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ON ROUGH BI-SEMI GENERALIZED CONTINUOUS MAPS IN ROUGH SET BITOPOLOGICAL SPACES

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Abstract: The purpose of this paper is to introduce and study the concepts of new class of maps, namely Rough bi-semi generalized continuous maps in Rough bitopological spaces. Also derive their characterizations in terms of Rough bi-semi generalized closed sets, Rough bi-semi-generalized closure and Rough bi-semi-generalized interior and obtain some of their properties.

Keywords and Phrases: Rough bi-sg closed Sets, Rough bi-sg open Sets, Rough bi-continuity, Rough bi-semi continuous function, Rough bi-sg continuous function.

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1. Introduction

In 1970, Levine [8] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. While in 1987, P.Bhattacharyya et.al. [1] have introduced the notion of semi generalized closed sets in topological spaces. The concept of semi-generalized mappings was studied by R. Devi et.al. [5] in 1993. The theory of rough sets, proposed by Pawlak [10], is an extension of set theory for the study of intelligent systems characterized by insufficient and incomplete information. The basic operators in rough set theory are approximations. In 1963, J.C.Kelly [7] initiated the study of bitopological spaces. Mean while in 1989, Fukutake [6] introduced semi open sets in bitopological spaces. In 2014, K.

Bhuvaneswari et al., [2] have introduced the notion of nano semi generalized closed sets in nano topological space. The aim of this paper is to define and analyze the properties of Rough bi-semi generalized continuity. Also establish various forms of continuities associated to Rough bi-semi generalized closed sets.

2. Preliminaries

Definition 2.1. [1] *A subset A of (X, τ) is called a semi generalized closed set (briefly sg-closed) if $scl(A) \subseteq U$ whenever $A \subseteq U$ and U is semi-open in X.*

Definition 2.2. [9] *A function $f : X \to Y$ is semi generalized continuous (sg-continuous) if $f^{-1}(V)$ is sg-closed set in X for every closed set V of Y, or equivalently, a function $f : X \to Y$ is sg-continuous if and only if the inverse image of each open set is sg-open set.*

Definition 2.3. [10] *Let U be the universe, R be an equivalence relation on U and the Rough topology $\tau_R(X) = \{U, \phi, \underline{R}(X), \overline{R}(X), BN_R(X)\}$, where $X \subseteq U$ which satisfies the following axioms:*
(i). U and $\Phi \in \tau_R(X)$.
(ii). The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
(iii). The intersection of the elements of any finite sub collection of $\tau_R(X)$ is in $\tau_R(X)$.
That is, $\tau_R(X)$ is a topology on U called the Nano/Rough Topology on U with respect to X. $(U, \tau_R(X))$ is called the Rough topological space. The elements of $\tau_R(X)$ are known as Rough open sets in U.

Definition 2.4. [2] *A subset A of $(U, \tau_R(X))$ is called Nano semi generalized closed set (Nsg-closed) if $Nscl(A) \subseteq V$ whenever $A \subseteq V$ A and V is Nano semi open in $(U, \tau_R(X))$.*

Definition 2.5. [3] *Let $(U, \tau_R(X))$ and $(V, \tau_{R'}(Y))$ be two Nano topological spaces. Then a map $f : (U, \tau_R(X)) \to (V, \tau_{R'}(Y))$ is Nano sg-continuous on U if the inverse image of every Nano open set in V is Nano sg-open in U.*

Definition 2.6. [4] *Let U be the universe, R be an equivalence relation on U and $\tau_{R_{1,2}}(X) = U\{\tau_{R_1}(X), \tau_{R_2}(X)\}$ where $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ and $X \subseteq U$. Then $(U, \tau_{R_{1,2}}(X))$ is said to be Rough/Nano bitopological space. Elements of the nano bitopology are known as nano (1, 2)* open sets in U. Elements of $[\tau_{R_{1,2}}(X)]^c$ are called nano (1, 2)* closed sets in $\tau_{R_{1,2}}(X)$.*

