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PROPERTIES OF NEUTROSOPHIC b-CONNECTED SPACES

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Abstract: In this paper, we study the neutrosophic b-open sets in a neutrosophic topological spaces and develop some of their properties. Using these neutrosophic b-open sets, we define the neutrosophic b-connected spaces and neutrosophic b-seperated sets and develop some of their important properties.

Keywords and Phrases: Neutrosophic topological spaces, neutrosophic b-open sets, neutrosophic b-connected spaces, neutrosophic b-seperated sets.

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1. Introduction

In many real life problems, we deals with uncertainties. Solving these problems by traditional mathematical models, we face difficulties. There are approaches such as fuzzy sets [15], intuitionistic fuzzy sets [2], vague sets [4], rough sets [7], and soft sets [6] used as mathematical tools to deal with these ambiguous data. Smarandache [13] studied the idea of neutrosophic sets as an approach for solving issues which deals with unreliable, indeterminacy and persistent data. Wang et al. [14] introduced single valued neutrosophic sets. Peng et al. [8] studied operations of neutrosophic numbers and introduced the idea of neutrosophic numbers. Neutrosophic topological space was introduced by Salama et.al. [9] in 2012 and this was studied in many articles such as [10]- [12].

Properties of the neutrosophic topological space is studied through the neutrosophic open sets, neutrosophic closed sets, neutrosophic interior operators and neutrosophic closure operators. There are many kind of open sets defined in the neutrosophic topological space. Here we considered one a kind of neutrosophic open set called neutrosophic b-open set. This notion was introduced by Evanzalin Ebenanjar et.al. [3].

Connectedness is one of the important topological properties which is used to classify and describe topological spaces. This is used as an important assumption in many applications such as the intermediate value theorem. Connectedness in neutrosophic topological spaces was defined by Ahu Acikgoz and Ferhat Esenbel in [1]. A neutrosophic connected space is defined to be a neutrosophic topological which does not has no proper neutrosophic clopen set (neutrosphic closed and open set) [1]. A neutrosophic semi-connected space was defined via neutrosophic semi-open set by Iswarya and Bageerathi in [5]. Here the neutrosophic semi-connected space and neutrosophic semi-disconnected space were defined and their properties were established. In this paper, we define the neutrosophic b-connected space via neutrosophic b-open set and develope some of their properties. We show that in a neutrosophic b-connected space, the empty set and the whole set are the only neutrosophic sets which are both neutrosophic b-open and neutrosophic b-closed. This result is the definition of the neutrosophic connected space in [1]. It was shown in [3] that every neutrosophic semi-open set is a neutrosophic b-open set. Therefore, it is clear that the neutrosophic b-connected space is finer than the neutrosophic semi-connected space.

2. Preliminaries

Definition 2.1. [9] *Let X be a fixed set. A neutrosophic set A is an object having the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$, where $\mu_A(x), \sigma_A(x)$ and $\gamma_A(x)$ represents the degree of membership function, the degree of indeterminacy function and the degree of non-membership function respectively for each element $x \in X$ to the set A.*

Definition 2.2. [9] *Let $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ be a neutrosophic set on X. The complement of the set A may be defined as in the following three different ways:*

1. $C(A) = \{\langle x, 1 - \mu_A(x), 1 - \sigma_A(x), 1 - \gamma_A(x)\rangle : x \in X\}$

2. $C(A) = \{\langle x, \gamma_A(x), \sigma_A(x), \mu_A(x)\rangle : x \in X\}$

3. $C(A) = \{\langle x, \gamma_A(x), 1 - \sigma_A(x), \mu_A(x)\rangle : x \in X\}$

Definition 2.3. [9] *Let X be a set, and let the neutrosophic sets A and B be in the form $A = \{\langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle : x \in X\}$ and $B = \{\langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle : x \in X\}$. Then, we may consider two possible definitions for subsets. That is, $A \subseteq B$ may be defined as in the following two different ways:*

