

SOME MORE PROPERTIES OF $(1,2)S_\beta$ -OPEN SETS IN BITOPOLOGICAL SPACES

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Abstract: The aim of this paper is to define some operators using $(1,2)S_\beta$ -open sets in bitopological spaces and study some of their properties.

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1. Introduction and Preliminaries

In the year 1963, Kelly initiated the systematic study of bitopology which is a triple (X, τ, σ) , where X is a non-empty set together with two distinct topologies τ, σ on X . Levine initiated semi-open sets and their properties in 1963. In 1983, Abd-El-monsef introduced the notion of β -open sets and β -continuity in topological spaces. In 2013, Alias B.Khalaf and Nehmat K. Ahmed introduced and defined a new class of semi-open sets called S_β -open sets in topological spaces. The aim of this paper is to define some operators of $(1,2)S_\beta$ -open sets in bitopological spaces and study some of their properties.

Definition 1.1. [5] Let A be a subset of a bitopological space (X, τ_1, τ_2) . Then A is said to be

- (i) $\tau_1\tau_2$ -open if $A \in \tau_1 \cup \tau_2$,
- (ii) $\tau_1\tau_2$ -closed if $A^c \in \tau_1 \cup \tau_2$,

(iii) $(1,2)\beta$ -open if $A \subseteq \tau_1\tau_2 - cl(\tau_1 - int(\tau_1\tau_2 - cl(A)))$, where τ_1 -Int(A) is the interior of A with respect to the topology τ_1 and $\tau_1\tau_2$ -Cl(A) is the intersection of all $\tau_1\tau_2$ -closed sets containing A .

(iv) $(1,2)\beta$ -Int(A) is the union of all $(1,2)\beta$ -open sets contained in A .

(v) $(1,2)\beta$ -Cl(A) is the intersection of all $(1,2)\beta$ -closed sets containing A .

Definition 1.2. [5] A subset A of X is said to be

(i) $(1,2)$ semi-open if $A \subseteq \tau_1\tau_2$ -Cl(τ_1 -Int(A)),

(ii) $(1,2)$ regular-open if $A = \tau_1$ -Int($\tau_1\tau_2$ -Cl(A)),

(iii) $(1,2)\beta$ -open if $A \subseteq \tau_1\tau_2$ -Cl(τ_1 -Int($\tau_1\tau_2$ -Cl(A))).

The set of all $(1,2)$ semi-open, $(1,2)$ regular-open, $(1,2)\beta$ -open are denoted as $(1,2)SO(X, \tau_1, \tau_2)$, $(1,2)RO(X, \tau_1, \tau_2)$, $(1,2)\beta O(X, \tau_1, \tau_2)$ or simply, $(1,2)SO(X)$, $(1,2)RO(X)$, $(1,2)\beta O(X)$ respectively.

Definition 1.3. [3] A subset A of X is said to be

(i) $(1,2)$ semi-closed if $\tau_1\tau_2$ -Int(τ_1 -Cl(A)) $\subseteq A$.

(ii) $(1,2)$ regular-closed if $A = \tau_1$ -Cl($\tau_1\tau_2$ -Int(A))

(iii) $(1,2)\beta$ -closed if $\tau_1\tau_2$ -Int($\tau_1 - Cl(\tau_1\tau_2$ -Int(A))) $\subseteq A$.

The set of all $(1,2)$ semi-closed, $(1,2)$ regular-closed, $(1,2)\beta$ -closed are denoted as $(1,2)SCL(X, \tau_1, \tau_2)$, $(1,2)RCL(X, \tau_1, \tau_2)$, $(1,2)\beta CL(X, \tau_1, \tau_2)$ or simply, $(1,2)SCL(X)$, $(1,2)RCL(X)$, $(1,2)\beta CL(X)$ respectively.

Remark 1.4. [5] For any subset A of X ,

(i) τ_1 -Int(A) $\subseteq \tau_1\tau_2$ -Int(A) and τ_2 -Int(A) $\subseteq \tau_1\tau_2$ -Int(A).

