South East Asian J. Math. & Math. Sc.
Vol.6 No.2(2008), pp.23–36

n-COLOR OVERPARTITIONS, TWISTED DIVISOR FUNCTIONS, AND ROGERS-RAMANUJAN IDENTITIES

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(*Received: August 01, 2007*)

Dedicated to Professor G.E. Andrews on his seventieth birthday

Abstract: In the early 90's Andrews discussed a certain q-series whose coefficients are determined by a twisted divisor function. We provide several other examples of this nature. All of these q-series can be interpreted combinatorially in terms of n-color overpartitions, as can some closely related series occurring in identities of the Rogers-Ramanujan type.

Keywords and Phrases: Partitions, overpartitions, n-color overpartitions, real quadratic fields, Bailey pairs, Rogers-Ramanujan identities
2000 AMS Subject Classification: 11P81, 05A17, 33D15

1. Introduction

In 1988 Andrews, Dyson, and Hickerson [16] made an extensive study of a q-series that first came to light in Ramanujan's lost notebook [12,13],

$$\sum_{n\geq 0} \frac{q^{n(n+1)/2}}{(1+q)(1+q^2)\cdots(1+q^n)} = 1+q-q^2+2q^3-2q^4+q^5+q^7-2q^8+2q^{10}+\cdots . \tag{1.1}$$

If $r(n)$ denotes the coefficient of q^n in this series, then $r(n)$ has a rather simple combinatorial interpretation - as the number of partitions of n into distinct parts with even rank minus the number with odd rank. Recall that the rank of a partition is the largest part minus the number of parts. On the other hand, Andrews, Dyson, and Hickerson showed that $r(n)$ is almost always 0 and assumes every integer infinitely often, facts which may be deduced from a multiplicative "exact" formula relating $r(24n+1)$ to the arithmetic of $\mathbb{Z}[\sqrt{6}]$.

*The first author was partially supported by an ACI "Jeunes Chercheurs et Jeunes Chercheuses"