1. $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \leq \sigma_B(x)$ *and* $\gamma_A(x) \geq \gamma_B(x) \forall x \in X$

2. $A \subseteq B \Leftrightarrow \mu_A(x) \leq \mu_B(x), \sigma_A(x) \geq \sigma_B(x)$ *and* $\gamma_A(x) \geq \gamma_B(x) \forall x \in X$

Definition 2.4. [9] *Let X be a set, and let $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle$, $B = \langle x, \mu_B(x), \sigma_B(x), \gamma_B(x)\rangle$ be neutrosophic sets. Then,*
1.$A \cap B$ *may be defined as in the following in two different ways:*

1. $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ *and* $\gamma_A(x) \vee \gamma_B(x)\rangle$

2. $A \cap B = \langle x, \mu_A(x) \wedge \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ *and* $\gamma_A(x) \vee \gamma_B(x)\rangle$

2.$A \cup B$ *may be defined as in the following in two different ways:*

1. $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \vee \sigma_B(x)$ *and* $\gamma_A(x) \wedge \gamma_B(x)\rangle$

2. $A \cup B = \langle x, \mu_A(x) \vee \mu_B(x), \sigma_A(x) \wedge \sigma_B(x)$ *and* $\gamma_A(x) \wedge \gamma_B(x)\rangle$

Here the notations $\wedge$ and $\vee$ means the minimum and the maximum respectively.

Definition 2.5. [9] *A neutrosophic topology for a set X is a family τ of neutrosophic subsets in X satisfying the following axioms :*

1. $0_N, 1_N \in \tau$,

2. $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$,

3. $\cup G_i \in \tau$ *for every* $\{G_i : i \in J\} \subseteq \tau$.

The pair (X, τ) is called a neutrosophic topological space.

The elements of τ are called neutrosophic open sets. The complement of a neutrosophic open set is called a neutrosophic closed set.

Definition 2.6. [9] *Let (X, τ) be neutrosophic topological space and let $A = \langle x, \mu_A(x), \sigma_A(x), \gamma_A(x)\rangle$ be a neutrosophic set in X. Then, the neutrosophic closure of A is defined by $Ncl(A) = \cap\{K : K$ is a neutrosophic closed set in X*

and $A \subseteq K\}$ *and the neutrosophic interior of* A *is defined by* $Nint(A) = \cup\{G : G$ *is a neutrosophic open set in* X *and* $G \subseteq A\}$.

It can be also shown that $Ncl(A)$ is neutrosophic closed set and $Nint(A)$ is a neutrosophic open set in X. Further, we have that A is a neutrosophic open set if and only if $A = Nint(A)$ and A is a neutrosophic closed set if and only if $A = Ncl(A)$.

Definition 2.7. [3] *A neutrosophic set* U *in a neutrosophic topological space* X *is called a neutrosophic b-open set if* $U \subseteq Nint(Ncl(U)) \cup Ncl(Nint(U))$.
Further, F *is called a neutrosophic b-closed set if* $F \supseteq Nint(Ncl(F)) \cap Ncl(Nint(F))$.

Example 2.1. Let $X = \{a, b\}$ and $A = \langle(0.3, 0.6, 0.4), (0.6, 0.3, 0.1)\rangle$, $B = \langle(0.2, 0.5, 0.7), (0.5, 0.2, 0.4)\rangle$, $D = \langle(0.3, 0.6, 0.4), (0.6, 0.3, 0.2)\rangle$, and $E = \langle(0.2, 0.6, 0.7), (0.5, 0.2, 0.5)\rangle$ be neutrosophic sets in X. Then, $\tau = \{0_N, A, B, 1_N\}$ is a neutrosophic topological space on X. Further,$A_1 = \langle(0.4, 0.6, 0.4), (0.8, 0.3, 0.4)\rangle$ is a neutrosophic b-open set in X.