(ii) $\tau_1\tau_2$ -Cl(A) $\subseteq \tau_1$ -Cl(A) and $\tau_1\tau_2$ -Cl(A) $\subseteq \tau_2$ -Cl(A).

(iii) $\tau_1\tau_2$ -Cl($A \cap B$) $\subseteq \tau_1\tau_2$ -Cl(A) $\cap \tau_1\tau_2$ -Cl(B).

(iv) $\tau_1\tau_2$ -Int(A) $\cup \tau_1\tau_2$ -Int(B) $\subseteq \tau_1\tau_2$ -Int($A \cup B$).

Theorem 1.5. [1] Let (X, τ_1, τ_2) be a bitopological space. If $A \in \tau_1$ and $B \in (1,2)SO(X)$, then $A \cap B \in (1,2)SO(X)$.

Theorem 1.6. [1] Let $A \subset Y \subset (X, \tau_1, \tau_2)$ and if A is τ_i -semi open in X , then A is τ_i -semi open in Y .

Definition 1.7. [8] A $(1,2)$ semi-open subset A of a bitopological space (X, τ_1, τ_2) is said to be $(1,2)S_\beta$ -open if for each $x \in A$ there exists a $(1,2)\beta$ -closed set F such that $x \in F \subseteq A$.

2. $(1,2)S_\beta$ -Operations

Definition 2.1. A subset N of a bitopological space (X, τ_1, τ_2) is called $(1,2)S_\beta$ -neighborhood, if there exists a $(1,2)S_\beta$ -open set U such that $A \subseteq U \subseteq N$.

If $A = \{x\}$, then N is $(1,2)S_\beta$ -neighborhood of x .

Definition 2.2. A point $x \in X$ is said to be a $(1,2)S_\beta$ -interior point of A , if there exists a $(1,2)S_\beta$ -open set U containing x such that $x \in U \subseteq A$.

The set of all $(1,2)S_\beta$ -interior points of A is said to be $(1,2)S_\beta$ -interior of A and it is denoted by $(1,2)S_\beta\text{-Int}(A)$.

Proposition 2.3. Let A be any subset of a bitopological space X . If a point x is in the $(1,2)S_\beta$ -interior of A , then there exists a $(1,2)\beta$ -closed set F of X containing x such that $F \subseteq A$.

Proof. Suppose that $x \in (1,2)S_\beta\text{-Int}(A)$. Then there exists a $(1,2)S_\beta$ -open set U of X containing x such that $U \subseteq A$. Since U is a $(1,2)S_\beta$ -open set, there exists a $(1,2)\beta$ -closed set F containing x such that $F \subseteq U \subseteq A$ and hence $x \in F \subseteq A$.

Remark 2.4. For any subset A of a bitopological space X , the following statements are true.

- (i) $(1,2)S_\beta$ -Interior of A is the union of all $(1,2)S_\beta$ -open sets contained in A .
- (ii) $(1,2)S_\beta\text{-Int}(A)$ is the largest $(1,2)S_\beta$ -open set contained in A .
- (iii) A is $(1,2)S_\beta$ -open set if and only if $A = (1,2)S_\beta\text{-Int}(A)$.

Proposition 2.5. If A and B are any two subsets of a bitopological space X , then

- (i) $(1,2)S_\beta\text{-Int}(\phi) = \phi$, and $(1,2)S_\beta\text{-Int}(X) = X$.
- (ii) $(1,2)S_\beta\text{-Int}(A) \subseteq A$.
- (iii) If $A \subseteq B$, then $(1,2)S_\beta\text{-Int}(A) \subseteq (1,2)S_\beta\text{-Int}(B)$.
- (iv) $(1,2)S_\beta\text{-Int}(A) \cup (1,2)S_\beta\text{-Int}(B) \subseteq (1,2)S_\beta\text{-Int}(A \cup B)$.
- (v) $(1,2)S_\beta\text{-Int}(A \cap B) \subseteq (1,2)S_\beta\text{-Int}(A) \cap (1,2)S_\beta\text{-Int}(B)$.
- (vi) $(1,2)S_\beta\text{-Int}(A - B) \subseteq (1,2)S_\beta\text{-Int}(A) - (1,2)S_\beta\text{-Int}(B)$.

