left[(\lambda_{40}-1) + R^2_{36}(\lambda_{04}-1) - 2R_{36}(\lambda_{22}-1)\right]$
$\hat{S}^2_{37} = s^2_y\left[\frac{S^2_x \beta_{2(x)} + S_x}{s^2_x \beta_{2(x)} + S_x}\right]$	$\gamma S^2_y R_{37}\left[R_{37}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S^4_y\left[(\lambda_{40}-1) + R^2_{37}(\lambda_{04}-1) - 2R_{37}(\lambda_{22}-1)\right]$
$\hat{S}^2_{38} = s^2_y\left[\frac{S^2_x M_d + \beta_{2(x)}}{s^2_x M_d + \beta_{2(x)}}\right]$	$\gamma S^2_y R_{38}\left[R_{38}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S^4_y\left[(\lambda_{40}-1) + R^2_{38}(\lambda_{04}-1) - 2R_{38}(\lambda_{22}-1)\right]$
$\hat{S}^2_{39} = s^2_y\left[\frac{S^2_x \beta_{2(x)} + M_d}{s^2_x \beta_{2(x)} + M_d}\right]$	$\gamma S^2_y R_{39}\left[R_{39}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S^4_y\left[(\lambda_{40}-1) + R^2_{39}(\lambda_{04}-1) - 2R_{39}(\lambda_{22}-1)\right]$
$\hat{S}^2_{40} = s^2_y\left[\frac{S^2_x \rho + \beta_{1(x)}}{s^2_x \rho + \beta_{1(x)}}\right]$	$\gamma S^2_y R_{40}\left[R_{40}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S^4_y\left[(\lambda_{40}-1) + R^2_{40}(\lambda_{04}-1) - 2R_{40}(\lambda_{22}-1)\right]$
$\hat{S}^2_{41} = s^2_y\left[\frac{S^2_x \beta_{1(x)} + \rho}{s^2_x \beta_{1(x)} + \rho}\right]$	$\gamma S^2_y R_{41}\left[R_{41}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S^4_y\left[(\lambda_{40}-1) + R^2_{41}(\lambda_{04}-1) - 2R_{41}(\lambda_{22}-1)\right]$
$\hat{S}^2_{42} = s^2_y\left[\frac{S^2_x S_x + \beta_{1(x)}}{s^2_x S_x + \beta_{1(x)}}\right]$	$\gamma S^2_y R_{42}\left[R_{42}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S^4_y\left[(\lambda_{40}-1) + R^2_{42}(\lambda_{04}-1) - 2R_{42}(\lambda_{22}-1)\right]$
$\hat{S}^2_{43} = s^2_y\left[\frac{S^2_x \beta_{1(x)} + S_x}{s^2_x \beta_{1(x)} + S_x}\right]$	$\gamma S^2_y R_{43}\left[R_{43}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S^4_y\left[(\lambda_{40}-1) + R^2_{43}(\lambda_{04}-1) - 2R_{43}(\lambda_{22}-1)\right]$

$\hat{S}^2_{44} = s_y^2\left[\frac{S_x^2 M_d + \beta_{1(x)}}{s_x^2 M_d + \beta_{1(x)}}\right]$	$\gamma S_y^2 R_{44}\left[R_{44}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S_y^4\left[(\lambda_{40}-1) + R_{44}^2(\lambda_{04}-1) - 2R_{44}(\lambda_{22}-1)\right]$
$\hat{S}^2_{45} = s_y^2\left[\frac{S_x^2 \beta_{1(x)} + M_d}{s_x^2 \beta_{1(x)} + M_d}\right]$	$\gamma S_y^2 R_{45}\left[R_{45}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S_y^4\left[(\lambda_{40}-1) + R_{45}^2(\lambda_{04}-1) - 2R_{45}(\lambda_{22}-1)\right]$
$\hat{S}^2_{46} = s_y^2\left[\frac{S_x^2 S_x + \rho}{s_x^2 S_x + \rho}\right]$	$\gamma S_y^2 R_{46}\left[R_{46}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S_y^4\left[(\lambda_{40}-1) + R_{46}^2(\lambda_{04}-1) - 2R_{46}(\lambda_{22}-1)\right]$
$\hat{S}^2_{47} = s_y^2\left[\frac{S_x^2 \rho + S_x}{s_x^2 \rho + S_x}\right]$	$\gamma S_y^2 R_{47}\left[R_{47}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S_y^4\left[(\lambda_{40}-1) + R_{47}^2(\lambda_{04}-1) - 2R_{47}(\lambda_{22}-1)\right]$
$\hat{S}^2_{48} = s_y^2\left[\frac{S_x^2 M_d + \rho}{s_x^2 M_d + \rho}\right]$	$\gamma S_y^2 R_{48}\left[R_{48}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S_y^4\left[(\lambda_{40}-1) + R_{48}^2(\lambda_{04}-1) - 2R_{48}(\lambda_{22}-1)\right]$
$\hat{S}^2_{49} = s_y^2\left[\frac{S_x^2 \rho + M_d}{s_x^2 \rho + M_d}\right]$	$\gamma S_y^2 R_{49}\left[R_{49}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S_y^4\left[(\lambda_{40}-1) + R_{49}^2(\lambda_{04}-1) - 2R_{49}(\lambda_{22}-1)\right]$
$\hat{S}^2_{50} = s_y^2\left[\frac{S_x^2 M_d + S_x}{s_x^2 M_d + S_x}\right]$	$\gamma S_y^2 R_{50}\left[R_{50}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S_y^4\left[(\lambda_{40}-1) + R_{50}^2(\lambda_{04}-1) - 2R_{50}(\lambda_{22}-1)\right]$
$\hat{S}^2_{51} = s_y^2\left[\frac{S_x^2 S_x + M_d}{s_x^2 S_x + M_d}\right]$	$\gamma S_y^2 R_{51}\left[R_{51}(\lambda_{04}-1) - (\lambda_{22}-1)\right]$	$\gamma S_y^4\left[(\lambda_{40}-1) + R_{51}^2(\lambda_{04}-1) - 2R_{51}(\lambda_{22}-1)\right]$