Definition 2.8. [3] *Let* (X, τ) *be a neutrosophic topological space and let* U *be a neutrosophic set over* X. *Then, the neutrosophic b-interior of* U *is defined to be the union of all neutrosophic b-open sets of* X *which are contained in* U. *That is,* $Nbint(U) = \cup\{G : G$ *is a neutrosophic b-open set in* X *and* $G \subseteq U\}$.

The neutrosophic b-closure of U is defined to be the intersection of all neutrosophic b-closed sets of Z which contains U. That is, $Nbcl(U) = \cap\{H : H$ is a neutrosophic closed set in Z and $K \supseteq U\}$.

3. Main Results

Proposition 3.1. *Every neutrosophic open set is a neutrosophic b-open set in a neutrosophic topological space.*
Proof. Let A be neutrosophic open set in a neutrosophic topological space X. Then, we have $A = Nint(A)$ and $A \subseteq Ncl(A)$. Now $Nint(A) \subseteq Nint(Ncl(A))$. This implies that $A \subseteq Nint(Ncl(A)) \cup Ncl(Nint(A))$. Hence A is a neutrosophic b-open set in X.

Theorem 3.1. *The union of two neutrosophic b-open sets is a neutrosophic b-open set in a neutrosophic topological space.*
Proof. Let A and B be two neutrosophic b-open sets in a neutrosophic topological space X. Then, $A \subseteq Nint(Ncl(A)) \cup Ncl(Nint(A))$ and $B \subseteq Nint(Ncl(B)) \cup Ncl(Nint(B))$. This implies that $A \cup B \subseteq Nint(Ncl(A) \cup Ncl(B)) \cup Ncl(Nint(A) \cup Nint(B)) \subseteq Nint(Ncl(A \cup B)) \cup Ncl(Nint(A \cup B))$. Hence $A \cup B$ is a neutrosophic b-open set in X.

Remark 3.1. *In general, the intersection of two neutrosophic b-open sets need not be a neutrosophic b-open set. This can be shown in the following example:*

Example 3.2. Let $X = \{a, b\}$ and let $A = \langle(0.3, 0.6, 0.4), (0.6, 0.3, 0.1)\rangle$ and $B = \langle(0.2, 0.5, 0.7), (0.5, 0.2, 0.4)\rangle$ be neutrosophic sets in X. Let $\tau = \{0_N, A, B, 1_N\}$ be a neutrosophic topological space on X. Now $A_1 = \langle(0.4, 0.6, 0.4), (0.8, 0.3, 0.4)\rangle$ and $A_2 = \langle(1, 0.9, 0.2), (0.5, 0.7, 0)\rangle$ are two neutrosophic b-open sets; but $A_1 \cap A_2$ is not a neutrosophic b-open set.

Definition 3.1. *Let X be a neutrosophic topological space and let A be a neutrosophic set in X. If there exist two neutrosophic b-open sets B and D such that $A = B \cup D$ and $B \cap D = 0_N$, then the neutrosophic set A is called a neutrosophic b-disconnected set in X. If there does not exist such two neutrosophic b-open sets, then the neutrosophic set A is called a neutrosophic b-connected set in X.*

Example 3.3. Let $X = \{a, b\}$ with $\tau = \{0_N, A, B, D, 1_N\}$ be a neutrosophic topological space, where $A = \langle(a, 1, 0.3, 0.3), (b, 0.4, 0.6, 0.5)\rangle$, $B = \langle(a, 0, 0.3, 1), (b, 0.4, 0.6, 0.5)\rangle$ and $D = \langle(a, 1, 0, 0.3), (b, 0, 0.6, 1)\rangle$. Then, A, B and D are neutrosophic b-open sets in X. Now $B \cap D = \langle(a, 0, 0, 1), (b, 0, 0, 1)\rangle = 0_N$ and $A = B \cup D$. Hence A is a neutrosophic b-disconnected set.