Proof. Follows from definition 2.2.

Note 2.6. The converse of (iii) (iv), (v), (vi) of the above proposition need not be always true and is shown in the following examples.

Example 2.7. Let $X = \{a, b, c, d\}$ with the topologies $\tau_1 = \{\phi, X, \{a\}, \{a, b\}\}$ and $\tau_2 = \{\phi, X, \{a\}, \{c\}, \{a, c\}\}$. Let $A = \{a, d\}$ and $B = \{a, b, c\}$, then $(1,2)S_\beta\text{-Int}(A) = \{a\}$, $(1,2)S_\beta\text{-Int}(B) = \{a, b\}$. It follows that $(1,2)S_\beta\text{-Int}(A) \subseteq (1,2)S_\beta\text{-Int}(B)$ but A is not a subset of B .

Example 2.8. Let $X = \{a, b, c, d\}$ with the topologies $\tau_1 = \{\phi, X, \{b\}, \{a, c\}, \{a, b, c\}, \{a, c, d\}\}$ and $\tau_2 = \{\phi, X\}$. Take $A = \{a, c\}$ and $B = \{b, d\}$. Then, $(1,2)S_\beta\text{-Int}(A) = \{a, c\}$, $(1,2)S_\beta\text{-Int}(B) = \{b\}$ and $(1,2)S_\beta\text{-Int}(A) \cup (1,2)S_\beta\text{-Int}(B) = \{a, b, c\}$ and $(1,2)S_\beta\text{-Int}(A \cup B) = X$. It follows that $(1,2)S_\beta\text{-Int}(A \cup B)$ is not a subset of $(1,2)S_\beta\text{-Int}(A) \cup (1,2)S_\beta\text{-Int}(B)$.

Example 2.9. Let $X = \{a, b, c\}$ with the topologies $\tau_1 = \{\phi, X, \{a, c\}\}$ and $\tau_2 =$

$\{\phi, X, \{b\}\}$. Then, $(1,2)S_\beta O(X) = \{\phi, X, \{a\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. If we take $A = \{a, b\}$, $B = \{b, c\}$, then $(1,2)S_\beta\text{-Int}(A) = \{a, b\}$, $(1,2)S_\beta\text{-Int}(B) = \{b, c\}$ and $(1,2)S_\beta\text{-Int}(A) \cap (1,2)S_\beta\text{-Int}(B) = \{b\}$, $(1,2)S_\beta\text{-Int}(A \cap B) = \phi$. It follows that $(1,2)S_\beta\text{-Int}(A) \cap (1,2)S_\beta\text{-Int}(B)$ is not a subset of $(1,2)S_\beta\text{-Int}(A \cap B)$.

Also, if we take $A = \{b, c\}$ and $B = \{c\}$, then $(1,2)S_\beta\text{-Int}(A) = \{b, c\}$, $(1,2)S_\beta\text{-Int}(B) = \{c\}$ and $(1,2)S_\beta\text{-Int}(A) - (1,2)S_\beta\text{-Int}(B) = \{b\}$ but $(1,2)S_\beta\text{-Int}(A - B) = \phi$. It follows that $(1,2)S_\beta\text{-Int}(A) - (1,2)S_\beta\text{-Int}(B)$ is not a subset of $(1,2)S_\beta\text{-Int}(A - B)$.

Definition 2.10. Let A be a subset of a bitopological space X . A point $x \in X$ is in $(1,2)S_\beta$ -Closure of A if and only if $A \cap U \neq \phi$, for every $(1,2)S_\beta$ -open set U containing x .

The intersection of all $(1,2)S_\beta$ -closed sets containing F is called $(1,2)S_\beta$ -Closure of F and is denoted by $(1,2)S_\beta\text{-Cl}(F)$.

Proposition 2.11. Let A be a subset of a bitopological space X . If $A \cap F \neq \phi$, for every $(1,2)S_\beta$ -closed set F of X containing x , then the point x is in $(1,2)S_\beta$ -Closure of A .