Definition 2.7. [4] *A subset A of $(U, \tau_{R_{1,2}}(X))$ is called nano (1, 2)* semi-generalized closed set (briefly N (1, 2)*sg-closed) if $N\tau_{1,2}$ $scl(A) \subseteq V$ whenever $A \subseteq V$ and V is nano (1, 2)* semi open in $(U, \tau_{R_{1,2}}(X))$.*

3. Rough Bi-semi generalized Continuity

In this section, the definition of Rough bi-semi generalized continuous maps are introduced and studied certain characterizations of Rough bi-semi generalized continuous maps.

Definition 3.1. *Let $(U, \tau_{R_{bi}}(X))$ and $(V, \tau'_{R_{bi}}(Y))$ be two Rough bitopological spaces. Then a function $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ is Rough bi-continuous (briefly R_{bi}-continuous) on U if the inverse image of every Rough bi-open set in V is Rough bi-open in U.*

Definition 3.2. *Let $(U, \tau_{R_{bi}}(X))$ and $(V, \tau'_{R_{bi}}(Y))$ be two Rough bitopological spaces. Then a function $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ is Rough bi-semi continuous (briefly R_{bi}-s continuous) function on U if the inverse image of every Rough bi-open set in V is Rough bi-semi open in U.*

Definition 3.3. *Let $(U, \tau_{R_{bi}}(X))$ and $(V, \tau'_{R_{bi}}(Y))$ be two Rough bitopological spaces. Then a function $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ is Rough bi-generalized continuous (briefly R_{bi}-g continuous) function on U if the inverse image of every Rough bi-closed set in V is Rough bi-generalized closed in U.*

Definition 3.4. *Let $(U, \tau_{R_{bi}}(X))$ and $(V, \tau'_{R_{bi}}(Y))$ be two Rough bitopological spaces. Then a function $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ is Rough bi-semi generalized continuous (briefly R_{bi}-sg continuous) function on U if the inverse image of every Rough bi-open set in V is Rough bi-semi generalized open in U.*

Example 3.5. Let $U = \{a, b, c, d\}$ with $U/R = \{\{c\}, \{d\}, \{a, b\}\}$. Let $X_1 = \{a, b\} \subseteq U$ and $\tau_{R_1}(X) = \{U, \phi, \{c\}, \{a, b, c\}, \{a, b\}\}, X_2 = \{b, d\} \subseteq U$ and $\tau_{R_1}(X) = \{U, \phi, \{d\}, \{a, b, d\}, \{a, b\}\}$. Then $\tau_{R_{bi}}(X) = \{U, \phi, \{c\}, \{d\}, \{a, b, c\}, \{a, b\}, \{a, b, d\}\}$ which are R_{bi} open sets. R_{bi} -sg open sets are $\{U, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ R_{bi} -sg closed sets are $\{U, \phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}, \{b, c, d\}\}$ Let $V = \{x, y, z, w\}$ with $V/R' = \{\{x\}, \{y, z\}, \{w\}\}$. Let $Y_1 = \{x, z\} \subseteq V$ and $\tau'_{R_1}(Y) = \{V, \phi, \{x\}, \{x, y, z\}, \{y, z\}\}, Y_2 = \{z, w\} \subseteq V$ and $\tau'_{R_2}(Y) = \{V, \phi, \{w\}, \{w, y, z\}, \{y, z\}\}$ then $\tau'_{R_{bi}}(Y) = \{V, \phi, \{x\}, \{w\}, \{y, z\}, \{w, y, z\}, \{x, y, z\}\}$ which are R_{bi}- open sets. Then define $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ as $f(a) = y, f(b) = z, f(c) = w, f(d) = x$. Then $f^{-1}(V) = U$, $f^{-1}(\phi) = \phi$, $f^{-1}\{x\} = \{d\}$, $f^{-1}\{w\} = \{c\}$, $f^{-1}\{y, z\} = \{a, b\}$, $f^{-1}\{w, y, z\} = \{a, b, c\}, f^{-1}\{x, y, z\} = \{a, b, d\}$. Then the inverse image of every R_{bi}- open set in V is R_{bi} sg open in U. Hence $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ is R_{bi} sg continuous.

