

## SUFFICIENT CONDITIONS FOR CERTAIN SUBCLASSES OF ANALYTIC FUNCTIONS

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**Abstract:** In this paper, sufficient conditions for normalized analytic functions defined on unit disk to be in the subclasses of close-to-convex, close-to-star and quasi-convex functions are obtained.

**Keywords and Phrases:** Normalized, analytic, close-to-convex, close-to-star, quasi-convex functions.

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### 1. Introduction and Definitions

Let  $\mathcal{A}$  denote the class of all analytic functions  $f : \Delta \rightarrow \mathbb{C}$ , where  $\Delta = \{z \in \mathbb{C} : |z| < 1\}$  which are normalized by  $f(0) = 0$  and  $f'(0) = 1$ . Then

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n. \quad (1)$$

Denote by  $\mathcal{S}$  the subclass of  $\mathcal{A}$  containing univalent functions. The well known Bieberbach conjecture [3] says that for functions  $f \in \mathcal{S}$  of the form (1),  $|a_n| \leq n$ ,  $\forall n \geq 2$ . This was settled positively by Louis de Branges [2] and henceforth is known as “de Branges Theorem.” However, in attempting to prove this result, researchers had defined various subclasses of  $\mathcal{S}$  and had verified the same. Some of the standard subclasses of  $\mathcal{S}$  introduced and studied for this purpose were subclasses