where, $R_i = \frac{S_x^2}{S_x^2 + w_i}$, $i = 1, 2, ..., 51$ and $w_1 = C_x$, $w_2 = \beta_{2(x)}$, $w_3 = \beta_{1(x)}$, $w_4 = \rho$, $w_5 = S_x$, $w_6 = M_d$, $w_7 = Q_1$, $w_8 = Q_3$, $w_9 = Q_r$, $w_{10} = Q_d$, $w_{11} = Q_a$, $w_{12} = D_1$, $w_{13} = D_2$, $w_{14} = D_3$, $w_{15} = D_4$, $w_{16} = D_5$, $w_{17} = D_6$, $w_{18} = D_7$, $w_{19} = D_8$, $w_{20} = D_9$, $w_{21} = D_{10}$, $w_{22} = \frac{C_x}{\beta_{2(x)}}$, $w_{23} = \frac{\beta_{2(x)}}{C_x}$, $w_{24} = \frac{C_x}{\beta_{1(x)}}$, $w_{25} = \frac{\beta_{1(x)}}{C_x}$, $w_{26} = \frac{C_x}{\rho}$, $w_{27} = \frac{\rho}{C_x}$, $w_{28} = \frac{C_x}{S_x}$, $w_{29} = \frac{S_x}{C_x}$, $w_{30} = \frac{C_x}{M_d}$, $w_{31} = \frac{M_d}{C_x}$, $w_{32} = \frac{\beta_{2(x)}}{\beta_{1(x)}}$, $w_{33} = \frac{\beta_{1(x)}}{\beta_{2(x)}}$, $w_{34} = \frac{\beta_{2(x)}}{\rho}$, $w_{35} = \frac{\rho}{\beta_{2(x)}}$, $w_{36} = \frac{\beta_{2(x)}}{S_x}$, $w_{37} = \frac{S_x}{\beta_{2(x)}}$, $w_{38} = \frac{\beta_{2(x)}}{M_d}$, $w_{39} = \frac{M_d}{\beta_{2(x)}}$, $w_{40} = \frac{\beta_{1(x)}}{\rho}$, $w_{41} = \frac{\rho}{\beta_{1(x)}}$, $w_{42} = \frac{\beta_{1(x)}}{S_x}$, $w_{43} = \frac{S_x}{\beta_{1(x)}}$, $w_{44} = \frac{\beta_{1(x)}}{M_d}$, $w_{45} = \frac{M_d}{\beta_{1(x)}}$, $w_{46} = \frac{\rho}{S_x}$, $w_{47} = \frac{S_x}{\rho}$, $w_{48} = \frac{\beta_{1(x)}}{M_d}$, $w_{49} = \frac{M_d}{\beta_{1(x)}}$, $w_{50} = \frac{S_x}{M_d}$, $w_{51} = \frac{M_d}{S_x}$.
Thus, the bias and MSE of 51 estimators may be written as,

$$B(\hat{S}_i^2) = \gamma S_y^2 R_i[R_i(\lambda_{04} - 1) - (\lambda_{22} - 1), i = 1, 2, ..., 51]$$

$$MSE(\hat{S}_i^2) = \gamma S_y^4[(\lambda_{04} - 1) + R_i^2(\lambda_{04} - 1) - 2R_i(\lambda_{22} - 1)], i = 1, 2, ..., 51 \quad (2.6)$$

2. Conclusion

In the present short review of the estimators of the population variance, we have considered the ratio type estimators of population variance using positively correlated auxiliary variable. Various ratio type estimators have been given in a chronological order. The biases and mean squared errors of all the estimators have been given up to the first order of approximation. The improvement in estimators can be justified through numerical examples. Since all the estimators of population variance have been given in chronological order, this short review will be very beneficial for the researchers working in the field of sampling especially for the estimation of population variance using information on a single auxiliary variable.

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