Theorem 3.6. *A function $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ is R_{bi}- sg continuous if and only if the inverse image of every R_{bi} - closed set in $(V, \tau'_{bi}(Y))$ is R_{bi} - sg* *closed in* $(U, \tau_{R_{bi}}(X))$.

Proof. Let $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ be R_{bi} - sg continuous and F be R_{bi} closed set in $(V, \tau'_{bi}(Y))$. That is $V - F$, is R_{bi} - open set in V. Since f is R_{bi} - sg continuous the inverse image of everyR_{bi}-open set in V is R_{bi}-sg open in U. Hence $f^{-1}(V - F)$ is R_{bi}-sg open in U. That is, $f^{-1}(V) - f^{-1}(F) = U - f^{-1}(F)$ is R_{bi}-sg open in U. Hence $f^{-1}(F)$ is R_{bi}-sg closed in U. Thus the inverse image of every R_{bi}-closed set $(V, \tau'_{bi}(Y))$ in is R_{bi}-sg closed $(U, \tau_{bi}(X))$ in f is R_{bi}-sg continuous.

Conversely, let the inverse image of every R_{bi}-closed set in $(V, \tau'_{R_{bi}}(Y))$ be R_{bi} -sg closed in $(U, \tau_{R_{bi}}(X))$. Let H be a R_{bi}-open set in V. Then $V - H$ is R_{bi}-closed in V and $f^{-1}(V - H)$ is R_{bi}-sg closed in U. That is, $f^{-1}(V) - f^{1}(H) = U - f^{-1}(H)$ is R_{bi}-sg closed in U. Hence $f^{-1}(H)$ is R_{bi}-sg open in U. Thus the inverse image of every R_{bi}-open set in $(V, \tau'_{bi}(Y))$ is R_{bi}-sg -open in $(U, \tau_{bi}(X))$. This implies that $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ is R_{bi}-sg continuous on U.

Theorem 3.7. *If the function* $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ *is* R_{bi}*- continuous, then it is* R_{bi}*-sg continuous but not conversely.*

Proof. Let $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ be R_{bi}-continuous on U. Also, every R_{bi}-closed set is R_{bi}-sg closed but not conversely. Since f is R_{bi}-continuous on $(U, \tau_{R_{bi}}(X))$, the inverse image of every R_{bi}-closed set in $(V, \tau'_{R_{bi}}(Y))$ is R_{bi}-closed in $(U, \tau_{R_{bi}}(X))$. Hence the inverse image of everyR_{bi}-closed set in V is R_{bi}-sg closed in U and so $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ is R_{bi}-sg continuous.
Conversely, every R_{bi}-sg closed sets are not R_{bi}-closed sets and hence R_{bi}-sg continuous function is not R_{bi}-continuous.

Theorem 3.8. *A function* $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ *is* R_{bi}*-sg continuous if and only if* $f(R_{bi}sgcl(A)) \subseteq R_{bi}cl(f(A))$ *for every subset A of* $(U, \tau_{R_{bi}}(X))$.

