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ON A LINEAR FORM FOR CATALAN'S CONSTANT

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Dedicated to George Andrews

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Abstract: It is shown how Andrews' multidimensional extension of Watson's transformation between a very-well-poised ${}_8\phi_7$-series and a balanced ${}_4\phi_3$-series can be used to give a straightforward proof of a conjecture of Zudilin and the second author on the arithmetic behaviour of the coefficients of certain linear forms of 1 and Catalan's constant. This proof is considerably simpler and more stream-lined than the first proof, due to the second author.

Keywords and Phrases: Catalan's constant, linear forms, hypergeometric series, Andrews' identity
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1. Introduction

Andrews' multidimensional extension [1, Theorem 4] of Watson's transformation between a very-well-poised ${}_8\phi_7$-series and a balanced ${}_4\phi_3$-series [6, (2.5.1); Appendix (III.18)] in its full beauty reads

$$\sum_{k=0}^{n} \frac{(a;q)_k\,(q\sqrt{a};q)_k\,(-q\sqrt{a};q)_k\,(b_1;q)_k\,(c_1;q)_k\cdots(b_{m+1};q)_k\,(c_{m+1};q)_k\,(q^{-n};q)_k}{(\sqrt{a};q)_k\,(-\sqrt{a};q)_k\,(qa/b_1;q)_k\,(qa/c_1;q)_k\cdots(qa/b_{m+1};q)_k\,(qa/c_{m+1};q)_k\,(q^{n+1}a;q)_k}$$

$$\times\left(\frac{a^{m+1}q^{m+1+n}}{b_1c_1\cdots b_{m+1}c_{m+1}}\right)^k$$

$$= \frac{(qa;q)_n\,(qa/b_{m+1}c_{m+1};q)_n}{(qa/b_{m+1};q)_n\,(qa/c_{m+1};q)_n} \sum_{0\le i_1\le i_2\le\cdots\le i_m\le n} \frac{a^{i_1+\cdots+i_{m-1}}q^{i_1+\cdots+i_m}}{(b_2c_2)^{i_1}\cdots(b_mc_m)^{i_{m-1}}}$$