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# PROPERTY-LOADED VERTEX COLORINGS OF A HYPERGRAPH

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***Dedicated to Prof. M.A. Pathan on his $75^{th}$ birth anniversary***

**Abstract:** Given a hypergraph $H = (X, \mathcal{E})$, an integer $k \geq 1$ and a property $\mathcal{P}$, of subsets of $X$, a $(\mathcal{P}, k)$-*coloring* of $H$ is a function $\pi : X \to \{1, 2, \ldots, k\}$=: $k$ such that for all $i \in k$ the induced subhypergraph $\langle \pi^{-1}(i) \rangle_H \in \bar{\mathcal{P}}$, where $\bar{\mathcal{P}}$ denotes the set of all subsets of $X$ that do not possess the property $\mathcal{P}$. The hypergraph $H$ is $(\mathcal{P}, k)$-*colorable* if and only if it has a $(\mathcal{P}, k)$-coloring. The $\mathcal{P}$-*chromatic number* $\chi_{\mathcal{P}}(H)$ of $H$ is then defined as the least $k$ such that $H$ has a $(\mathcal{P}, k)$-coloring. In this note, we initiate a study of $\chi_{\mathcal{P}}(H)$ for hereditary properties $\mathcal{P}$. For non-hereditary properties, the study appears challenging.

**Keywords:** hypergraph, coloring, domination, stability, hereditary property, supra-hereditary property, $\mathcal{P}$-*chromatic*, enclaveless set.

**AMS Subject Classification:** 80 05 C

## 1. Introduction

For all terminology and notation in the theories of graphs and hypergraphs we refer the reader to Harary [5] and Berge [4], respectively. The hypergraphs considered here are more general in that, unlike in [4], they may have *isolates*, that is, the set $Y$ of vertices that are not contained any edge of the hypergraph; this fundamental difference was first noticed and hypergraphs were treated accordingly in [1].

Hypergraphs are a natural generalization of undirected graphs in which edges may consist of more than 2 vertices. More precisely, a (finite) hypergraph $H = (V, E)$ is a pair $\{X, H\}$ where $H = \{E_1, E_2, \ldots, E_q\}$ is a set of subsets of $X$ such that $E_i \neq \emptyset$ for all $i$, and $\bigcup_{i=1}^{q} E_i = X$, consisting of $p$ vertices and $q$ edges; if $p = 0$ then $H$ is called the *null hypergraph* and is denoted by $K_0$. The elements of $V$ are called vertices and the elements of $E$ are called hyper- edges, or simply edges of