

## A STUDY OF AN UNDIRECTED GRAPH ON A FINITE SUBSET OF NATURAL NUMBERS

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**Abstract:** Let  $G_n = (V, E)$  be an undirected simple graph, whose vertex set comprises of the natural numbers which are less than  $n$  but not relatively prime to  $n$  and two distinct vertices  $u, v \in V$  are adjacent if and only if  $\gcd(u, v) > 1$ . Connectedness, completeness, minimum degree, maximum degree, independence number, domination number and Eulerian property of the graph  $G_n$  are studied in this paper.

**Keywords and Phrases:** Clique, connected graph, complete graph, prime counting function.

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### 1. Introduction and Preliminaries

Let  $G = (V, E)$  be a simple graph where  $V$  is the set of vertices and  $E$  is the set of edges. For any vertex  $u \in V$ , the *degree* of a vertex  $u$  denoted by  $\deg(u)$  is the number of edges incident on  $u$ . The maximum (minimum) degree of  $G$  is denoted by  $\Delta(G)$  ( $\delta(G)$ ). For graph theory terminology and notation in general we follow [8]. Let  $n \in \mathbb{N}$  be a composite number. It would be interesting to know the structural properties of the subset of the natural numbers which are less than  $n$  but not relatively prime to  $n$ . Thus proceeding in that direction, in this paper we define an undirected simple graph  $G_n = (V, E)$ , whose vertex