Proof. Suppose that U is any $(1,2)S_\beta$ -open set containing x . Then by definition 1.7, there exists $(1,2)S_\beta$ -closed set F such that $x \in F \subseteq U$. So by hypothesis, $A \cap F \neq \phi$ which implies that $A \cap U \neq \phi$, for every $(1,2)S_\beta$ -open set U containing x . Therefore, $x \in (1,2)S_\beta\text{-Cl}(A)$.

Remark 2.12. For any subset F of a bitopological space X , the following statements are true.

- (i) $(1,2)S_\beta\text{-Cl}(F)$ is the intersection of all $(1,2)S_\beta$ -closed set in X containing F .
- (ii) $(1,2)S_\beta\text{-Cl}(F)$ is the smallest $(1,2)S_\beta$ -closed set containing F .
- (iii) F is $(1,2)S_\beta$ -closed set if and only if $F = (1,2)S_\beta\text{-Cl}(F)$.

Theorem 2.13. If F and E are any two subsets of a bitopological space X , then

- (i) $(1,2)S_\beta\text{-Cl}(\phi) = \phi$ and $(1,2)S_\beta\text{-Cl}(X) = X$.
- (ii) For any subset F of X , $F \subseteq (1,2)S_\beta\text{-Cl}(F)$.
- (iii) If $F \subseteq E$, then $(1,2)S_\beta\text{-Cl}(F) \subseteq (1,2)S_\beta\text{-Cl}(E)$.
- (iv) $(1,2)S_\beta\text{-Cl}(F) \cup (1,2)S_\beta\text{-Cl}(E) \subseteq (1,2)S_\beta\text{-Cl}(F \cup E)$.
- (v) $(1,2)S_\beta\text{-Cl}(F \cap E) \subseteq (1,2)S_\beta\text{-Cl}(F) \cap (1,2)S_\beta\text{-Cl}(E)$.

Proof. Follows from definition 2.10.

Note 2.14. From the above theorem, $(1,2)S_\beta\text{-Cl}(F) \cup (1,2)S_\beta\text{-Cl}(E) \neq (1,2)S_\beta\text{-Cl}(F \cup E)$ and $(1,2)S_\beta\text{-Cl}(F \cap E) \neq (1,2)S_\beta\text{-Cl}(F) \cap (1,2)S_\beta\text{-Cl}(E)$, it is shown in the following example.

Example 2.15. Let $X = \{a, b, c, d\}$ with the topologies $\tau_1 = \{\phi, X, \{a\}, \{b\}$,

$\{a, b\}$ and $\tau_2 = \{\phi, X, \{a, b, d\}\}$, then $(1,2)S_\beta\text{-CL}(X) = \{\phi, X, \{a\}, \{b\}, \{c\}, \{d\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. If we take $F = \{b\}$ and $E = \{a\}$ then $(1,2)S_\beta\text{-Cl}(F) = \{b\}$ and $(1,2)S_\beta\text{-Cl}(E) = \{a\}$ then $(1,2)S_\beta\text{-Cl}(F) \cup (1,2)S_\beta\text{-Cl}(E) = \{a, b\}$ but $(1,2)S_\beta\text{-Cl}(F \cup E) = X$. Hence, $(1,2)S_\beta\text{-Cl}(F) \cup (1,2)S_\beta\text{-Cl}(E) \neq (1,2)S_\beta\text{-Cl}(F \cup E)$.

Again if we take $F = \{a, b\}$ and $E = \{a, c, d\}$, we get $(1,2)S_\beta\text{-Cl}(F) = X$ and $(1,2)S_\beta\text{-Cl}(E) = \{a, c, d\}$, then $(1,2)S_\beta\text{-Cl}(F) \cap (1,2)S_\beta\text{-Cl}(E) = \{a, c, d\}$, but $(1,2)S_\beta\text{-Cl}(F \cap E) = \{a\}$. Hence, $(1,2)S_\beta\text{-Cl}(F \cap E) \neq (1,2)S_\beta\text{-Cl}(F) \cap (1,2)S_\beta\text{-Cl}(E)$.

