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ON MODULAR RELATIONS FOR THE FUNCTIONS ANALOGOUS TO ROGER'S-RAMANUJAN FUNCTIONS WITH APPLICATIONS TO PARTITIONS

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Dedicated to Professor G. E. Andrews, on the occasion of his Seventieth birthday

Abstract: In this paper, we establish modular relations involving the functions, $S(q) := \sum_{n=0}^{\infty} \frac{q^{n^2+n}}{(q^2;q^2)_n}$ and $T(q) := \sum_{n=0}^{\infty} \frac{q^{n^2}}{(q^2;q^2)_n}$, which are analogous to Ramanujan's modular identities. Furthermore, we extract some partition results from them.

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1. Introduction

The famous Roger's-Ramanujan functions $G(q)$ and $H(q)$ are

$$G(q) := \sum_{n=0}^{\infty} \frac{q^{n^2}}{(q;q)_n} = \frac{1}{(q;q^5)_\infty (q^4;q^5)_\infty} \tag{1.1}$$

and

$$H(q) := \sum_{n=0}^{\infty} \frac{q^{n(n+1)}}{(q;q)_n} = \frac{1}{(q^2;q^5)_\infty (q^3;q^5)_\infty}, \tag{1.2}$$

where, as customary

$$(a;q)_n = \prod_{k=0}^{n-1} (1 - aq^k),$$