South East Asian J. of Mathematics and Mathematical Sciences
Vol. 15, No. 2 (2019), pp. 123-126

ISSN (Print): 0972-7752

ON METRIZABLE SPACES

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(Received: January 29, 2019)

Abstract: In this paper we have discussed the some results on topological metric spaces.

Keywords and Phrases: Topological spaces, basis, metrizable, sequence.

2010 Mathematics Subject Classification: 46A19, 14P25.

1. Introduction

In this paper we proved the equivalence of some metrization theorems, modified single sequence theorem ,modified double sequence theorem , we also defined metric topologies, Before that, however, we want to give a name to those topological spaces whose topologies are metric topologies.

Definition *A topological space (X, T) is said to be metrizable if there is a metric d on X that generates T. Due to the fact that very different looking metrics can generate the same topology, we usually talk about metrizable spaces rather than about metric spaces. The particular details of a metric are often not important to us. We care about the topologies they generate. As a topological property, metrizability is very well-behaved.*

2. Main Results

Theorem 2.1. *If a topological space R*

1.1 *is a T_0 space.*

1.2 *has a neighbourhood basis $\{W_n(p) : n = 1, 2, 3, ...\}$ at each point p of R.*

1.3 $\{q \in W_n(p)\} \Leftrightarrow \{p \in W_n(q)\}$ *for* $p, q \in R$ *and* $n \in N$.

Proof. conditions 1.1 and 1.3 imply that R is T_1-space. We may assume with out loss of generality that

$$W_m(p) \subset W_n(p) \text{ for } m \geq n \quad and\ m(n,p) \geq n \tag{2.1}$$

Let

$$W_n(p) = \{W_n(p) : p \in R\} \tag{2.2}$$

Then each of the $W_1, W_2 ... W_n$ is a cover of R.
Set

$$T_n(p) = S(p, W_n : p \in W_n) = S(p, W_n) \tag{2.3}$$

and

$$V_n(p) = W_{m(n,p)}(p) \tag{2.4}$$

We prove that $T_n(p)$ is a neighbourhood basis.
Let us consider $U(p)$ be a neighbourhood of p.
Since $\{W_n(p) : n = 1, 2, 3 ...\}$ is a neighbourhood basis, there exists $n_1 \in N$.
Such that

$$W_{n_1}(p) \subset U(p) \tag{2.5}$$

Than from (2.1) there exists $m_1 = m_1(n_1, p)$.
Such that

$$\{q \in W_{m(n_1,p)}(p)\} \Rightarrow W_{m_1}(n_1, p)(q) \subset W_{n_1}(p). \tag{2.6}$$

According to the results (2.1),(2.2),(2.3),(2.4),(2.5),(2.6),the results is proved.

Theorem 2.2. *If a topological space* R.

1.1 *is a* T_0 *space.*

1.2 *has a neighbourhood basis* $\{U_n(p) : n = 1, 2, 3, ...\}$ *and a sequence neighbourhood of* $\{H_n(p), n \in N\}$ *at each of* $p \in R$.

1.3 $\{q \in H_n(p)\} \Leftrightarrow \{p \in H_n(q)\}$.

1.4 $\{q \in H_n(p)\} \Leftrightarrow H_n(q) \subset U_n(p)$ *then* R *is metrizable.*

Proof. Using the results of theorem 2.1 which implies the conditions of (1.1) ,(1,2),(1.3),(1.4).
Thus $T_n(p)$ is a neighbourhood and $V_n(p)$ is a sequence of neighbourhood. Now

from (2.1) given $n \in N$ and $p, q \in R$ then there exists $m(n,p)$ and $m(n,p)$ such that $V_n(p) = W_{m(n,p)}(p).V_n(p) = W_{m(n,p)}(q)$ and

$$\{p \in W_{m(n,p)}(p)\} \Rightarrow W_{m(n,p)}(q) \subset T_n(q). \tag{2.7}$$

and $\{q \in W_{m(n,p)}(p)\} \Rightarrow W_{m(n,p)}(q) \subset T_n(p)$.

$$\{q \in W_{m(n,p)}(p)\} \Rightarrow W_{m(n,p)}(q) \subset T_n(p). \tag{2.8}$$

Case 1. Let

$$m(n,p) \leq m(n,p) \tag{2.9}$$

Then

$$V_n(p) = W_{m(n,p)}(q) \subset W_{m(n,p)}(q) \subset W_n(p) \subset T_n(p) \tag{2.10}$$

Hence from (2.8)(2.9)(2.10).

$$\{q \in V_n(p)\} \Rightarrow V_n(q) \subset T_n(p) \tag{2.11}$$

Case 2. Let

$$m(n,p) \geq m(n,p) \tag{2.12}$$

Then

$$W_{m(n,p)}(p) \subset W_{m(n,p)}(q) \tag{2.13}$$

and

$$W_{m(n,p)}(q) \subset W_{m(n,p)}(q) \tag{2.14}$$

If

$$q \in V_n(p) = W_{m(n,p)}(p) \subset W_n(p).$$

where

$$m(n,p) > n \tag{2.15}$$

Then which follows the required the results of theorem 2.

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