

RELATIONSHIPS BETWEEN MOCK THETA FUNCTIONS AND q -CONTINUED FRACTIONS

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Abstract: The main object of this paper is to present five new interrelationships between mock theta functions and q -continued fractions.

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1. Introduction and Definitions

Three months before his death in early 1920 Ramanujan sent a letter to Hardy of 17 functions, which he called mock theta functions, his functions being separated into three groups, four of order three, ten of order five and three of order seven. These mock theta functions are q -series which converge of $|q| < 1$ and have certain properties as the theta functions when q tends to a root of unity.

Throughout this paper, we denote by \mathbb{N} , \mathbb{Z} , and \mathbb{C} the set of positive integers, the set of integers and the set of complex numbers respectively. We also let

$$\mathbb{N}_0 := \mathbb{N} \cup \{0\} = \{0, 1, 2, \dots\}.$$

The q -shifted factorial $(a; q)_n$ is defined (for $|q| < 1$) by

$$(a; q)_n := \begin{cases} 1 & (n = 0) \\ \prod_{k=0}^{n-1} (1 - aq^k) & (n \in \mathbb{N}), \end{cases}$$