Proof. Let $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ be R_{bi}-sg continuous and $A \subseteq U$ such that $f(A) \subseteq V$. Hence $R_{bi}cl(f(A))$ is R_{bi}-closed in V. Since f is R_{bi}-sg continuous, $f^{-1}R_{bi}cl(f(A))$ is also R_{bi}-sg closed in $(U, \tau_{R_{bi}}(X))$. Since , $f(A) \subseteq R_{bi}cl(f(A))$ it follows that $A \subseteq f^{-1}(R_{bi}cl(f(A)))$. Thus, $f^{-1}(R_{bi}cl(f(A)))$ is a R_{bi}-sg closed set containing A. But $R_{bi}sgcl(A)$ is the smallest R_{bi}-sg closed set containing A. Hence $R_{bi}sgcl(A) \subseteq f^{-1}(R_{bi}cl(f(A)))$ which implies $f(R_{bi}sgcl(A)) \subseteq R_{bi}cl(f(A))$.
Conversely, let $f(R_{bi}sgcl(A)) \subseteq R_{bi}cl(f(A))$ for every subset A of $(U, \tau_{R_{bi}}(X))$. Let F be a R_{bi}-closed set in $(V, \tau'_{R_{bi}}(Y))$. Now $f^{-1}(F) \subseteq U$ and thus $f(R_{bi}\ sgcl(f^{-1}\ (F))) \subseteq R_{bi}\ cl(f(f^{-1}\ (F))) = R_{bi}\ cl(F)$. It follows that $R_{bi}sgcl(f^{-1}(F)) \subseteq f^{-1}(R_{bi}cl(F)) = f^{-1}(F)$ as F is R_{bi}- closed.
Hence $R_{bi}sgcl(f^{-1}(F))) \subseteq f^{-1}(F) \subseteq R_{bi}sgcl(f^{-1}(F))$. Thus $R_{bi}sgcl(f^{-1}(F)) = f^{-1}(F)$ which implies that $f^{-1}(F)$ is R_{bi}-sg closed in U for every R_{bi}-closed set F in V. Hence, it follows that $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ is R_{bi}-sg continuous.

Theorem 3.9. *If the function $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ is R_{bi}-continuous, then it is R_{bi}-semi continuous.*

Proof. Let the function $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ be R_{bi}-continuous on U. Let A be a R_{bi}- open set in $(V, \tau'_{R_{bi}}(Y))$. By the given hypothesis $f^{-1}(A)$ is R_{bi}-open in $(U, \tau_{R_{bi}}(X))$. As every R_{bi}-open set is R_{bi}-semi-open, $f^{-1}(A)$ is R_{bi}-semi open in $(U, \tau_{R_{bi}}(X))$. Hence the R_{bi}-continuous function is R_{bi}-semi continuous.

Theorem 3.10. *Let $(U, \tau_{R_{bi}}(X))$ and $(V, \tau'_{R_{bi}}(Y))$ be two Rough bitopological spaces where $X \subseteq U$ and $Y \subseteq V$. Then $\tau'_{R_{bi}}(Y) = \{V, \phi, L'_R(Y), U'_R(Y), B'_R(Y)\}$ and its basis is given by $B'_R = \{V, \phi, L'_R(Y), B'_R(Y)\}$. A function $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ is R_{bi}-sg continuous if and only if the inverse image of every member of B'_R is R_{bi}-sg open in U.*

Proof. Let $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ be R_{bi}-sg continuous function on U and let $B \in B'_R$. Then B is R_{bi}-open in V. Since f is R_{bi}-sg continuous, $f^{-1}(B)$ is R_{bi}-sg open in U and $f^{-1}(B) \in \tau_{R_{bi}}(X)$. Hence the inverse image of every member of B'_R is R_{bi}-sg open in U.

Conversely, let the inverse image of every member of B'_R be R_{bi}-sg open in U. Let G be R_{bi}-open set in V. Now $G = \cup\{B : B \in B_1\}$ where $B_1 \subset B'_R$. Then $f^{-1}(G) = f^{-1}\Big[\cup\{B : B \in B_1\}\Big] = \cup\{f^{-1}B : B \in B_1\}$ where each $f^{-1}(B)$ is R_{bi}-sg open in U and their union which is $f^{-1}(G)$ is also R_{bi}-sg open in U. By definition of R_{bi}-sg continuous function, $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ is R_{bi}-sg continuous on U.

