

**APPROXIMATION, EXISTENCE AND UNIQUENESS OF THE
INTEGRABLE LOCAL SOLUTION OF NONLINEAR HYBRID
FUNCTIONAL INTEGRAL EQUATIONS**

Janhavi B. Dhage and Bapurao C. Dhage

Kasubai, Gurukul Colony,
Ahmedpur - 413515, Latur, Maharashtra, INDIA
E-mail : jbdhage@gmail.com, bcdhage@gmail.com

(Received: Jul. 03, 2025 Accepted: Nov. 30, 2025 Published: Dec. 30, 2025)

Abstract: In this paper, we prove a couple of approximation results for existence and uniqueness of the integrable local solutions of nonhomogeneous nonlinear hybrid functional integral equations under weaker partial compactness, partial Lipschitz and usual monotonicity type conditions. We employ the Dhage monotone iteration method based on the recent hybrid fixed point theorems of Dhage (2024) while establishing our main results of this paper. Our hypotheses and abstract results are also illustrated with a couple of numerical examples.

Keywords and Phrases: Functional integral equation, Hybrid fixed point principle, Dhage iteration method; Approximation result, Existence and uniqueness theorem.

2020 Mathematics Subject Classification: Primary 45G10, Secondary 47H10.

1. Introduction

Theoretical approximation results for existence and uniqueness of continuous and integrable local solutions for nonlinear differential and integral equations can be obtained under usual Lipschitz condition on the nonlinearity or monotonicity condition blending with the existence of upper and lower solutions of the related nonlinear problems. These results are achieved by the applications of Banach fixed point theorem or by monotone iteration method given in Ladde *et al.* [25] or generalized iteration method as depicted in Hekila and Lakshmikantham [26].