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A NOTE ON UNIQUENESS OF UNIFORM NORM PROPERTY IN THE BEURLING ALGEBRA $L^1(G_1 \times G_2, \omega)$

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Abstract: Let G_1 and G_2 be LCA groups with identities being e_1 and e_2, and let ω be a (Borel measurable) weight function on $G_1 \times G_2$. Let $\overline{\omega}(s,t) = \omega(s,e_2)\omega(e_1,t)$ $((s,t) \in G_1 \times G_2)$. Then $\overline{\omega}$ is also a weight function on $G_1 \times G_2$. In this small note, it is proved that the Beurling algebra $L^1(G_1 \times G_2, \omega)$ has unique uniform norm property iff $L^1(G_1 \times G_2, \overline{\omega})$ has the same property. This result is important because the above statement does not hold true for some properties.

Keywords and Phrases: LCA Group, Weight, Beurling Algebra, and Unique Uniform Norm Property (UUNP).

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1. Introduction

An algebra norm (not necessarily complete) $|\cdot|$ on an algebra $\mathcal{A}$ is a *uniform norm* if it satisfies the square property $|a^2| = |a|^2$ $(a \in A)$. For example, if $\mathcal{A}$ is semisimple and commutative, then the spectral radius $r_{\mathcal{A}}(\cdot)$ is a uniform norm on $\mathcal{A}$. By the spectral radius formula, we can show that any two equivalent uniform norms on $\mathcal{A}$ are identical. If $\mathcal{A}$ admits at least one uniform norm, then it must be commutative and semisimple. So throughout $\mathcal{A}$ is assumed to be semisimple and commutative. The $\mathcal{A}$ has *unique uniform norm property (UUNP)* if it admits exactly one uniform norm. This property was introduced and studied extensively by Bhatt and Dedania (see [2], [3], [5]). Dabhi and Dedania proved one surprising