Theorem 3.11. *A function $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ is R_{bi}-sg continuous if and only if $R_{bi}sgcl(f^{-1}(B)) \subseteq f^{-1}(R_{bi}cl(B))$ for every subset B of V.*

Proof. Let $B \subseteq V$ and $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ be R_{bi}-sg continuous function. Then $R_{bi}cl(B))$ is R_{bi}-closed in $(V, \tau'_{R_{bi}}(Y))$ and hence $f^{-1}R_{bi}cl(B))$ is R_{bi}-sg closed in $(U, \tau_{R_{bi}}(X))$. Thus $R_{bi}sgcl(f^{-1}R_{bi}cl(B))) = f^{-1}R_{bi}cl(B)$. As $B \subseteq R_{bi}cl(B)$, it follows that $f^{-1}(B) \subseteq f^{-1}R_{bi}cl(B)$ and so $R_{bi}sgcl(f^{-1}(B)) = R_{bi}sgcl(f^{-1}(R_{bi}cl(B))) = f^{-1}(R_{bi}cl(B))$. Hence, $R_{bi}sgcl(f^{-1}(B)) \subseteq f^{-1}(R_{bi}cl(B))$ Conversely, let $R_{bi}sgcl(f^{-1}(B)) \subseteq f^{-1}(R_{bi}cl(B))$ for every subset $B \subseteq V$. Now, let B be aR_{bi}-closed set in $(V, \tau'_{R_{bi}}(Y))$ then $R_{bi}cl(B) = B$. By the given hypothesis, $R_{bi}sgcl(f^{-1}(B)) \subseteq f^{-1}(R_{bi}cl(B))$ and hence $R_{bi}sgcl(f^{-1}(B)) \subseteq f^{-1}(B)$. But $f^{-1}(B) \subseteq R_{bi}sgcl(f^{-1}(B))$ and thus $R_{bi}sgcl(f^{-1}(B)) = f^{-1}(B)$. Thus $f^{-1}(B)$ is R_{bi}-sg closed set in $(U, \tau_{R_{bi}}(X))$ for every R_{bi}-closed set B in $(V, \tau'_{R_{bi}}(Y))$. Hence $f : (U, \tau_{R_{bi}}(X)) \to (V, \tau'_{R_{bi}}(Y))$ is R_{bi}-sg -continuous.

The following theorem establishes a criteria forR_{bi}-sg continuous functions in terms of inverse image of R_{bi}- interior of a subset of $(V, \tau'_{R_{bi}}(Y))$.

Theorem 3.12. *A function $f:(U,\tau_{R_{bi}}(X))\to(V,\tau'_{R_{bi}}(Y))$ is R_{bi}-sg continuous if and only if $f^{-1}(R_{bi}\ Int(B))\subseteq R_{bi}sgInt(f^{-1}(B))$ for every subset B of $(V,\tau'_{R_{bi}}(Y))$*

Proof. Let $f:(U,\tau_{R_{bi}}(X))\to(V,\tau'_{R_{bi}}(Y))$ be a R_{bi} - sg continuous function and let $B\subseteq V$. Then $R_{bi}\ Int(B)$ is R_{bi}-open in V. Now $f^{-1}(R_{bi}\ Int(B))$ is R_{bi}-sg open in $(U,\tau_{R_{bi}}(X))$. It follows that $R_{bi}sgInt(f^{-1}(R_{bi}\ Int(B)))=f^{-1}(R_{bi}\ Int(B))$. Also for $B\subseteq V$, $R_{bi}\ Int(B)\subseteq B$ is true always. Then $f^{-1}(R_{bi}\ Int(B))\subseteq f^{-1}(B)$. It follows that $R_{bi}sgInt(f^{-1}(R_{bi}\ Int(B)))\subseteq R_{bi}sgInt(f^{-1}(B))$ and hence $f^{-1}(R_{bi}\ Int(B))\subseteq R_{bi}sgInt(f^{-1}(B))$.