Example 3.4. Let $X = \{a, b\}$ with $\tau = \{0_N, A, B, D, E, 1_N\}$ be a neutrosophic topological space, where $A = \langle(a, 0.6, 0.4, 0.2), (b, 0.3, 0.5, 0.4)\rangle$, $B = \langle(a, 0.5, 0.3, 0.4), (b, 0.6, 0.2, 0.5)\rangle$, $D = \langle(a, 0.4, 0.1, 0.7), (b, 0.8, 0, 0.5)\rangle$ and $E = \langle(a, 0.6, 0.4, 0.2), (b, 0.8, 0.5, 0.4)\rangle$. Then, A, B, D and E are neutrosophic b-open sets in X. Now $B \cap D = \langle(a, 0.4, 0.1, 0.7), (b, 0.6, 0, 0.5)\rangle \neq 0_N$ and $A \neq B \cup D = \langle(a, 0.5, 0.3, 0.4), (b, 0.8, 0.2, 0.5)\rangle$. Hence A is a neutrosophic b-connected set.

Definition 3.2. *A neutrosophic topological space X is neutrosophic b-disconnected space if there exist neutrosophic b-open sets A and B in X, with $A \neq 0_N$,$B \neq 0_N$ such that $A \cup B = 1_N$ and $A \cap B = 0_N$. If X is not a neutrosophic b-disconnected space, then it is said to be a neutrosophic b-connected space.*

Example 3.5. Let $X = \{a, b, c\}$ with $\tau = \{0_N, A, B, 1_N\}$ be a neutrosophic topological space, where $A = \langle(a, 1, 1, 0), (b, 0, 0, 1), (c, 0, 0, 1)\rangle$, $B = \langle(a, 0, 0, 1), (b, 1, 1, 0), (c, 1, 1, 0)\rangle$ are neutrosophic b-open sets in X. Then, $A \neq 0_N$, $B \neq 0_N$, $A \cup B = \langle(a, 1, 1, 0), (b, 1, 1, 0), (c, 1, 1, 0)\rangle = 1_N$ and $A \cap B = \langle(a, 0, 0, 1), (b, 0, 0, 1), (c, 0, 0, 1)\rangle = 0_N$. Hence X is a neutrosophic b-disconnected space.

Example 3.6. Let $X = \{a\}$ with $\tau = \{0_N, A, B, 1_N\}$ be a neutrosophic topological space, where $A = \langle(a, 0.3, 0.5, 0.2)\rangle$, $B = \langle(a, 0.2, 0.4, 0.5)\rangle$ are neutrosophic b-open sets in X. Then, $A \neq 0_N, B \neq 0_N, A \cup B = A \neq 1_N$ and $A \cap B = B \neq 0_N$. Hence

X is neutrosophic b-connected space.

Theorem 3.7. *Let (X, τ) be a neutrosophic b-connected space and $\sigma \subseteq \tau$. Then, (X, σ) is also a neutrosophic b-connected space.*
Proof. Suppose (X, σ) is a neutrosophic b-disconnected space. Then, there exist two neutrosophic b-open sets A, B in σ with $A \neq 0_N, B \neq 0_N$ such that $A \cup B = 1_N$ and $A \cap B = 0_N$. Since $\sigma \subseteq \tau$, we have A, B in τ with $A \neq 0_N, B \neq 0_N$ such that $A \cup B = 1_N$ and $A \cap B = 0_N$. This shows that (X, τ) is a neutrosophic b-disconnected space which is a contradiction. Hence (X, σ) is neutrosophic b-connected space.

Theorem 3.8. *For a neutrosophic topological space X, the following conditions are equivalent:*

1. *X is a neutrosophic b-connected space,*

2. *the only neutrosophic sets of X which are both neutrosophic b-open and neutrosophic b-closed are 0_N and 1_N.*

Proof. First suppose that X be a neutrosophic b-connected space and let A be a neutrosophic set in X which is both neutrosophic b-open and neutrosophic b-closed. Then, A and $C(A)$ are disjoint neutrosophic b-open sets and $1_N = A \cup C(A)$. Since X is neutrosophic b-connected, either $A = 0_N$ or $C(A) = 0_N$. That is, either $A = 0_N$ or $A = 1_N$.

