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A note on some summations due to Ramanujan, their generalization and some allied series

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Abstract: In this short note, we aim to discuss some summations due to Ramanujan, their generalizations and some allied series.
Keywords and Phrases: generalized hypergeometric series, Gauss summation theorem, Karlsson-Minton summation formula.

1. Introduction

We start with the following summations due to Ramanujan [6]

$$1+\frac{1}{5}\left(\frac{1}{2}\right)^2+\frac{1}{9}\left(\frac{1\cdot 3}{2\cdot 4}\right)^2+\cdots=\frac{\pi^2}{4\Gamma^2(\frac{3}{4})} \tag{1.1}$$

and

$$1+\frac{1}{5^2}\left(\frac{1}{2}\right)+\frac{1}{9^2}\left(\frac{1\cdot 3}{2\cdot 4}\right)+\cdots=\frac{\pi^{5/2}}{8\sqrt{2}\Gamma^2(\frac{3}{4})}. \tag{1.2}$$

As pointed out by Berndt [1] the above summations can be obtained quite simply by putting (i) $a=b=\frac{1}{2}, c=\frac{1}{4}$ and (ii) $a=\frac{1}{2}, b=c=\frac{1}{4}$ in Dixon's summation theorem [8, p.52] for the ${}_3F_2$ series, viz.

$${}_3F_2\left[\begin{array}{l} a,b,c;1 \\ 1+a-b,1+a-c \end{array}\right]=\frac{\Gamma(1+\frac{1}{2}a)\Gamma(1+a-b)\Gamma(1+a-c)\Gamma(1+\frac{1}{2}a-b-c)}{\Gamma(1+a)\Gamma(1+\frac{1}{2}a-b)\Gamma(1+\frac{1}{2}a-c)\Gamma(1+a-b-c)}$$

valid provided $Re(\frac{1}{2}a-b-c)>-1$.

A similar series evaluation

$$1+\frac{1}{5}\left(\frac{1}{2}\right)+\frac{1}{9}\left(\frac{1.3}{2.4}\right)+...=\frac{\pi^{3/2}}{2\sqrt{2}\Gamma^2(\frac{3}{4})} \tag{1.3}$$