Definition 2.16. Let A be a subset of a bitopological space X . A point $x \in X$ is said to be $(1,2)S_\beta$ -limit point of A if for each $(1,2)S_\beta$ -open set U containing x , $U \cap (A - \{x\}) \neq \phi$. The set of all $(1,2)S_\beta$ -limit points of A is called $(1,2)S_\beta$ -derived set of A and is denoted by $(1,2)S_\beta\text{-D}(A)$.

Proposition 2.17. Let A be any subset of X . If $F \cap (A - \{x\}) \neq \phi$, for every $(1,2)\beta$ -closed set F containing x , then $x \in (1,2)S_\beta\text{-D}(A)$.

Proof. Let U be any $(1,2)S_\beta$ -open set containing x . Then there exists $(1,2)\beta$ -closed set F such that $x \in F \subseteq U$. By hypothesis, $F \cap (A - \{x\}) \neq \phi$. Hence $U \cap (A - \{x\}) \neq \phi$. Therefore, $x \in (1,2)S_\beta\text{-D}(A)$.

Proposition 2.18. If a subset A of a bitopological space X is $(1,2)S_\beta$ -closed, then A contains the set of all its $(1,2)S_\beta$ -limit points.

Proof. Suppose that A is $(1,2)S_\beta$ -closed set, then $X - A$ is $(1,2)S_\beta$ -open set. Thus A is $(1,2)S_\beta$ -closed set if and only if each point of $X - A$ has $(1,2)S_\beta$ -neighborhood contained in $X - A$, that is, if and only if no point of $X - A$ is $(1,2)S_\beta$ -limit point of A or equivalently that A contains each of its $(1,2)S_\beta$ -limit points.

Proposition 2.19. Let A and B be two subsets of a bitopological space X . If $A \subseteq B$, then $(1,2)S_\beta\text{-D}(A) \subseteq (1,2)S_\beta\text{-D}(B)$.

Proof. Let $x \in (1,2)S_\beta\text{-D}(A)$. Then by definition 2.16, for all $(1,2)S_\beta$ -open set U containing x such that $U \cap (A - \{x\}) \neq \phi$. Since $A \subseteq B$, $U \cap (B - \{x\}) \neq \phi$. Therefore $x \in (1,2)S_\beta\text{-D}(B)$ which implies that $(1,2)S_\beta\text{-D}(A) \subseteq (1,2)S_\beta\text{-D}(B)$.

Remark 2.20. Let A and B be subsets of a bitopological space X . Then we have the following properties:

- (i) $(1,2)S_\beta\text{-D}(\phi) = \phi$.
- (ii) $(1,2)S_\beta\text{-D}(A) \cup (1,2)S_\beta\text{-D}(B) \subseteq (1,2)S_\beta\text{-D}(A \cup B)$.
- (iii) $(1,2)S_\beta\text{-D}(A \cap B) \subseteq (1,2)S_\beta\text{-D}(A) \cap (1,2)S_\beta\text{-D}(B)$.
- (iv) If A is $(1,2)S_\beta$ -closed, then $(1,2)S_\beta\text{-D}(A) \subseteq A$.

Theorem 2.21. Let X be any bitopological space and A be a subset of X , then

- (i) $A \cup (1,2)S_\beta\text{-}D(A)$ is $(1,2)S_\beta\text{-}closed$.
- (ii) $(1,2)S_\beta\text{-}D((1,2)S_\beta\text{-}D(A))\text{-}A \subseteq (1,2)S_\beta\text{-}D(A)$.
- (iii) $(1,2)S_\beta\text{-}D(A \cup (1,2)S_\beta\text{-}D(A)) \subseteq A \cup (1,2)S_\beta\text{-}D(A)$.

Proof. (i) Let $x \notin A \cup (1,2)S_\beta\text{-}D(A)$. Then $x \notin A$ and $x \notin (1,2)S_\beta\text{-}D(A)$. This implies that there exists a $(1,2)S_\beta$ -open set N_x in X which contains no point of A other than x . But $x \notin A$, so N_x contains no point of A , which implies that $N_x \subseteq X\text{-}A$. Again N_x is a $(1,2)S_\beta$ -open set and it is a neighborhood of each of its points. Also N_x does not contain any point of A implies no point of N_x can be $(1,2)S_\beta$ -limit point of A . Therefore, no point of N_x can belong to $(1,2)S_\beta\text{-}D(A)$ and this implies that $N_x \subseteq X\text{-}(1,2)S_\beta\text{-}D(A)$. Hence it follows that $x \in N_x \subseteq (X\text{-}A) \cap (X\text{-}(1,2)S_\beta\text{-}D(A)) \subseteq X\text{-}(A \cup (1,2)S_\beta\text{-}D(A))$. Therefore, $A \cup (1,2)S_\beta\text{-}D(A)$ is $(1,2)S_\beta\text{-}closed$.