Now suppose X is a neutrosophic b-disconnected space. Then, there exist two disjoint neutrosophic b-open sets A and B with $A \neq 0_N$ and $B \neq 0_N$ such that $1_N = A \cup B$. Since $A = C(B), A$ is neutrosophic b-closed. By hypothesis, $A = 0_N$ or $A = 1_N$ which is a contradiction to that A is a non-empty neutrosophic b-open set. Therefore, X is a neutrosophic b-connected space.

Definition 3.3. *Two neutrosophic sets A and B in X with $A \neq 0_N$ and $B \neq 0_N$ are called neutrosophic b-separated sets if $Nbcl(A) \cap B = A \cap Nbcl(B) = 0_N$.*

Theorem 3.9. *Any two disjoint neutrosophic b-closed sets with $A \neq 0_N$ and $B \neq 0_N$ are neutrosophic b-separated sets.*
Proof. Let A and B be neutrosophic b-closed sets with $A \neq 0_N$ and $B \neq 0_N$. Then, $Nbcl(A) \cap B = A \cap Nbcl(B) = A \cap B = 0_N$. Thus A and B are neutrosophic b-separated sets.

However, in general the neutrosophic b-separated sets need not be neutrosophic b-closed sets which can be shown by the following example:

Example 3.10. Let $X = \{a, b\}$ with $\tau = \{0_N, A, B, D, E, 1_N\}$ be a neutrosophic topological space, where $A = \langle(a, 0.5, 1, 0.3), (b, 0.6, 1, 0.3)\rangle$, $B = \langle(a, 0, 0.2, 1), (b, 0, 0, 1)\rangle$, $D = \langle(a, 1, 0.7, 0), (b, 1, 0.5, 0)\rangle$, $E = \langle(a, 0.5, 0.7, 0.3), (b, 0.6, 0.5, 0.3)\rangle$, and $C(A) = \langle(a, 0.3, 0, 0.5), (b, 0.3, 0, 0.6)\rangle$, $C(B) = \langle(a, 1, 0.8, 0), (b, 1, 1, 0)\rangle$, $C(D) = \langle(a, 0, 0.3, 1), (b, 0, 0.5, 1)\rangle$ and $C(E) = \langle(a, 0.3, 0.3, 0.5), (b, 0.3, 0.5, 0.6)\rangle$. Then $Nbcl(C(A)) = C(A)$ and $Nbcl(B) = C(D)$. Thus $Nbcl(C(A)) \cap B = C(A) \cap B = 0_N$ and $C(A) \cap Nbcl(B) = C(A) \cap C(D) = 0_N$. Therefore, $C(A)$ and B are neutrosophic b-separated sets but B is not a neutrosophic b-closed set.

Theorem 3.11. *The neutrosophic sets A and B in neutrosophic topological space X are neutrosophic b-separated if and only if there exist neutrosophic b-open sets U and V in X such that $A \subseteq U, B \subseteq V$ and $A \cap V = 0_N$ and $B \cap U = 0_N$.*
Proof. Let A and B be neutrosophic b-separated. Then, $A \cap Nbcl(B) = 0_N = Nbcl(A) \cap B$. Take $V = C(Nbcl(A))$ and $U = C(Nbcl(B))$. Then U and V are neutrosophic b-open sets such that $A \subseteq U, B \subseteq V$ and $A \cap V = 0_N$ and $B \cap U = 0_N$.

Conversely, let U and V be neutrosophic b-open sets such that $A \subseteq U, B \subseteq V$ and $A \cap V = 0_N$ and $B \cap U = 0_N$. Then, $A \subseteq C(V), B \subseteq C(U)$ and $C(V)$ and $C(U)$ are neutrosophic b-closed sets. This implies that $Nbcl(A) \subseteq Nbcl(C(V)) = C(V) \subseteq C(B)$ and $Nbcl(B) \subseteq Nbcl(C(U)) = C(U) \subseteq C(A)$. That is, $Nbcl(A) \subseteq C(B)$ and $Nbcl(B) \subseteq C(A)$. Therefore $A \cap Nbcl(B) = 0_N = Nbcl(A) \cap B$. Hence A and B are neutrosophic b-separated sets.

