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ON THE COMBINATORICS OF POLYNOMIAL GENERALIZATIONS OF ROGERS-RAMANUJAN TYPE IDENTITIES

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Dedicated to Professor G. E. Andrews on his seventieth birthday

Abstract: In this paper following some ideas introduced by Andrews (Combinatorics and Ramanujan's "lost" notebook, London Mathematical Society Lecture Note Series, No. 103, Cambridge University Press, London, 1985, pp. 1-23) and results given by Santos (Computer algebra and identities of the Rogers-Ramanujan type. Ph.D. Thesis, Pennsylvania State University, 1991) we give a polynomial generalization for the Fibonacci sequence from which we get new formula and combinatorial interpretation for the Fibonacci Numbers.

Keywords and Phrases: Rogers-Ramanujan type identities, Fibonacci numbers, partitions
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1. Introduction

In [8] Lucy Slater presented a list of 130 q-series identities including the 3 listed below that are the ones of numbers 18, 14 and 20 respectively with the first two being the famous Rogers Ramanujan identities.

$$\sum_{n=0}^{\infty}\frac{q^{n^2}}{(q;q)_n}=\prod_{n=1}^{\infty}\frac{1}{(1-q^{5n-1})(1-q^{5n-4})} \tag{1.1}$$

$$\sum_{n=0}^{\infty}\frac{q^{n^2+n}}{(q;q)_n}=\prod_{n=1}^{\infty}\frac{1}{(1-q^{5n-2})(1-q^{5n-3})} \tag{1.2}$$

$$\sum_{n=0}^{\infty}\frac{q^{n^2}}{(q^4;q^4)_n}=\frac{(-q;q^2)_\infty}{(q^2;q^2)_\infty}\prod_{n=1}^{\infty}(1-q^{5n-2})(1-q^{5n-3})(1-q^{5n}) \tag{1.3}$$

where

$$(a;q)_n=(1-a)(1-aq)\dots(1-aq^{n-1}),$$