J. of Ramanujan Society of Math. and Math. Sc. **ISSN : 2319-1023**
Vol.5, No.2 (2016), pp. 135-138

A STUDY OF ANALYTIC K-TORSE FORMING VECTOR FIELD

Rajeev Kumar Singh,

Department of Mathematics,
PBPG College, Pratapgarh-230001, (UP), India
E-mail: dr.rajeevthakur2012@gmail.com

Abstract: In this paper, we discuss about the K-torse forming vector field analytic.

Keywords and Phrases: Kaehlerian space, vector field, covariant differentiation, Christoffel symbols.

Mathematics Subject Classification: 53B20, 53C55.

1. Introduction, Notations and Definitions

In a Kaehlerian space, a vector field V is said to be K-torse forming vector field [1], provided it satisfies the equation

$$\nabla_j V^h = a\delta_j^h + bF_j^h + \alpha_j V^h + \beta_j \tilde{V}^h \tag{1}$$

where $\tilde{V}^h = F_r^h V^r$, a and b are suitable functions and α, β are certain 1- forms and ∇_j stands for the operator of covariant differentiation with respect to the Christoffel symbols of the Kaehlerian manifolds.

In order that the K-torse forming vector field analytic, it has been proved that it is necessary and sufficient that

$$\beta_j = -F_j^r \alpha_r = \tilde{\alpha}_j$$

and so for an analytic K-torse forming vector field, we have

$$\nabla_j V^h = a\delta_j^h + bF_j^h + \alpha_j V^h + \tilde{\alpha}_j \tilde{V}^h \tag{2}$$

Now on substituting from

$$\Gamma_{ji}^h = \left\{ \begin{matrix} h \\ ji \end{matrix} \right\} + \delta_j^h p_i + \delta_i^h p_j - g_{ji} p^h + F_j^h q_i - F_{ji} q^h$$

into

$$D_j V^h = \partial_j K^h + \Gamma^h_{ji} V'$$

We find that

$$D_K V^h = \nabla_K V^h + \delta^h_K p_m V^m + p_k V^h - V_K p^h + F^h_K q_m V^m + q_K \tilde{V}^h - F_{Km} q^h V^m$$

and consequently on substituting from (2) in the above relation, we find after simplification

$$D_K V^h = c\delta^h_K + dF^h_K + \gamma_K V^h + \tilde{\gamma}_K \tilde{V}^h - (V_K p^h + F_{Km} q^h V^m) \tag{3}$$

where we have put

$$c = a + p_m V^m$$

$$d = b + q_m V^m$$

$$\beta_K = \alpha_K + p_K$$

and

$$\tilde{\beta}_K = -F^r_K \beta_r = F^r_K(\alpha_r + p_r) = \tilde{\alpha}_K + q_K$$

2. Results and Discussion

Theorem 1:

In order that an analytic K-torse forming vector field V with respect to $\left\{ \begin{matrix} h \\ ji \end{matrix} \right\}$ is analytic K-torse with respect to the complex conformal connection also, it is necessary and sufficient that the outer product $V_K p^h$ of V with associated 1- forms p of complex conformal connection is pure in h and K.

As the purity of tensor $V_K p^h$ in the indices h and K implies the hybridness of $V_K p^h$ in the same indices and vice versa too we have.

Theorem 2:

In order that an analytic K-torse forming vector field V with respect to $\left\{ \begin{matrix} h \\ ji \end{matrix} \right\}$ is analytic K-torse forming with respect to the complex conformal connection also, it is necessary and sufficient that the tensor $V_k p_h$ is hybrid in the indices h and K. On substituting from

$$\overset{1}{\Gamma}{}^h_{ji} = \left\{ \begin{matrix} h \\ ji \end{matrix} \right\} + \delta^h_j p_i - g_{ji} p^h + F^h_j q_i + F^h_i q_j - F_{ji} q^h$$

