

CHARACTERIZATION OF IDEMPOTENCY IN POWER-ASSOCIATIVE RINGS

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Abstract: In this paper, we extend Mosaic's result for idempotency in associative rings to power-associative rings. We provide a necessary and sufficient condition for idempotency and give some examples.

Keywords and Phrases: Ring, power-associative, idempotent.

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1. Introduction

In this article we provide a necessary and sufficient condition for idempotency in power-associative rings, hence extending Mosaic's result in [5]. Mosaic gives the relation between idempotent and tripotent elements in an associative ring R , generalizing the result on matrices by Trenkler and Baksalary [8]. Namely, for any $x \in R$, where 2, 3 are invertible, x is idempotent if and only if x is tripotent and $1 - x$ is tripotent or $1 + x$ is invertible.

In [1], we pointed out that even though $\mathbb{O}/\mathbb{Z}_p^{1,2}$ is not associative, the result does hold in some cases. For example, consider the tripotent $x = 4 + 3e_1 + e_2 + 4e_3$ in \mathbb{O}/\mathbb{Z}_7 , which is also an idempotent. It is not hard to check this directly or using