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ON NEW FOUR-TERM RECURRENCE RELATIONS FOR THE 3-j COEFFICIENT

R. Subramanian, K. V. Murugan and K. Srinivasa Rao*

Department of Mathematics,
Sri Sairam Engineering College,
Chennai - 600044, INDIA

E-mail : subramanian.math@sairam.edu.in, murugan.math@sairam.edu.in

*The Institute of Mathematical Sciences, Chennai - 600113, INDIA

E-mail : ksrao18@gmail.com

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Abstract: In our earlier article [9] "On new three-term recurrence relations for the 3-j coefficient", we derived six new three-term recurrence relations of fundamental importance in the Quantum Theory of Angular Momentum. In this article, we derive new four term recurrence relations for the 3-j coefficient, as a direct consequence of the recurrence relations for the ${}_3F_2(\mathbf{a};\ \mathbf{b};\ z)$ given in Tamara Antonova, Roman Dymtryshyn and Serhii Sharyn (2021) [1]. The derived 4-term recurrence relations for the 3-j coefficient are **new**.

Keywords and Phrases: Generalized hypergeometric series, Angular momentum coupling coefficient, Clebsch-Gordan, or 3-j coefficient, recurrence relations.

2020 Mathematics Subject Classification: 33C20, 33C90.

1. Introduction

It has been shown [7] that

$$\begin{aligned}\begin{pmatrix} j_1 & j_2 & j_3 \\ m_1 & m_2 & m_3 \end{pmatrix} &= \delta_{m_1+m_2+m_3,0}(-1)^{j_1-j_2-m_3}\prod_{i,k=1}^{3}\left[\frac{R_{ik}!}{(J+1)!}\right]^{1/2} \\ &\quad\times(-1)^{\sigma(pqr)}\ [\Gamma(1-A,1-B,1-C,D,E)]^{-1} \\ &\quad\times\ {}_3F_2(A,\ B,\ C;\ D,\ E;\ 1)\end{aligned} \tag{1}$$