(ii) Let $x \in (1,2)S_\beta\text{-}D((1,2)S_\beta\text{-}D(A))\text{-}A$. Then by definition 2.16, U is a $(1,2)S_\beta$ -open set containing x such that $U \cap ((1,2)S_\beta\text{-}D(A)\text{-}\{x\}) \neq \phi$. Now let $y \in (U \cap (1,2)S_\beta\text{-}D(A)\text{-}\{x\})$. Then $y \in U$ and $y \in (1,2)S_\beta\text{-}D(A)$, so $U \cap (A\text{-}\{y\}) \neq \phi$. Let $z \in (U \cap (A\text{-}\{y\}))$. Then, for $z \in A$ and $x \notin A$, $z \neq x$. Hence $U \cap (A\text{-}\{x\}) \neq \phi$. Therefore, $x \in (1,2)S_\beta\text{-}D(A)$.

(iii) Let $x \in (1,2)S_\beta\text{-}D(A \cup (1,2)S_\beta\text{-}D(A))$. If $x \in A$, then the result is obvious. So, let $x \in (1,2)S_\beta\text{-}D(A \cup (1,2)S_\beta\text{-}D(A))\text{-}A$. Then, by definition 2.16, for each $(1,2)S_\beta$ -open set U containing x such that $U \cap (A \cup (1,2)S_\beta\text{-}D(A)\text{-}\{x\}) \neq \phi$. Thus, either $U \cap (A\text{-}\{x\}) \neq \phi$ or $U \cap (1,2)S_\beta\text{-}D(A)\{x\} \neq \phi$. Now, $U \cap (A\text{-}\{x\}) \neq \phi$ implies that $x \in (1,2)S_\beta\text{-}D(A)$. Therefore $(1,2)S_\beta\text{-}D(A \cup (1,2)S_\beta\text{-}D(A)) \subseteq A \cup (1,2)S_\beta\text{-}D(A)$.

Theorem 2.22. Let A be a subset of a bitopological space X , then $(1,2)S_\beta\text{-}Cl(A) = A \cup (1,2)S_\beta\text{-}D(A)$.

Proof. Since $(1,2)S_\beta\text{-}D(A) \subseteq (1,2)S_\beta\text{-}Cl(A)$ and $A \subseteq (1,2)S_\beta\text{-}Cl(A)$ implies $A \cup (1,2)S_\beta\text{-}D(A) \subseteq (1,2)S_\beta\text{-}Cl(A)$. Again since $(1,2)S_\beta\text{-}Cl(A)$ is the smallest $(1,2)S_\beta$ -closed set containing A , $A \cup (1,2)S_\beta\text{-}D(A)$ is $(1,2)S_\beta$ -closed. Hence $(1,2)S_\beta\text{-}Cl(A) \subseteq A \cup (1,2)S_\beta\text{-}D(A)$. Thus $(1,2)S_\beta\text{-}Cl(A) = A \cup ((1,2)S_\beta\text{-}D(A))$.

Theorem 2.23. Let X be any bitopological space and A be a subset of X . Then, $(1,2)S_\beta\text{-}Int(A) = A\text{-}((1,2)S_\beta\text{-}D(X\text{-}A))$.

Proof. Obvious.

3. Conclusion

In this work, we have defined some operators of $(1,2)S_\beta$ -open sets in bitopological spaces and studied their properties. This work will lead to find another operators like $\bigwedge_{(1,2)S_\beta}$ -set and $\bigvee_{(1,2)S_\beta}$ -set of corresponding sets. Also, these findings

will help to carry out more theoretical research for future researchers.

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