Conversely, let $f^{-1}(R_{bi}Int(B))\subseteq R_{bi}sgInt(f^{-1}(B))$ for every subset B of V. Let B be R_{bi}-open in V and hence $R_{bi}Int(B)=B$. Given $f^{-1}(R_{bi}Int(B))\subseteq R_{bi}sgInt(f^{-1}B)$. That is, $f^{-1}(B)\subseteq R_{bi}sgInt(f^{-1}(B))$. Also, $R_{bi}sgInt(f^{-1}(B))\subseteq f^{-1}(B)$. Hence $f^{-1}(B)\subseteq R_{bi}sgInt(f^{-1}(B))$ which implies that $f^{-1}(B)$ is R_{bi}-sg open in U for every R_{bi}- open set B of V. Therefore $f:(U,\tau_{R_{bi}}(X))\to(V,\tau'_{R_{bi}}(Y))$ is R_{bi}-sg continuous.

Theorem 3.13. *Let $(U,\tau_{R_{bi}}(X))$ and $(V,\tau'_{R_{bi}}(Y))$ be any two Rough bitopological spaces with respect to $X\subseteq U$ and $Y\subseteq V$ respectively. Then for any function $f:(U,\tau_{R_{bi}}(X))\to(V,\tau'_{R_{bi}}(Y))$ the following are equivalent.*
(i). f is R_{bi}-sg continuous.
(ii). The inverse image of every R_{bi}-closed set in V is R_{bi}-sg closed in $(U,\tau_{R_{bi}}(X))$.
(iii). $f(R_{bi}sgcl(A))\subseteq R_{bi}cl(f(A))$ for every subset A of $(U,\tau_{R_{bi}}(X))$.
(iv). The inverse image of every member of B'_R is R_{bi}-sg open in $(U,\tau_{R_{bi}}(X))$.
(v). $R_{bi}sgcl((f^{-1}(B))\subseteq f^{-1}(R_{bi}cl(B))$ for every subset B of $(V,\tau'_{R_{bi}}(Y)$.

Proof of the Theorem 3.13 is obvious from Theorems 3.6– 3.11.

Remark 3.14. *The family of all R_{bi}-sg open sets in $(U,\tau_{R_{bi}}(X))$ is denoted by $\tau_{R_{bi}}^{R_{bi}\ sg}(X))$.*

Theorem 3.15. *A function $f:(U,\tau_{R_{bi}}(X))\to(V,\tau'_{R_{bi}}(Y))$ is R_{bi} -sg continuous if and only if $f:(U,\tau_{R_{bi}}^{R_{bi}\ sg}(X))\to(V,\tau'_{R_{bi}}(Y))$ is R_{bi}-continuous.*

Proof. Let $f:(U,\tau_{R_{bi}}(X))\to(V,\tau'_{R_{bi}}(Y))$ be a R_{bi} - sg continuous function. By the given hypothesis, $f^{-1}(A)\in\tau_{R_{bi}}^{R_{bi}\ sg}(X))$ for every set $A\in\tau'_{R_{bi}}$. Hence it follows that $f:(U,\tau_{R_{bi}}^{R_{bi}\ sg}(X)))\to(V,\tau'_{R_{bi}}(Y))$ is R_{bi}-continuous.

Conversely, assume that $f:(U,\tau_{R_{bi}}^{R_{bi}\ sg}(X)))\to(V,\tau'_{R_{bi}}(Y))$ the function is R_{bi}-continuous. Then $f^{-1}(G)\in\tau_{R_{bi}}^{R_{bi}\ sg}(X))$ for every set $G\in\tau'_{R_{bi}}(Y)$ and thus the function $f:(U,\tau_{R_{bi}}(X))\to(V,\tau'_{R_{bi}}(Y))$ be a R_{bi} is R_{bi}-sg continuous.

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