Proposition 3.2. *Every pair of neutrosophic b-separated sets are always disjoint.*
Proof. Let A and B be neutrosophic b-separated sets. Then, $A \cap Nbcl(B) = 0_N = Nbcl(A) \cap B$. Now $A \cap B \subseteq A \cap Nbcl(B) = 0_N$. Therefore, $A \cap B = 0_N$. Hence A and B are disjoint.

The converse of the above proposition need not be true in general which can be shown by the following example:

Example 3.12. Let $X = \{a, b\}$ with $\tau = \{0_N, A, B, D, 1_N\}$ be a neutrosophic topological space, where $A = \langle(a, 0.3, 0, 0.5), (b, 0, 0.3, 1)\rangle$, $B = \langle(a, 0, 0.4, 1), (b, 0.6, 0, 0.8)\rangle$, $D = \langle(a, 0.3, 0.4, 0.5), (b, 0.6, 0.3, 0.8)\rangle$, $C(A) = \langle(a, 0.5, 1, 0.3), (b, 1, 0.7, 0)\rangle$, $C(B) = \langle(a, 1, 0.6, 0), (b, 0.8, 1, 0.6)\rangle$ and $C(D) = \langle(a, 0.5, 0.6, 0.3), (b, 0.8, 0.7, 0.6)\rangle$. Then, $A \cap B = 0_N$. Thus A and B are disjoint neutrosophic sets. Now $Nbcl(A) = C(D)$ and $Nbcl(B) = C(D)$. Thus $Nbcl(A) \cap B = C(D) \cap B = B \neq 0_N$ and $A \cap Nbcl(B) = A \cap C(D) = A \neq 0_N$. Therefore, A and B are not neutrosophic b-separated sets.

Theorem 3.13. *A neutrosophic subset G of a neutrosophic topological space X is neutrosophic b-connected if and only if G is not the union of any two neutrosophic b-separated sets.*

Proof. Let X be a neutrosophic b-connected space and let $Y = A \cup B$, where A and B are neutrosophic b-separated sets. Then, $Nbcl(A) \cap B = A \cap Nbcl(B) = 0_N$. Since $A \subseteq Nbcl(A), A \cap B \subseteq Nbcl(A) \cap B = 0_N$. Also $Nbcl(A) \subseteq C(B) = A$ and $Nbcl(B) \subseteq C(A) = B$. Therefore, A and B are neutrosophic b-closed sets. Hence $A = C(B)$ and $B = C(A)$ are disjoint neutrosophic b-open sets. That is, G is not neutrosophic b-connected, which is a contradiction. Hence G is not the union of any two neutrosophic b-separated sets.

Conversely, assume that G is not the union of any two neutrosophic b-separated sets. Let X be a neutrosophic b-disconnected space. Then, $Y = A \cup B$ where, A and B are non-empty disjoint neutrosophic b-open sets in X. Since $A \subseteq C(B)$ and $B \subseteq C(A), Nbcl(A) \cap B \subseteq C(B) \cap B = 0_N$ and $A \cap Nbcl(B) \subseteq A \cap C(A) = 0_N$. Thus A and B are neutrosophic b-separated sets which is a contradiction. Therefore, G is neutrosophic b-connected.