into

$$\overset{1}{D_j}\; V^h = \partial_j V^h + \overset{1}{\Gamma^h_{ji}}\; V^i$$

and using equation (2) we find

$$\overset{1}{D_j}\; V^h = a\delta^h_K + F^h_K(b + q_m V^m) + V^h(\alpha_K + p_K) + \tilde{V}^h(\tilde{\alpha}_K + q_K)$$

$$+F^h_K q_m V^m - (\delta^r_K \delta^h_t - F^r_K F^h_t)V_r p^t \tag{4}$$

Thus if we assume that the vector field is analytic K-torse forming with respect to special semi symmetric metric F-connection $\overset{1}{\Gamma^h_{ji}}$, then we find that

$$F^h_K q_m V^m - (\delta^r_K \delta^h_t - F^r_K F^h_t)V_r p^t = 0$$

or

$$F^h_K q_m V^m - (O^{rh}_{kt})V_r p^t = 0$$

or

$$q_m V^m = (O^{rh}_{kt} V_r p^t) F^k_h \tag{5}$$

and conversely if (5) holds, we find that $\overset{1}{D_K}\; V^h$ is given by

$$\overset{1}{D_K}\; V^h = a\delta^K_h + F^h_K(b + q_m V^m) + v^h(\alpha_K - p_K) + \tilde{V}^h(\tilde{\alpha}_K + q_K) \tag{6}$$

Hence vector field is analytic K-torse forming vector field respect to the special semi symmetric metric F-connection $\overset{1}{\Gamma^h_{ji}}$ also and therefore we have

Theorem 3:

In order that an analytic K-torse forming vector field with respect to $\left\{ \begin{matrix} h \\ ji \end{matrix} \right\}$ is analytic K-torse forming with respect to special semi symmetric metric F connection $\overset{1}{\Gamma^h_{ji}}$ it is necessary and sufficient that equation (5) holds good.

Theorem 4:

A process similar to above by considering the special semi symmetric metric tensor of second kind i.e. $\overset{2}{\Gamma}{}^h_{ji}$ given by

$$\overset{2}{\Gamma}{}^h_{ji} = \left\{ \begin{matrix} h \\ ji \end{matrix} \right\} + \delta^h_j p_i - g_{ji} p^h + F^h_j q_i - F_{ji} q^h$$

yields

$$\overset{2}{D_j} V^h = c\delta^h_j + dF^h_j + \alpha_j V^h + \tilde{\alpha}_j \tilde{V}^h - {}^*O^{mh}_{jr} V_m p^r \quad (7)$$

where the constant c and d are same as given in (3). From (7) we find that when

$${}^*O^{mh}_{jr} V_m p^r = 0 \quad (8)$$

The tensor $V_m p^r$ is pure in m and r or $V_m p^r$ is hybrid in m and r, then

$$\overset{2}{D_j} V^h = c\delta^h_j + dF^h_j + \alpha_j V^h + \tilde{\alpha}_j \tilde{V}^h \quad (9)$$

which shows that the vector field is analytic K-torse forming with respect to $\overset{2}{\Gamma}{}^h_{ji}$ also.

Conversely when equation (9) holds i.e. the vector field become analytic K-torse forming with respect to special semi symmetric metric connection of second kind theorem, from (7) we find that (8) must hold.

References

[1] Yamagachi, S., Holomorphic subplaner curves in Kaehlerian manifold TRU Mathematics, 13-2 (1977), 41-55.

[2] Yamagachi, S. and Sato, S., Remarks on analytic HSP transformation in Kaehler manifolds, TRU Maths. 13-2 (1977), 57-63.

[3] Yano, K., On complex conformal connections, Kodai Math sem. Rep. 26 (1975), 137-151.

[4] Yano, K. and Imai, K., On semi symmetric metric F-connection tensor, H.S. 29 (1975), 134-138.

[5] Yano, K., On semi symmetric metric connection, Revue Roumanine de Mathematiques pures et. appliquces 15 (1970), 1578-1581.