Theorem 3.14. *A neutrosophic topological space X is neutrosophic b-connected if and only if 1_N is not the union of any two neutrosophic b-separated sets.*

Proof. Let X be a neutrosophic b-connected space and let $1_N = A \cup B$, where A and B are neutrosophic b-separated sets. Then, $Nbcl(A) \cap B = A \cap Nbcl(B) = 0_N$. Since $A \subseteq Nbcl(A), A \cap B \subseteq Nbcl(A) \cap B = 0_N$. Thus $A \cap B = 0_N$. Also $Nbcl(A) \subseteq C(B) = A$ and $Nbcl(B) \subseteq C(A) = B$. Therefore, A and B are neutrosophic b-closed sets. Hence $A = C(B)$ and $B = C(A)$ are disjoint neutrosophic b-open sets. That is, 1_N is not neutrosophic b-connected which is a contradiction. Hence 1_N is not the union of any two neutrosophic b-separated sets.

Conversely, assume that 1_N is not the union of any two neutrosophic b-separated sets. Let X be a neutrosophic b-disconnected space. Then, $1_N = A \cup B$ where A and B are non-empty disjoint neutrosophic b-open sets in X. Since $A \subseteq C(B)$ and $B \subseteq C(A)$, we have $Nbcl(A) \cap B \subseteq C(B) \cap B = 0_N$ and $A \cap Nbcl(B) \subseteq A \cap C(A) = 0_N$. Thus A and B are neutrosophic b-separated sets which is a contradiction. Therefore, X is neutrosophic b-connected.

Theorem 3.15. *If $A \subseteq H \cup K$ where A is a neutrosophic b-connected set and H, K are neutrosophic b-separated sets, then either $A \subseteq H$ or $A \subseteq K$.*

Proof. Suppose $A \nsubseteq H$ and $A \nsubseteq K$. Let $A_1 = H \cap A$ and $A_2 = K \cap A$. Then, A_1 and A_2 are non-empty neutrosophic sets and $A_1 \cup A_2 = (H \cap A) \cup (K \cap A) = (H \cup K) \cap A = A$. Since $A_1 \subseteq H, A_2 \subseteq K$ and H, K are neutrosophic b-separated sets, $Nbcl(A_1) \cap A_2 \subseteq Nbcl(H) \cap K = 0_N$ and $A_1 \cap Nbcl(A_2) \subseteq H \cap Nbcl(K) = 0_N$. Therefore, A_1, A_2 are neutrosophic b-separated sets such that $A = A_1 \cup A_2$. Hence A is neutrosophic b-disconnected which is a contradiction. Thus either $A \subseteq H$ or $A \subseteq K$.

Theorem 3.16. *If A and B are the neutrosophic b-connected space in the neutrosophic topological space X such that $A \cap B \neq 0_N$, then $A \cup B$ is a neutrosophic b-connected space in X.*

Proof. Let $A \cup B$ be neutrosophic b-disconnected. Then, there exist two neutrosophic b-separated sets H, K such that $A \cup B = H \cup K$. Since H and K are neutrosophic b-separated, H and K are neutrosophic sets and $H \cap K \subseteq Nbcl(H) \cap K = 0_N$. Since $A \subseteq A \cup B = H \cup K, B \subseteq A \cup B = H \cup K$ and A, B are neutrosophic b-connected. Then, $A \subseteq H$ or $A \subseteq K$ and $B \subseteq H$ or $B \subseteq K$. If $A \subseteq H$ and $B \subseteq H$, then $A \cup B \subseteq H$. Thus $A \cup B = H$. Since H and K are disjoint, $K = 0_N$ which is a contradiction to that $K \neq 0_N$. Similarly, if $A \subseteq K$ and $B \subseteq K$, we get a contradiction.

Further, if $A \subseteq H$ and $B \subseteq K$, then $A \cap B \subseteq H \cap K = 0_N$ and then $A \cap B = 0_N$. This is a contradiction to that $A \cap B \neq 0_N$. Similarly, we get a contradiction if $A \subseteq K$ and $B \subseteq H$. Therefore $A \cup B$ is neutrosophic b-connected in X.

4. Conclusion

In the paper, we studied the notion of neutrosophic b-open sets and discussed some of its properties in the neutrosophic topological spaces. Then, we defined and investigated some properties of neutrosophic b-connected spaces and neutrosophic b-separated sets.

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